LEARNING MATERIALS

SEMESTER : 3RD SEMESTER

BRANCH : MECHANICAL ENGINEERING

SUBJECT : STRENGTH OF MATERIAL (TH-2)

FACULTIES : (1) ER. SUBODHAKANTA GARNAIK (LECT. IN MECH. ENGG.)
(2) ER. RASABIHARI SAHU (LECT. IN MECH. ENGG.)

PURNA CHANDRA INSTITUTE OF ENGINEERING & TECHNOLOGY
AT/P.O.-CHHENDIPADA, DIST.-ANGUL.

Axial Loading :- $\xrightarrow{\text{Analogus}}$ Torsion :-

→ ⊥ to C.S. area, uniform.

→ $\sigma = \frac{P}{A}$

→ $\sigma = E \times \epsilon$ / E - Mod of elasticity

→ $\frac{PL}{AE} = \delta L$

→ $\delta L = \int \frac{P dx}{AE}$ i.e. $\int \frac{\sigma}{E} dx$

For statically indeterminate

→ $(\delta L)_{overall} = 0$ or

Comp. eqns.

→ Strength, $\hat{P}_{series} = \text{Min}(\hat{P}_1, \hat{P}_2, \ldots)$

→ $\hat{\sigma}_{parallel} = \hat{P} = \hat{P}_1 + \hat{P}_2 + \hat{P}_3$

→ Thermal Stress & Strain $\rightarrow 0$

$\delta L_T = \alpha L \Delta T$

→ Acting about the longitudinal axis.

→ $\frac{T}{J} = \frac{\tau}{r}$; solid $\tau = \frac{16T}{\pi d^3}$; $\tau = \frac{16T}{\pi d_o^3 (1-\beta^4)}$ $\beta = \frac{d_i}{d_o}$

→ $\tau = G \times \gamma$; G - Rigidity Modulus

→ $\theta = \frac{TL}{GJ}$; $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

→ $\delta\theta = \int \frac{T dx}{GJ}$; Non uniform dist, Linear

For statically indeterminate

→ $(\theta_{overall}) = 0$ & $\theta_1 = \theta_2$

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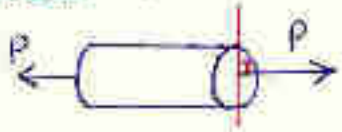
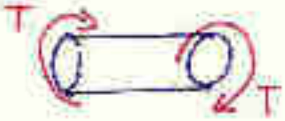
(MOS) (SOM) (MOM):-

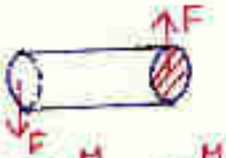
A load Constitute two forces always equal and opposite. A single force is not a load at all



⊥ to the C.S area (Causes elongation/compression)

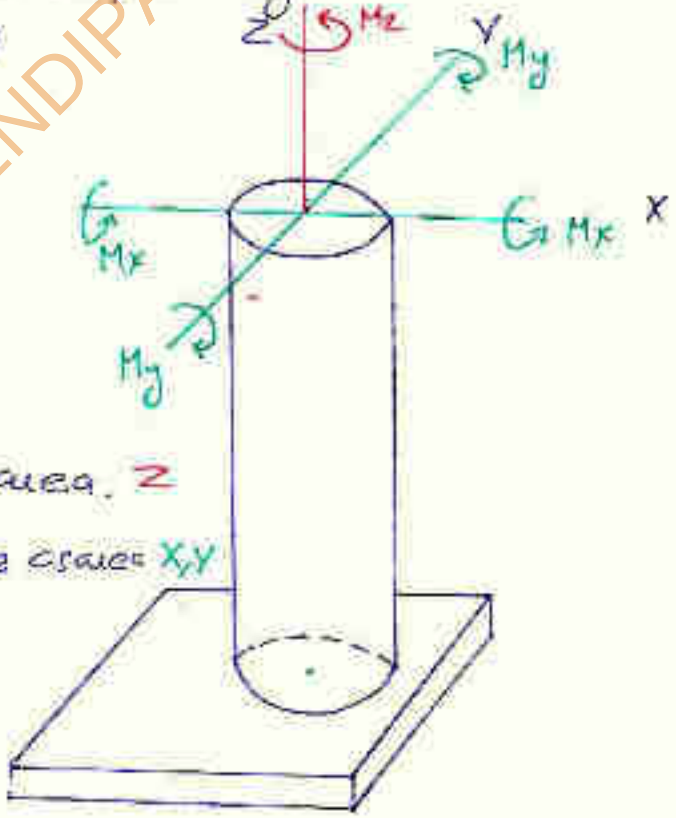


Equal and opp. along the longitudinal axis (Couple) (Causes twisting)



Equal and opposite parallel to the C.S.A (Causes shearing)

Equal and opposite couple along the transverse direction. (Causes bending)



Mz – twisting moment

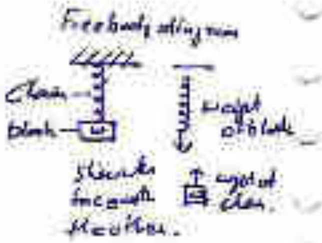
Mx, My – Bending moment

Longitudinal axis – axis ⊥ C.S. area. Z

Transverse axis – axis along the c.s.area X,Y

→Axial Loading:-

Types of Equations:-

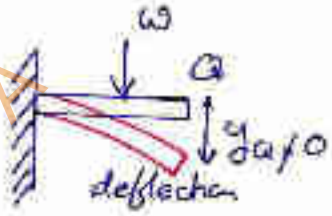


i) $\sum F_x = 0$

$\sum F_y = 0$ } Equilibrium equations

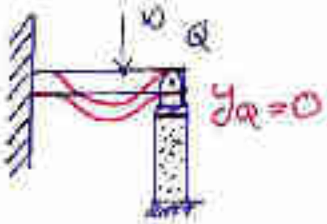
$\sum M = 0$

ii) Compatibity equations- Mathematical equation that obtained by imposing certain contraints over deformations are known as Compatibity equations.



III) Stress- Strain Relation

Hooke's law

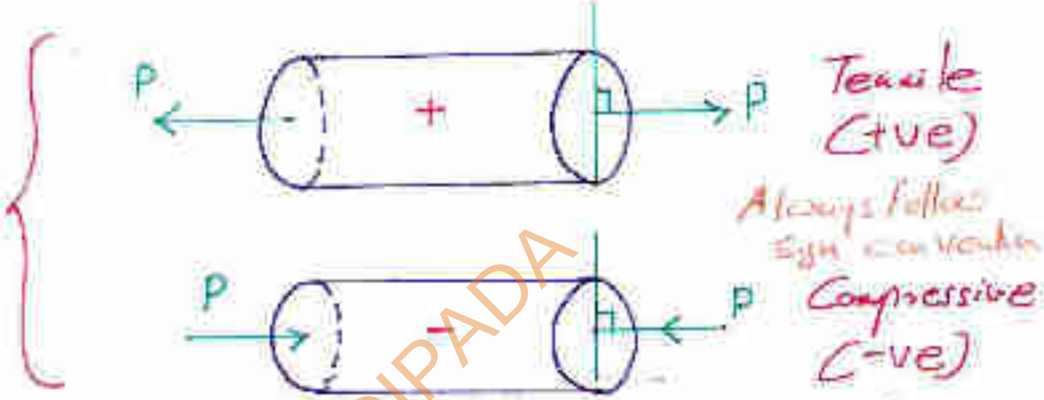


1 Axial Loading:-

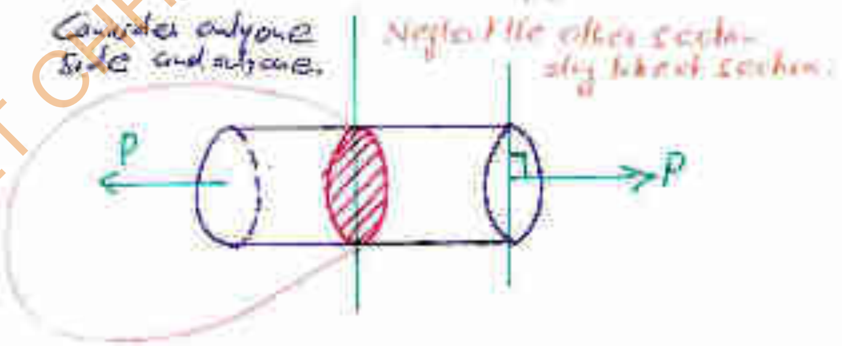
1.1 Load-

Axial loading- Equal and opposite acting perpendicular to the cross sectional area of a member constitute its axial loading.

2 types



Load at a section*-

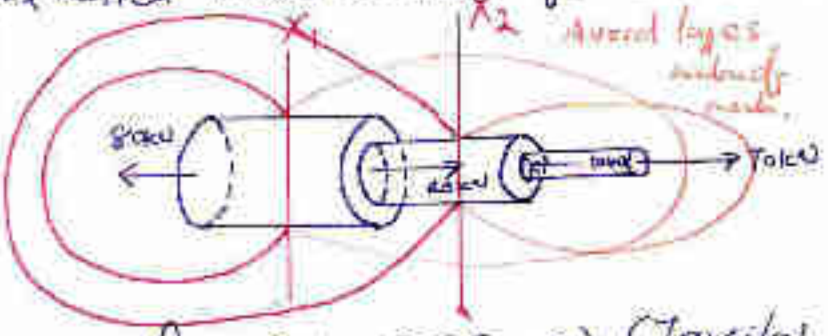


Axial load At a section is the algebraic sum of all the longitudinal forces transmitted to that section from either side

$P_{X_2} = 80-20 = 60 kN$ (left)

$P_{X_2} = 70-10 = 60 kN$ (right)

Both are pull.



$P_{X_1} =$ 80kN ($\Sigma F_x = 0$) (Tensile)

Or $P_{X_1} = 70-10+20 =$ 80kN (following convention)

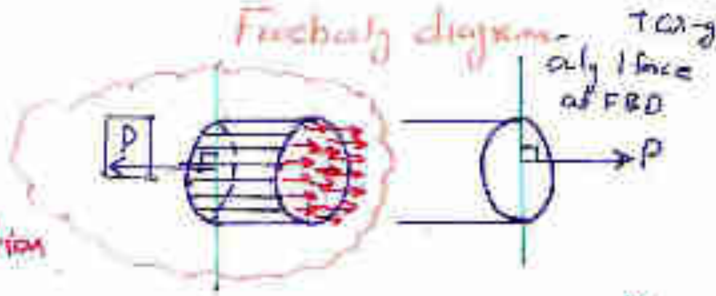
* Load at a section = Σ (Algebraic sum of longitudinal from Either Side)

1.2. Stress & Strain

Molecular have [illegible] & in one particular [illegible] T(λ-gap).

a) Stress

- Nature = Normal (σ)
- Uniform = Distribution
- Magnitude = P/A



Axial loading has the normal stress "Normal" means perpendicular to the cross-sectional area.

Distribution is "Uniform."

At Equilibrium,

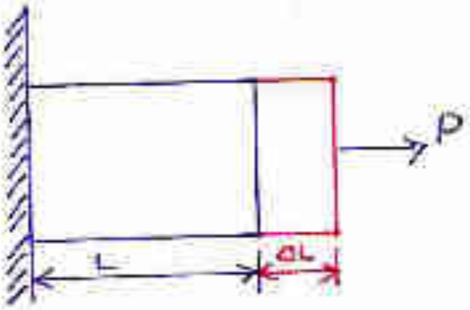
External force = (Total Internal force)
↓
Spread across the full cross sectional area.

So consider the unit area, such that internal force will be σ (per unit area)

For total area $\sigma \times A$

$$\therefore P = \sigma \times A \longrightarrow \text{Total Internal force}$$

b) Strain:- Axial loading causes some linear deformation known by the name "elongation" (ΔL)



Elongation caused in unit length of member called "strain." Other names are

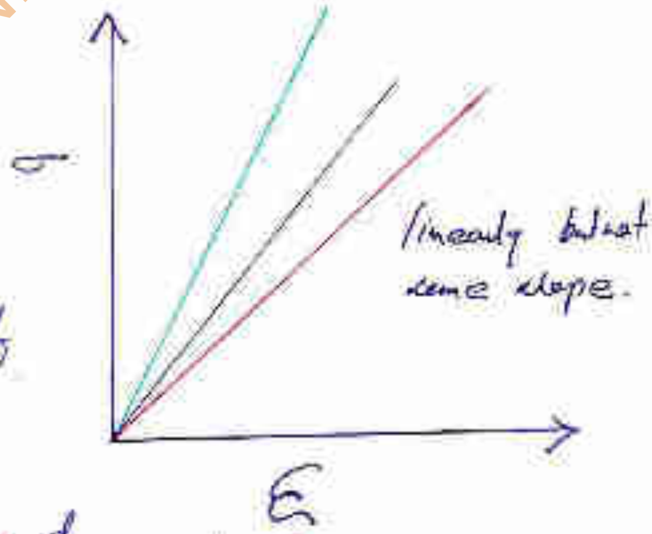
→ Linear strain.
→ Longitudinal strain.
→ Axial strain
→ Direct strain
→ Normal strain
→ Engineering strain.

$$\varepsilon = \frac{\Delta L}{L}$$

1.3) Hooke's Law:-

$y = mx$ (at origin)

$\sigma = E.\varepsilon$



linearly but not same slope.

"E" represents the slope of σ - ε curve &

"E" represents the resistance of material to linear deformation.

$\sigma\downarrow = \frac{\varepsilon}{E\uparrow}$

$\sigma = E \times \varepsilon$

1.4. Elongation due to Axial Loading:-

At Equilibrium; $\sigma = \frac{P}{A}$

At Hookes law; $\sigma = E \cdot \varepsilon$

$$\left[\sigma = \frac{P}{A} = E \cdot \frac{\Delta L}{L}\right] \rightarrow \text{Axial loading Equation.}$$

$$\delta L = \frac{P \times L}{E \cdot A} \longrightarrow \text{Elongation when uniform.}$$

Based on principle of Superposition

i) $\delta L = \frac{PL}{EA}$ (uniform)

ii) $\delta L = \int \frac{Px}{AE}\, dx$ (variable) [P vary]

iii) $\delta L = \delta L_1 + \delta L_2 + \delta L_3 + \ldots.$ (stepped)

$$\delta L = \frac{P}{E \times (A/L)}$$

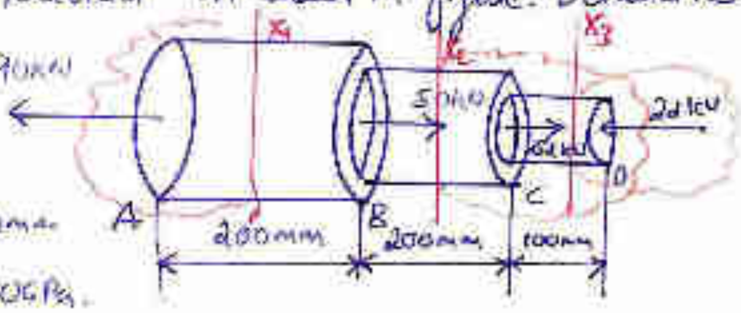
$GPa = 10^9\, Pa = 10^3\, MPa$

Q For the stepped member loaded as shown in figure. Determine the

i) Maximum stress

ii) Total elongation.

~~iii)~~ $d_1 = 50mm$, $d_2 = 40mm$, $d_3 = 30mm$; $E = 200GPa$.



Remarks

i) B/w A&B Load can be calculated for section wise segment.

$P = 90 KN$ (Tensile)

$$\sigma_{x_1} = \frac{P_{x_1}}{A_{x_1}} = \frac{40 \times 10^3}{\pi \times 50^2/4} = 45.83\, N/mm^2 \text{ or } 45.83\, MPa.$$

$$\sigma_{x_2} = \frac{P_{x_2}}{A_{x_2}} = \frac{62 - 22 \times 10^3}{\pi \times 40^2/4} = 31.83\, MPa.$$

$$\sigma_{x_3} = \frac{P_{x_3}}{A_{x_3}} = \frac{-22 \times 10^3}{\pi \times 30^2/4} = -31.12\, MPa$$

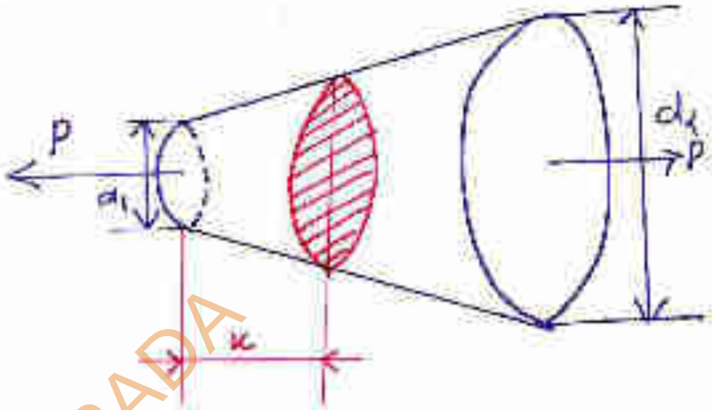
$\delta L = \delta L_1 + \delta L_2 + \delta L_3 \qquad \delta L = \frac{PL}{AE} = \frac{\sigma . L}{E}$

$\delta L = \frac{45.83 \times 200}{E} + \frac{31.83 \times 200}{E} + \frac{-31.12 \times 100}{E}$

$\delta L = \left[45.83 \times 200 + 31.83 \times 200 - 31.12 \times 100\right] / 200 \times 10^3 \text{ MPa.}$

$\delta L = 0.0621 \text{ mm}$

→ $\delta L = \int \frac{P dx}{A.E} = \frac{P}{E} \int \frac{dx}{A}$



$A = \frac{\pi d^2}{4}$

$\delta L = \frac{P}{\cancel{E}} \int \frac{dx}{\pi d^2/4}$

$\delta L = \frac{4P}{E.\pi} \int \frac{dx}{d^2}$

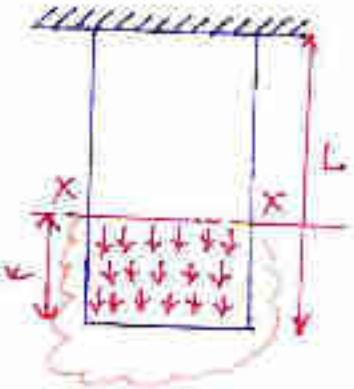
$d = d_1 + mx \qquad dm = \frac{d_2 - d_1}{L}$

$\delta L = \frac{4P}{E.\pi} \int_0^L \frac{dx}{(d_1 + mx)^2}$

$\delta L = \frac{4.P\,L}{E.\pi d_1 d_2}$

→ $\sigma = ?$; $\delta L = ?$

$\sigma_x = \frac{P}{A} = \frac{(\rho \cancel{A} x) g}{\cancel{A}} = \rho x g$



$\sigma_x = \rho x g$: at $x = L$; $\sigma_x = \rho . g . L$

$\sigma_x = \rho . g . x$ Vary with length but the distribution is uniform

$\delta L = \int \frac{P dx}{AE} = \int \frac{\sigma dx}{E}$

$\delta L = \int \frac{\rho g x dx}{E} = \frac{1}{E} \int_0^L \rho g x dx$

$$\delta L = \frac{\rho g}{E}\left[\frac{x^2}{2}\right]_0^L$$

$$\delta L = \frac{\rho g L^2}{2E}$$

$$\boxed{\delta L = \frac{\rho g . L^2}{2E}}$$

$$\delta L = \frac{\rho g L}{2E} \times \frac{A \times L}{A} = \frac{m g \times L}{2AE}$$

$$\boxed{\delta L = \frac{W . L}{2A . E}}$$

→

~~$\sigma_x = \frac{P}{A}$~~

~~$P = \frac{W \times x}{g}$~~

~~$\sigma_x = \frac{W \times x}{gA}$~~

~~$\delta L = \int \frac{\sigma}{E} dx$~~

~~$\delta L = \frac{1}{E} \int \frac{W x}{gA} dx$~~

~~$\delta L = \frac{W}{gAE} \int x dx$~~

~~$\delta L = \frac{W . L^2}{2gAE}$~~

~~$P = m \times \omega^2$~~

~~$P = m x \omega^2$~~

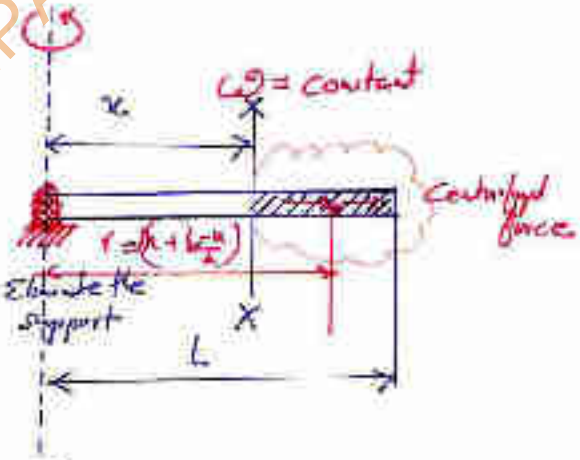
~~$P = \rho A x \omega^2$~~

~~$P = \rho A x \omega^2$~~

~~$\sigma = P/A$~~

~~$\sigma = \frac{\rho x \omega^2 A}{A}$~~

~~$\sigma = \rho x \omega^2$~~



$$\sigma_x = \frac{P_x}{A_x}$$

~~$P_x = m \times (L-x) \times \omega^2$~~

$$P_x = \rho \times (L-x) \times A \times (L-x) \times \omega^2$$

$$\sigma_x = \frac{P_x}{A_x}$$

$$\sigma_x = \frac{\rho A \times (L-x) \times (x + \frac{L-x}{2}) \times \omega^2}{A}$$

$$\sigma_x = \rho \times (L-x) * \left(x\omega^2 + \frac{(L-x)\omega^2}{2}\right)$$

$$\sigma_x = \rho (L-x) \times \left(x\omega^2 + \frac{(L-x)\omega^2}{2}\right)$$

$$\sigma_x = \rho L - \rho x \times \left(x\omega^2 + \omega^2 L - \frac{x\omega^2}{2}\right)$$

$$\sigma_x = \rho L - \rho x \times \left(\omega^2 L x + \frac{x\omega^2}{2}\right)$$

$\sigma_x = \frac{\rho\omega^2}{2}[L^2-x^2]$

$$\boxed{\sigma_x = \frac{\rho\omega^2}{2}[L^2-x^2]}$$

$\delta L = \int_0^L \frac{P\,dx}{AE}$

$\delta L = \int_0^L \frac{\sigma}{E}\,dx$

$\delta L = \frac{1}{E}\int_0^L \frac{\rho\omega^2}{2}[L^2-x^2]\,dx.$

$\delta L = \frac{1}{E}\times\frac{\rho\omega^2}{2}\int_0 {}^2-x^2\,dx$

$\delta L = \frac{\rho\omega^2}{2E}\int_0^L L^2-x^2\,dx.$

$\delta L = \frac{\rho\omega^2}{2E}\left(L^2-\frac{x^2}{3}\right)x\Big]_0^L$

$\delta L = \frac{\rho\omega^2}{2E}\left(\left(L^2-\frac{L^2}{3}\right)L\right)$

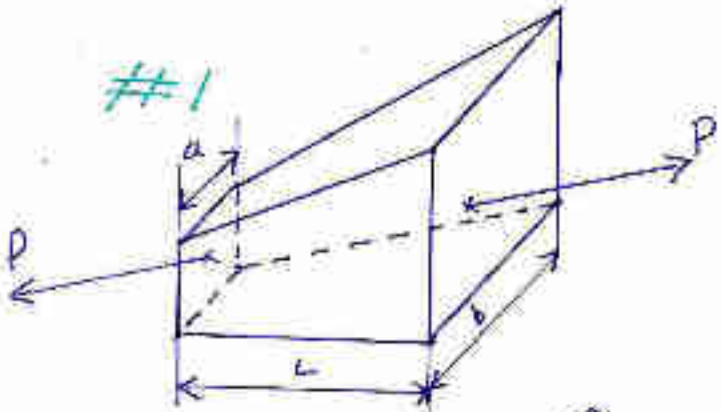
$\delta L = \frac{\rho\omega^2L^3}{3E}$

$$\boxed{\delta L = \frac{\rho\times\omega^2\times L^3}{3E}}$$ Take only one side

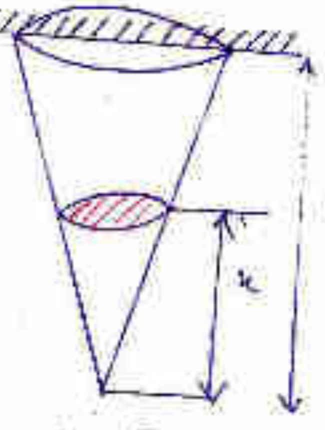
→ $\delta L = ?$

$\delta L = \int \frac{P\,dx}{AE}$

#1

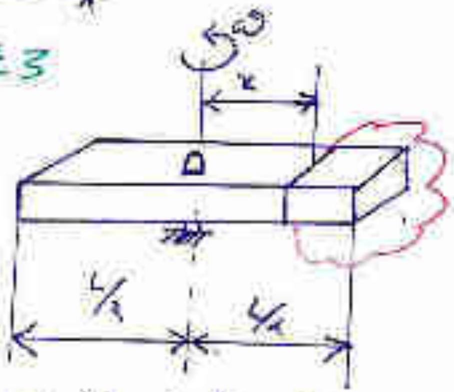


#2



$\sigma_x = ?$ $\delta L = ?$ (in ρ, g, L, E)

#3



$\sigma_x = ?$; $\delta L = ?$

1.5. STATICALLY INDETERMINATE PROBLEMS:-

* More supports than required
↓
2 types of support cause use of compatibility Equation.

* Degree of indeterminacy = Max. No. of Unknowns - Max. No. Eqn. equations

$R_1 + R_2 = P$ (Equilibrium)

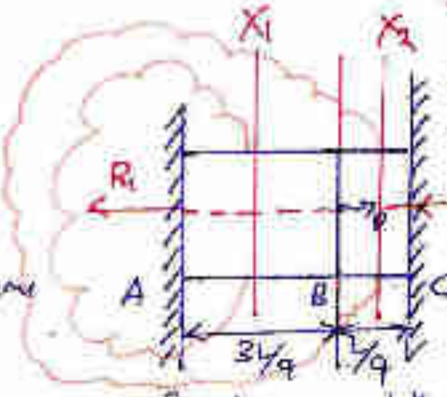
2 unknown one equation,

No moments $\Sigma F_x = 0$ (only)

So statically indeterminate problem

Compatibility come to play

overall chge not allowed.



Reaction comes to the play
Reaction must be known

$$\delta L_{overall} = 0.$$

$$\delta L_{AB} + \delta L_{BC} = 0$$

2 type Eqns used in any Statically indeterminate problem

→ Equilibrium Equation.
→ Compatibility Equation.

Load on $AB = R_1$; $L_{AB} = 3L/4$

Load on $BC = R_1 - P$; $L_{BC} = L/4$ $[R_1 - P = -R_2]$

$$\delta L_{AB} + \delta L_{BC} = 0$$

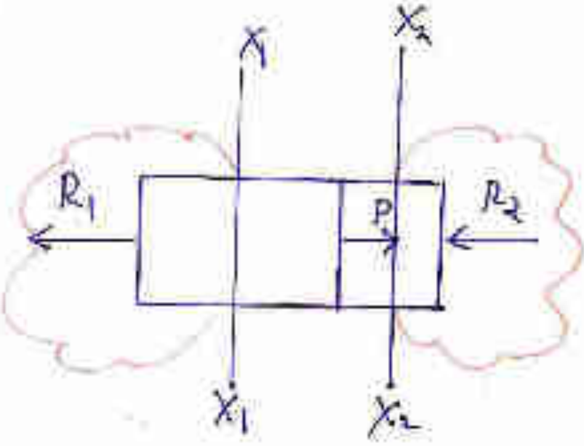
$$\frac{R_1 \times \frac{3L}{4}}{EA} + \frac{(R_1 - P) \times \frac{L}{4}}{EA} = 0$$

$$R_1 \times \frac{3\cancel{L}}{4} + \frac{R_1 \times \cancel{L}}{4} - \frac{P\cancel{L}}{4} = 0$$

$$3 \times \frac{R_1}{4} + \frac{R_1}{4} - \frac{P}{4} = 0$$

$$P = (2+2) R_1$$

$$\underline{\underline{R_1 = P/4}}$$



$$R_1 = P - R_1 = 3P/4$$

$$\sigma_{x_1} = \frac{R_1}{A} = \frac{P/4}{A} = P/4A \text{ (tensile)}$$

$$\sigma_{x_2} = \frac{R_2}{A} = \frac{-3P/4}{A} = -3P/4A \text{ (Compression)}$$

#1) $\delta L = \int \frac{P dx}{AE}$

$\delta L = \frac{P}{E}\int\frac{dAx}{A}$

Aischaying thylout so that
area at the left section = a^2
area at the right section = b^2.
Let the area at the section x_1 be z^2
here $z = a + mx$.

$m = \frac{b-a}{L}$

$\therefore z^2 = a + mx$

$$\delta L = \frac{P}{E}\int_0^L \frac{dx}{(a+mx)^2}$$

$$\delta L = \frac{P}{E}\left[\frac{-1}{(a+mx)m}\right]_0^L$$

$$\delta L = \frac{P}{E}\left[\frac{-1}{m(a+mL)} + \frac{1}{m(a+0)}\right]$$

$$\delta L = \frac{P}{E}\left[\frac{-\cancel{ma}+mL+\cancel{ma}}{\cancel{m}a(a+mL)}\right]$$

$$\delta L = \frac{P}{E}\left[\frac{L}{a(a+\frac{b-a}{L}L)}\right]$$

$$\delta L = \frac{P}{E}\left[\frac{L}{a(a+b-a)}\right]$$

$$\delta L = \frac{PL}{E.a.b}$$

#2) Here consider the section at x for the lowermost point the

Volume = $\frac{1}{3}\pi\times r^2\times h$

$P = W = mg = \rho\times V\times g$

$P = \frac{\rho}{3}\times\pi r^2\times h\times g$.

$$\sigma_x = \frac{P_x}{A_x} = \frac{\rho}{3}\times\frac{\cancel{\pi r^2}\times h\times g}{\cancel{\pi r^2}} = \rho/3\times h\times g$$

here $h = x$ $\therefore \sigma_x = \rho/3\times x\times g$

#2. So $\delta L = \frac{P\,dx}{AE}$

$$\delta L = \frac{\sigma\,dx}{E}$$

$$\delta L = \frac{1}{E}\int_0^L \sigma\,dx.$$

$$\delta L = \frac{1}{E}\int_0^L \frac{\rho}{3}\,xg\,dx.$$

$$\delta L = \frac{1}{E}\times\frac{\rho}{3}\times g\int_0^L x\,dx.$$

$$\delta L = \frac{1}{E}\times\frac{\rho}{3}\times g\,\frac{L^2}{2}.$$

$$\underline{\underline{\delta L = \frac{\rho}{6E}\times g\times L^2}}$$

We $\delta L = \frac{\rho}{6E}\times g\times L^2$

M/y are dividing by $\frac{1}{3}\times\pi r^2 h$.

$$\therefore\ \delta L = \frac{\rho}{6E}\times g\times L^2\times\frac{\frac{1}{3}\pi\times r^2 L}{\frac{1}{3}\pi r^2\times L}$$

$$\delta L = \frac{W\times L^{\cancel{2}}}{\cancel{6}\times\frac{1}{\cancel{3}}\pi r^2\times\cancel{L}\times E}$$

$$\delta L = \frac{W\times L}{2\pi r^2\times E}$$

$$\delta L = \frac{W\times L}{\cancel{2}\pi\times\frac{d^2}{\cancel{4}2}\times E}$$

$$\underline{\underline{\delta L = \frac{2WL}{\pi d^2\times E}}}$$

#3. $P = m\times r\times\omega^2$

$$P = \rho\times A\times\left(\frac{L}{2}-x\right)\times\left(x+\frac{\frac{L}{2}-x}{2}\right)\times\omega^2$$

$$P = \rho A\left(\frac{L}{2}-x\right)\times\left(x+\frac{L-2x}{4}\right)\times\omega^2$$

$$P = \rho A\left(\frac{L}{2}-x\right)\times\left(\frac{4x+L-2x}{4}\right)\times\omega^2$$

$$P = \rho A\left(\frac{L}{2}-x\right)\times\left(\frac{2x+L}{4}\right)\times\omega^2$$

$$P = \rho A\left(\frac{L}{2}-x\right)\times\left(\frac{L}{4}+\frac{x}{2}\right)\times\omega^2$$

$$\sigma_x = \frac{\rho\cancel{A}\left(\frac{L}{2}-x\right)\left(\frac{L}{4}+\frac{x}{2}\right)\times\omega^2}{\cancel{A}}$$

$$\sigma_x = \rho\left(\frac{L}{2}-x\right)\left(\frac{L}{4}+\frac{x}{2}\right)\times\omega^2$$

$$\sigma_x = \rho\left(\frac{L^2}{8}+\frac{x\cancel{L}}{\cancel{4}}-\frac{x\cancel{L}}{\cancel{4}}-\frac{x^2}{2}\right)\times\omega^2$$

$$\sigma_x = \rho\left(\frac{L^2}{8}-\frac{x^2}{2}\right)\times\omega^2$$

$$\delta x = \int\frac{\sigma_x}{E}\,dx.$$

$$\delta x = \frac{\rho\omega^2}{8E}\int_0^{L/2}(L^2-4x^2)\,dx$$

$$\delta x = \frac{\rho\omega^2}{8E}\left[L^2 x - \frac{4x^3}{3}\right]_0^{L/2}$$

$$\delta x = \frac{\rho\omega^2}{8E}\left[L^2\times\frac{L}{2} - \frac{4\times L^3/8L}{3}\right]$$

$$\delta x = \frac{\rho\omega^2}{8E}\left[\frac{L^3}{2} - \frac{L^3}{6}\right]$$

$$\delta x = \frac{\rho\omega^2}{8E}\left[\frac{3L^3}{6} - \frac{L^3}{6}\right]$$

$$\delta x = \frac{\rho\omega^2}{8E}\left[\frac{2\times L^3}{\cancel{6}\,3}\right]$$

$$\delta x = \frac{\rho\omega^2}{24E}\times L^3$$

Here the elongation of only one side for both sides overall elongation.

$$\delta x = 2\times\frac{\rho\omega^2}{24E}\times L^3$$

$$\delta x = \frac{\rho\omega^2}{12\times E}\times L^3$$

$$\delta n = \frac{\rho\omega^2}{8E}\left[L^2x - \frac{4x^3}{3}\right]_0^{L/2}$$

$$\delta n = \frac{\rho\omega^2}{8E}\left[L^2\times\frac{L}{2} - \frac{4\times L^3/8}{3}\right]$$

$$\delta n = \frac{\rho\omega^2}{8E}\left[\frac{L^3}{2} - \frac{L^3}{86}\right]$$

$$\delta n = \frac{\rho\omega^2}{8G}\left[\frac{36L^3}{6} - \frac{L^3}{86}\right]$$

$$\delta n = \frac{\rho\omega^2}{8E}\left[\frac{2\times L^3}{8.3}\right]$$

$$\delta n = \frac{\rho\omega^2\times L^3}{24E}$$

Here the elongation of only one side for both sides overall elongation.

$$\delta n = 2\times\frac{\rho\omega^2}{24E}\times L^3$$

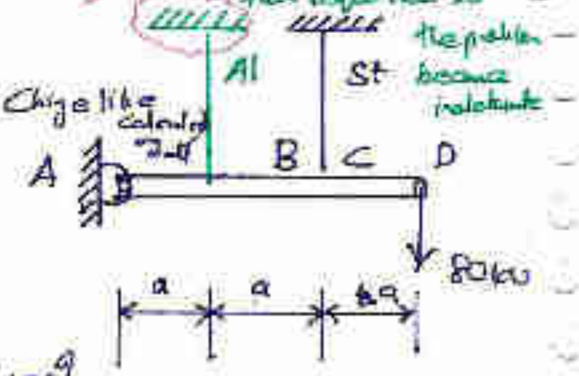
$$\delta n = \frac{\rho\omega^2}{12\times E}\times L^3$$

18/7/2017

(hinge support allow rotation)

? If the beam ABCD is rigid, if the elasticity of steel is 3 times the elasticity of aluminium. Determine
i) Reaction at the hinge?
ii) Stress at steel and aluminium wires?

[Take A_{St} as 500 mm² and $A_{Al} = 750$ mm²]



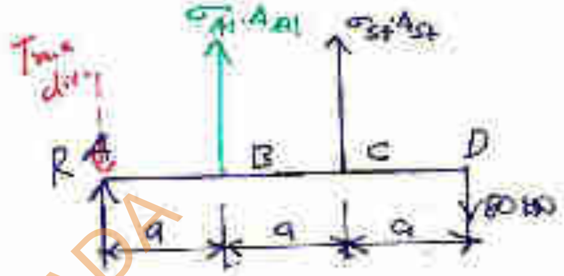
Here Al provides extra support than required so the problem become indeterminate

(Fixed support doesn't allow rotation)

i) Equilibrium : $\Sigma F = 0$

$R + \sigma_{Al} \cdot A_{Al} + \sigma_{St} \cdot A_{St} = 80,000$

$R + \sigma_{Al} \cdot 750 + \sigma_{St} \cdot 500 = 80,000$ —— (i)



$\Sigma M_A = 0 \Rightarrow$

$(750 \times \sigma_{Al} \times \cancel{a}) + (500 \times \sigma_{St} \times 2\cancel{a}) = 80000 \times 3\cancel{a}$

$750\sigma_{Al} + 500\sigma_{St} = 80,000$

$3\sigma_{Al} + 4\sigma_{St} = 960$ —— (ii)

2 unknowns depends the equation will be indeterminate.

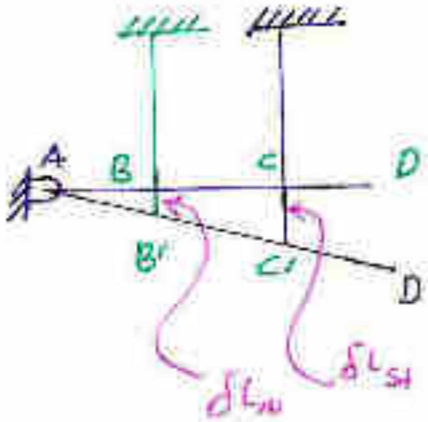
Compatibility:

$$\frac{\delta L_{AB}}{a} = \frac{\delta L_{St}}{2a}$$

$$\frac{\sigma_{Al} \cdot \cancel{L}}{E_{Al} \times \cancel{a}} = \frac{\sigma_{St} \cdot \cancel{L}}{E_{St} \times 2\cancel{a}}$$

$$\frac{\sigma_{Al}}{\cancel{E_{Al}}} = \frac{\sigma_{St}}{3 \times \cancel{E_{Al}} \times 2}$$

$\sigma_{St} = 6\sigma_{Al}$ —— (iii)



$3\sigma_{A1} + 4\times6\sigma_{A1} = 960$

$3\sigma_{A1} + 24\sigma_{A1} = 960.$

$27\sigma_{A1} = 960$

$\sigma_{A1} = 35.55$ MPa

$\sigma_{St} = 6\sigma_{A1} = 213.33$ MPa

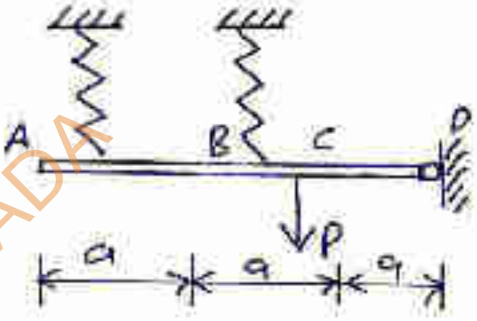
$R = \underline{\underline{-53.32 \text{ kN}}}$

Bk No. 15
Q. 4

9. Supports more than required.
Indeterminate.

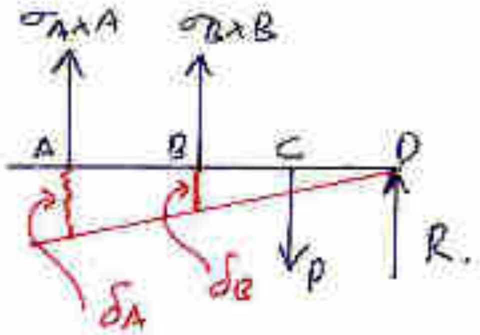
Stiffness of spring $A = 2k.$

Stiffness of spring $B = k$



Equilibrium equation.

$\sigma_A \times A + \sigma_B \times B + R = P.$



$\delta L = \frac{F}{k}$ $[F = kx]$

$\Sigma M_A = 0.$

$P \times A = (\sigma_B \times B) \times 2a + (\sigma_A \times A) \times 3a.$

Compatibility = 0

$$\frac{\delta_B}{2a} = \frac{\delta_A}{3a}$$

$$\frac{P_B L_B / A_B E_B}{2a} = \frac{P_A L_A / A_A E_A}{3a}$$

$$\frac{F_A / 2k}{3x} = \frac{F_B / k}{2x}$$

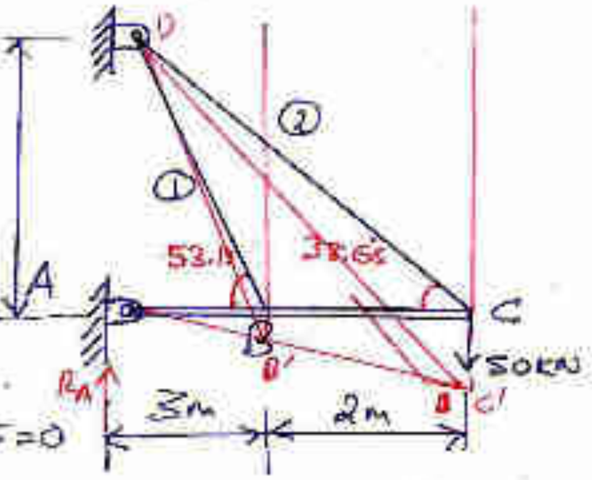
$$\frac{F_A}{\cancel{2}F_B} = \frac{3F_B}{\cancel{2}}$$

$$\frac{F_A}{F_B} = 3$$

? ABC is rigid. Wires 1 and wires 2 are of same material

$A_1 = A_2 = 600 mm^2$

$\sigma_1 = ?$ $\sigma_2 = ?$



~~$\Sigma F_x = 0$.~~

~~$\sigma_1 \times A_1 \times \cos 53.13 + \sigma_2 \times A_2 \times \cos 38.65 = 0$~~

~~$\Sigma F_y = 0$.~~

~~$R_1 + \sigma_1 A_1 \sin 53.13 + \sigma_2 A_2 \times \sin 38.65 = 50 \times 10^3$~~

~~$50 \times 10^3 = F_1 \times \sin 53.13 + F_2 \times \sin 38.65$~~

~~$F_2 \times \cos 53.13 = -F_1 \sin 38.65$~~

~~$F_2 = -1.3 \times F_1$~~

~~$50 \times 10^3 = F_1 \times \sin 53.13 - 1.3 \times F_1 \sin 38.65$~~

~~$\frac{\delta L}{3} = \frac{\delta L}{5}$~~ ~~$x + y$~~

~~$\frac{\frac{\sigma_1 A_1 \times L}{A_1 E}}{3} = \frac{\sigma_2 A_2 \times L}{3x} + \frac{5y}{} = 50 \times 10^3 \times 5$~~

~~$\Sigma M_A = 0$ $\sigma_1 A_1 \times \sin 53.13 \times 3 + \sigma_2 A_2 \times \sin 38.65 \times 5 = 50 \times 50 \times 10^2$~~

~~$2.39 \times \sigma_1 A_1 + 3.12 \sigma_2 A_2 = 250000$~~

~~$\Sigma F_y = 0$~~ Indeterminate 2 equations.

$M_A = 0$ $\sigma_1 A_1 \sin 53.13 \times 3 + \sigma_2 A_2 \times \sin 38.65 \times 5 = 50 \times 10^3 \times 5$

$479.99 \sigma_1 \times 3 + 374.73 \sigma_2 \times 5 = 250 \times 10^3$

$1439.97 \sigma_1 + 1873 \sigma_2 = 250 \times 10^3$

$$\frac{\delta L_B}{3} = \frac{\delta L_C}{5} \quad \left(\frac{PL}{AE} = \frac{\sigma L}{E}\right)$$

$\Delta ABB' \sim \Delta ACC'$

$\frac{BB'}{3} = \frac{CC'}{5}$

BB' is the elongation

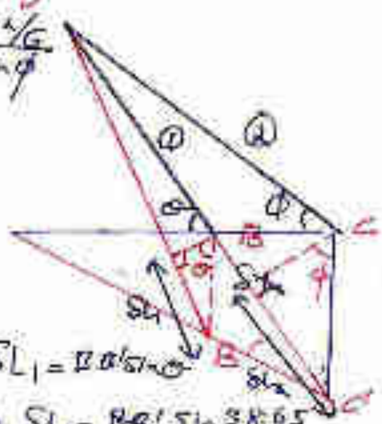
$$\frac{\frac{\sigma_1 \times 0.7999 \times 5}{E}}{3} = \frac{\frac{\sigma_2 \times 0.645 \times \sqrt{41} \times 0.625}{E}}{5}$$

$$\frac{\sigma_1 L_1 / E}{3 \sin\theta} = \frac{\sigma_2 L_2 / E}{5 \sin\phi}$$

$\sigma_1 = 0.6 \sigma_2$

$\therefore 1439.97 \times 0.6 \sigma_2 + 1872 \times \sigma_2 = 250 \times 10^3$

$\sigma_2 = 91.31 MPa$, $\sigma_1 = 59.786 MPa$



$\delta L_1 = BB' \sin\theta$

$\|ly$ $\delta L_2 = \frac{BB'}{CC'} \sin 38.65$

1.6. STRENGTH-

The maximum load that a component of given material and dimension can resist is known as strength of that component. Every material has the limit value.

Stress is not a function of material (P/A).

Permissible /Allowable stress is a function of material.

Safe: $\sigma_{max} \leqslant \hat{\sigma}$

$$\frac{P}{A} \leq \hat{\sigma}$$

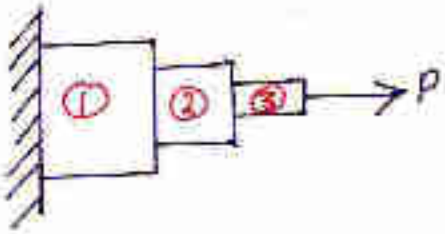
$$P \leqslant \hat{\sigma} . A$$

$$\hat{P} = \hat{\sigma} . A$$

$$\underset{\text{Max. value of load that can be applied}}{\hat{P}} = \underset{\text{Function of materia}}{\hat{\sigma}} . \overset{\text{C.S.}}{A} \longrightarrow \text{Strength Formula}$$

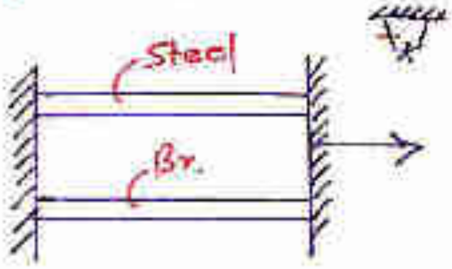
→ Seiries - (Load acting at different material so that

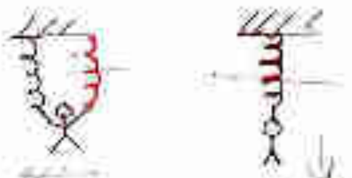
Max. load can be applied ∴ $\hat{P} = \min(\hat{P}_1, \hat{P}_2, \hat{P}_3)$



→ Parallel - (Load acting at different material in parallel

So Max. load applied ∴ $\hat{P} = \hat{P}_1 + \hat{P}_2 + \text{-------}$





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Q. 9

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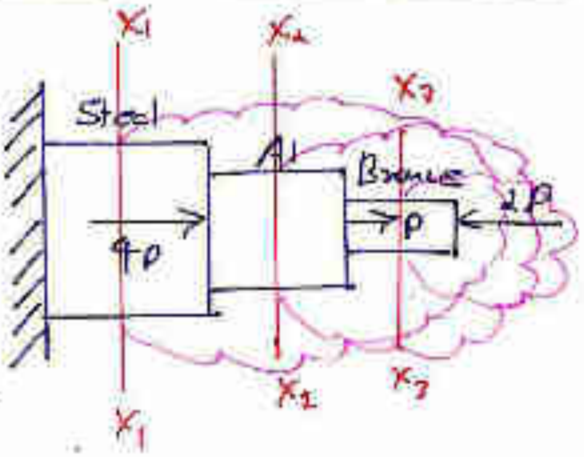
At secn ①



$A_S = 500\,mm^2$ $\quad \sigma_{St} = 140\,MPa$

$A_{Al} = 400\,mm^2$ $\quad \sigma_{Al} = 90\,MPa$

$A_B = 200\,mm^2$ $\quad \sigma_B = 100\,MPa$.

$\hat{\sigma}_{St} . A_{St} \geq 3P$

$3P \leq \hat{\sigma}_{St} . A_{St}$

$$P \leq \frac{140 \times 500}{3}$$

$P_{St} \leq 23.33\,kN$

$\hat{\sigma}_{Al} = 90\,MPa.$

$|-P| \leq \times 90 \times 400$

$P_{Al} \leq 36\,kN$

$\hat{\sigma}_{Br} = 100\,MPa.$

$|-2P| \leq 100 \times 200$

$P_{Br} \leq 10\,kN$

$\hat{P} = \min(\hat{P}_{St}, \hat{P}_{Al}, \hat{P}_{Br})$

$\hat{P} = \min(23.33, 36, 10)$

$\hat{P} = \underline{\underline{10\,kN}}$

Actual load acting on steel $= (4+1-2)P$

$P_{Steel} = 3P$

Actual load acting on Al $= P - 2P = -P$

Actual load acting on Br $= -2P$

$P_{Bronze} = |-2P| = 2P$

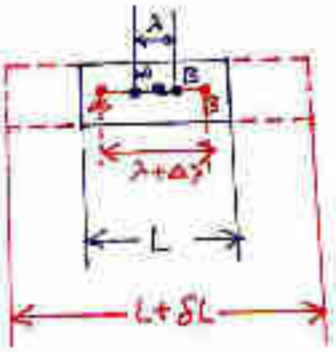
1.7. Thermal Stress & Strain:-

a. Thermal Strain

$$\delta L_{TH} \overset{=}{\cancel{\propto}} \alpha \times L \times \Delta T$$

Not against the will of the material (No force are needed) →

$$\delta L_{TH} = \alpha \times L \times \Delta T$$

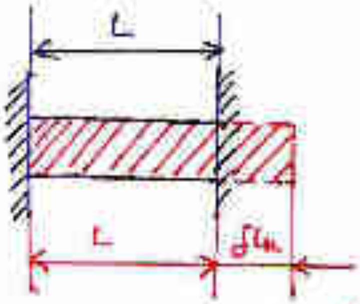


~~b.~~ Thermal Strain, $\epsilon_{TH} = \frac{\delta L_{TH}}{L} = \alpha \times \Delta T$

No resistance to the change, so there will be "no stress"

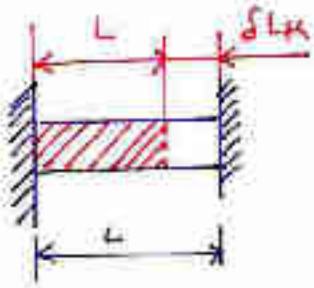
b. Thermal Stress- If a member is free to expand/contract no thermal stress would be developed.

Compressive thermal stress is developed if thermal expansion is either partially or completely restricted.



Compressive thermal stress

Tensile thermal stress is developed if thermal contraction either partially or completely restricted



Tensile thermal stress

→ Compatibility + Equilibrium

REMOVE

→ Remove the support / free the structure to determine the length.

RESTORE : $\delta L_{\sigma} = \frac{\sigma L}{E}$

→ Restore the member by introducing a suitable stress into the system

→ RELATE : $\delta L_{TH} = \delta L_{\sigma} + \Delta$

$$\delta L_{TH} = \frac{\sigma L}{E} + \Delta$$

$$\alpha L \Delta T = \frac{\sigma L}{E} + \Delta$$

* For complete restriction, Nothing allowed so $\Delta = 0$

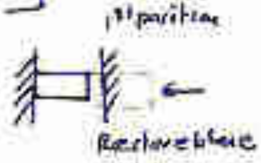
$$\therefore \delta T_{H} = \delta L_{\sigma} + \overset{=0}{\cancel{\Delta}}$$

$$\delta L_{TH} = \delta L_{\sigma}$$

? An aluminium 500mm long is fixed at the left end while a rigid wall is available at the right end at a clearance of 0.5mm. $\alpha_{Al} = 2\overset{4}{\cancel{8}}\times10^{-6}/°C$; $E_{Al} = 70GPa$. Determine the magnitude and nature of the stress set up in the rod if the temperature increased by 100°C

$L = 500$ mm ; $\Delta = 0.5$mm ; $\alpha_{Al} = 24\times10^{-6}/°C$; $E_{Al} = 70\times10^{3}$MPa.

Remove ; $\delta L_{TH} = \alpha \cdot L \times \Delta T$ $[\Delta T = 100°C]$



$$\delta L_{TH} = 24\times10^{-6} \times 500 \times 100$$

$$\delta L_{TH} = 1.2 mm$$

$$\delta L_{TH} = \delta L_{\sigma} + \Delta$$

$$\delta L_{TH} = \frac{\sigma L}{E} + \Delta =$$

$$1.2 = \frac{\sigma \times 500}{70\times10^{3}} + 0.5$$

$$\underline{\sigma = 98 MPa}$$

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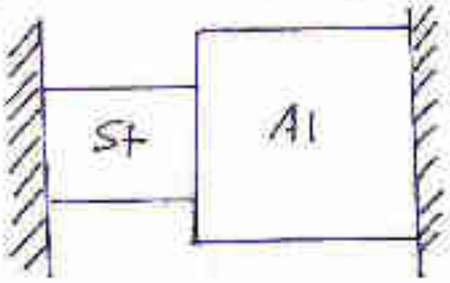
9. $E_s = 200GPa$; $\alpha_s = 11\times10^{-6}/°C$; $L_s = 1m$; $A_s = 100mm^2$

$E_a = 70GPa$; $\alpha_a = 29\times10^{-6}/°C$; $L_a = 1m$; $A_a = 200mm^2$.

$\Delta T = 58 - 38 = \underline{\underline{20°C}}$

St, Al alone partially restricted
Overall completely restricted



At the junction, Al push the steel, Even if the temperature rise steel will contract and Al will expand.

Steel = Contract Al = Expand
$\alpha\downarrow$ $D\downarrow$ $\alpha\uparrow$ $D\uparrow$

For overall complete restriction

$$(\delta L_{TH})_{overall} = (\delta L_\sigma)_{overall}$$

$$\alpha_{St}\cdot L\cdot\Delta T + \alpha_{Al}\cdot L\,\Delta T = \frac{\sigma_s L}{E_{St}} + \frac{\sigma_{Al} L}{E_{Al}}$$

$$11\times10^{-6}\times 20 + 29\times10^{-6}\times 20 = \frac{\sigma_s}{200{,}000} + \frac{\sigma_{Al}}{70{,}000}$$

$$9800\times\cancel{10^{3}} = \sigma_s\times 70\times\cancel{10^{3}} + \sigma_{Al}\times 200\times\cancel{10^{3}}$$

$$9800 = \sigma_s\times 70 + \sigma_{Al}\times 200$$

$$\sigma_s + 2.85\sigma_{Al} = 140$$

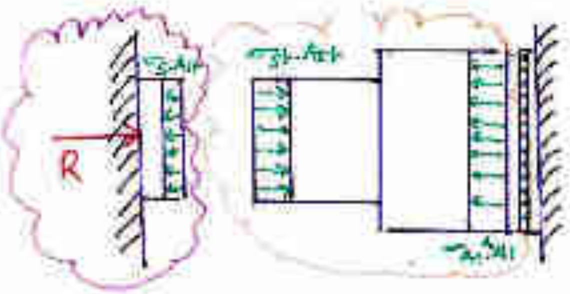
$$\underline{\underline{70\sigma_s + 20\sigma_{Al} = 980}}$$

Need of Equilibrium

$\therefore\ \sigma_{St}\cdot A_{St} = \sigma_{Al}\cdot A_{Al}$

$\sigma_{St}\times 100 = \sigma_{Al}\times 200$

$\sigma_{St} = 2\sigma_{Al}$



Cont*

$7\sigma_s + 20\sigma_{Al} = 980$

$7(2\sigma_{A1}) + 20\sigma_{A1} = 980$

$14\sigma_{A1} + 20\sigma_{A1} = 980$

$\underline{\sigma_{A1} = 28.82 \text{ MPA}}$

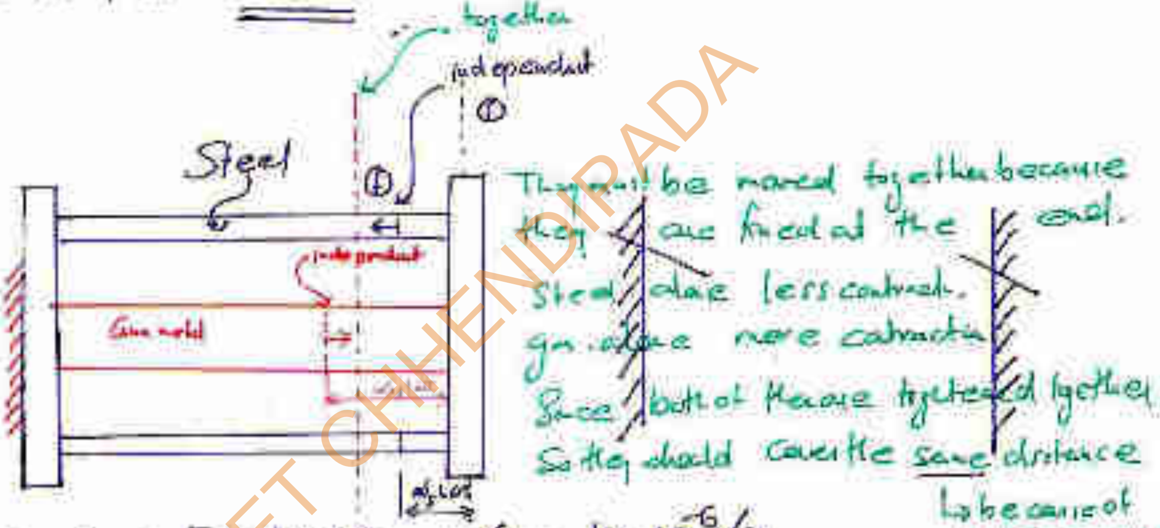
$\sigma_S = 2\times\sigma_{A1} = 2\times 28.82 = 57.64 \text{ MPa}$

$R = \sigma_{St}\cdot A_{St}$

$R = 57.64\times 100$

$R = 5764 \text{ N} = \underline{5.7 \text{ kN}}$

Pg. No-17
Q = 3, 9 ?



They must be moved together because they are fixed at the end.
Steel alone less contraction.
gm. alone more contraction.
Since both of them are tightened together
So they should cover the same distance
to be cause of mutual movement.

Bolt, bhai

$A_g = 200\text{mm}^2$; $E_g = 100\text{GPa}$; $\alpha_g = 10\times10^{-6}/°F$

$A_S = 100\text{mm}^2$; $E_s = 200\text{GPa}$; $\alpha_S = 6\times10^{-6}/°F$

$[① \rightarrow ⓕ]_{St}$ $[① \rightarrow ⓕ]_{gm}$

$$\alpha_{St} L\Delta T + \frac{\sigma_{St} L}{E_{St}} = \alpha_g L\times\Delta T - \frac{\sigma_{gm}\times L}{E_{gm}}$$

$$6\times10^{-6}\times 200 + \frac{\sigma_{St}}{100\times10^3} = 10\times10^{-6}\times200 - \frac{\sigma_{gm}}{100\times10^3}$$

$$1.2\times10^{-3} + \frac{\sigma_{St}}{\overset{2}{\cancel{200}}\times10^3} = 2\times10^{-3} - \frac{\sigma_{gm}}{\underset{1}{\cancel{100}}\times10^3}$$

$$2\times120 + \sigma_{St} = (400 - \sigma_{gm}).2$$

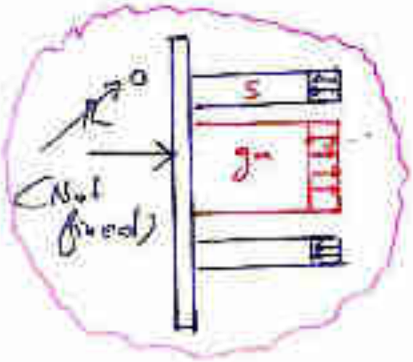
~~$\sigma_{St} + \sigma_{gm} = 200 - 120 = 160$~~

$\sigma_{St} + \sigma_{gm}\times 2 = 160$

$\sigma_{St} + 2\times\sigma_{gm} = 160$

At Equilibrium

(Nut fixed) $\cancelto{0}{R} = \sigma_{st} A_{st} \sigma_{gm} \times A_{gm}.$



$A_{gm} \cancel{R} = \times\sigma_{st} = \sigma_{gm} \times A_{st}$

$200 \times \sigma_{gt} = \sigma_{st} \times 100$

$\sigma_{st} = 2 \times \sigma_{gt}$

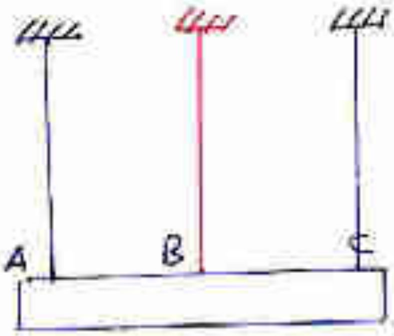
$2\sigma_{gt} + 2\sigma_{gt} = 160$

$4 \times \sigma_{gt} = 160$

$\sigma_{gt} = 160/4 = 40$ MPa.

$\sigma_{st} = 2 \times \sigma_{gt} = 80$ MPa

#2) horizontal rigid platform ABC is supported with the help of central copper wire and two steel wires placed symmetrically at the end.



$\alpha_c = 18 \times 10^{-6}/°C$ $L_s = L_{sc}$

$\alpha_s = 12 \times 10^{-6}/°C$ $A_s = 400\,mm^2$

$A_c = 600\,mm^2$

$E_c = 100$ GPa

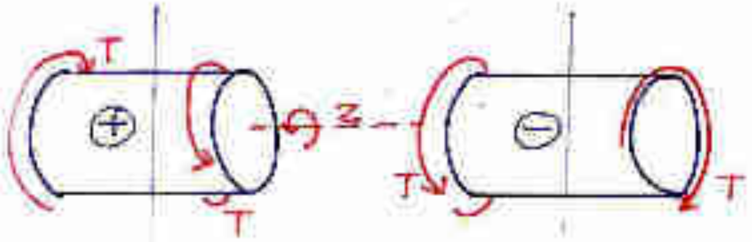
$E_s = 200$ GPa

i) Determine the stresses induced in steel and Cu wires of the platform abc is weightless and temperature is uniformly increased by 50°C?

ii) Determine the stress and steel and copper wires if the platform has a weight of 12 kN? and temperature of the structure is raised by 50°C?

2. TORSION :-

2.1. LOAD:



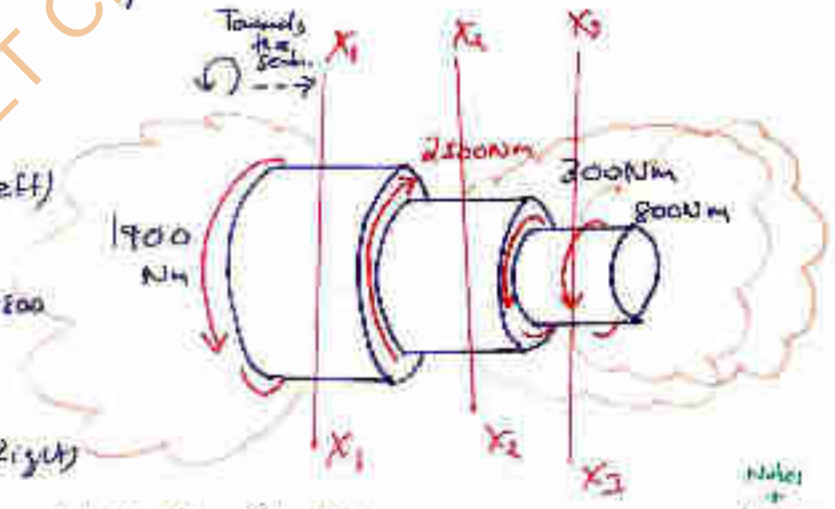
* **Definition:** Equal and opposite couples acting about - longitudinal axis of a member constitute its torsion.

* **Convention:** Twisting moment is treated as positive if the right hand thumb goes away from the section when right hand fingers are culed rotated basically in the sense of twisting moment.

Σm: [No sign]

* **@ section** Otherwise it is treated as negtive

Twisting Moment: Twisting Moment at a section is the algebraic sum of all the longitudinal couples transmitted to that section from either side.



X_1: $T_{x_1} = -1900$ Nm (Left)

X_{2l}: $T_{x_{2r}} = -2500+300+800$

$T_{x_{2r}} = -1900$ Nm

$T_{x_1} = -1900$ Nm (Right)

X_2: $T_{x_2} = 300+800 = 1100$ Nm. (Left)

$T_{x_2} = 2500-1900 = 1100$ Nm (Right)

X_3 : $T_{x_3} = 800$ Nm (Right)

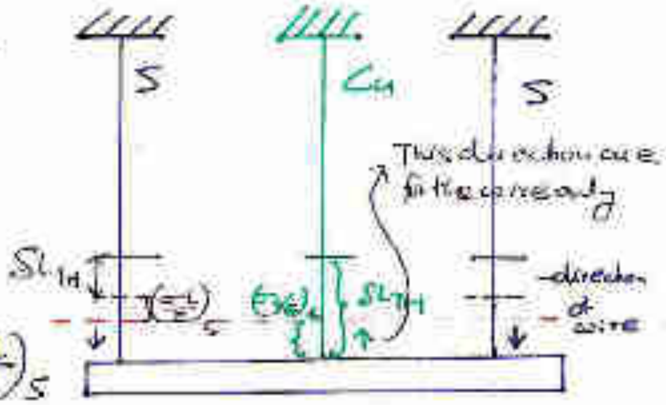
$T_3 = 2500-1900-300 = 800$ Nm (Left)

2. $\alpha_c = 18\times10^{-6}/^\circ C$; $E_c = 100\,GPa$

$\alpha_s = 12\times10^{-6}/^\circ C$; $E_s = 200\,GPa$.

$A_c = 600\,mm^2$; $\Delta T = 50^\circ C$

$A_s = 400\,mm^2$



i) $\left(\delta L_{TH} - \cancel{\alpha}\frac{\sigma L}{E}\right)_{Cu} = \left(\delta L_{TH} + \frac{\sigma L}{E}\right)_S$

$18\times10^{-6}\times\cancel{L}\times50 - \frac{\sigma_c\times\cancel{L}}{E_c} = 12\times10^{-6}\times50\times\cancel{L} + \frac{\sigma_s\times\cancel{L}}{E_s}$

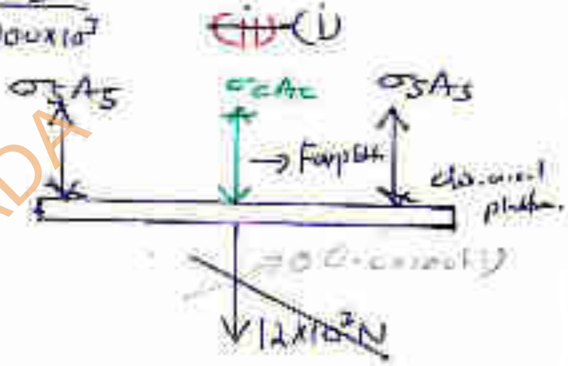
$18\times10^{-6}\times50 - \frac{\sigma_c}{100\times10^3} = 12\times10^{-6}\times50 + \frac{\sigma_s}{200\times10^3}$

$\frac{90-\sigma_c}{100\times10^3} = \frac{120+\sigma_s}{2\times100\times10^3}$

$2(90-\sigma_c) = 120+\sigma_s$

$\sigma_s + 2\sigma_c = 180-120$

$\sigma_s + 2\sigma_c = 60$



In the first case No cycle applied

S. Act $\sigma_c A_c = 2\sigma_s A_s$

$\sigma_c\times600 = 2\sigma_s\times400$

$\sigma_c = 4/3\,\sigma_s$

$\sigma_c = 4/3\,\sigma_s$

$\sigma_c = 4/3\times16.36$

$\sigma_c = 21.81\,MPa$

$\therefore \sigma_s + 2\times\frac{4}{3}\sigma_s = 60$

$\underline{\underline{\sigma_s = 16.36\,MPa}}$ $\therefore \sigma_s = 16.36\,MPa$; $\sigma_c = 21.81\,MPa$.

~~$\sigma_c\times600+12\times10^3 = 2\sigma_s\times400$~~

ii) ~~σ_c~~$\times600 \neq 2\sigma_s\times400+12000$ ~~$\sigma_s = 3/4\,\sigma_c + 15$~~

$\sigma_c = \frac{4}{3}\sigma_s+20$ $\sigma_c\times\frac{3}{4}+\frac{12000}{400\times2}\times\frac{4}{3}$ $\sigma_c = \frac{4}{3}\times5.45+20$

~~$\sigma_c = 27.27\,MPa.$~~

$\therefore \sigma_s + 2\left(\frac{4}{3}\sigma_s+20\right) \neq 60$

$\sigma_s + \frac{8}{3}\sigma_s + 40 = 60$ ~~$3/4\,\sigma_c + 15 + 2\sigma_c = 60$~~

$11/3\,\sigma_s \neq 20$; $\sigma_s \neq 5.45\,MPa$ ~~$\sigma_c = 16.36\,MPa$~~

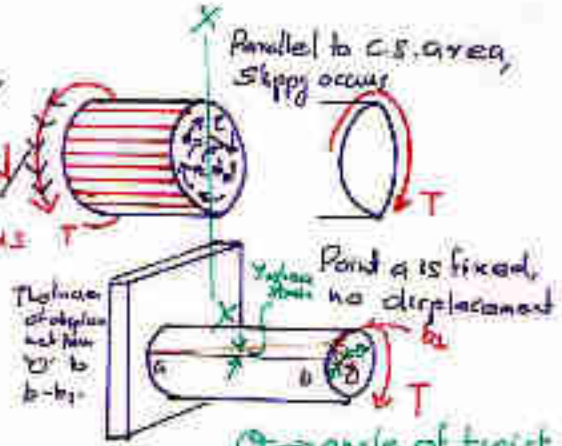
~~$\sigma_s = 27.27\,MPa$~~

$\sigma_s \neq 5.45\,MPa$; $\sigma_c \neq 27.27\,MPa$

17/01/2020

2.2 Torsional Stress:

- → Nature : Torsional shear/ Shear/ tangential/ Frictional / Viscous
- → Distribution : Nonuniform/ Linear.
- → Magnitude : $\tau = \frac{T}{J} \times r$



θ → angle of twist

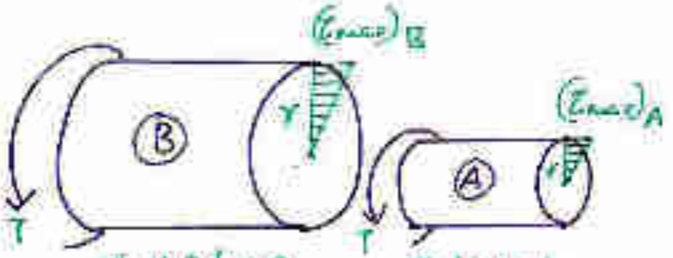
angular change is called shear strain.

$$T = \int \underbrace{\tau . dA}_{\text{force}} \times r$$

$$T = \int kr . dA \times r$$

$$T = k\boxed{\int r^2 dA} \rightarrow \text{Polar MOI} = \frac{\pi d^4}{32}$$

$$T = kJ$$



because dia is less J is less so T/J is values more in A compared to B.

Shaft B have less than τ_{max}

Shaft A has more value of τ_{max}

$J\uparrow d\uparrow \left[\frac{T}{J}\right]$ slope ↓

$J\downarrow d\downarrow \frac{T}{J}\uparrow$ slope

For the same torque the max. shear stress will be at 'A' than 'B' because $\tau = \frac{T}{J} \times r$

a) Max Shear (Solid Shaft):

$$\tau = \frac{T}{J} \times r$$

$$\tau_{max} = \frac{T}{J} \times r_{max}$$

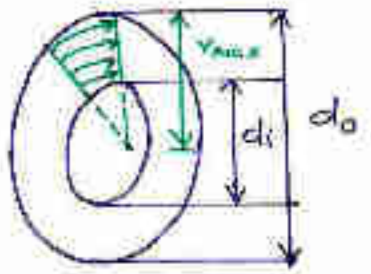
$$\tau_{max} = \frac{T}{\pi d^4/32} \times \frac{d}{2}$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

Max. Shear (Hollow Shaft):-

$$\tau_{max} = \frac{T}{J} \times r_{max}$$



→ r_{max} should be meaured from the Centre.

$$\tau_{max} = \frac{T \times}{\frac{\pi d_o^4}{32} \frac{\pi d_i^4}{32}} \frac{d_o}{2}$$

$$\tau_{max} = \frac{T}{\frac{\pi d_o^4}{32}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} \times d_o/2$$

$$\tau_{max} = \frac{16\,T}{\pi d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} \quad \beta$$

$$\tau_{max} = \frac{16 \times T}{\pi d_o^3 \left[1 - \beta^4\right]}$$

$\beta = \frac{d_i}{d_o}$ [Always will be ≤ 1]

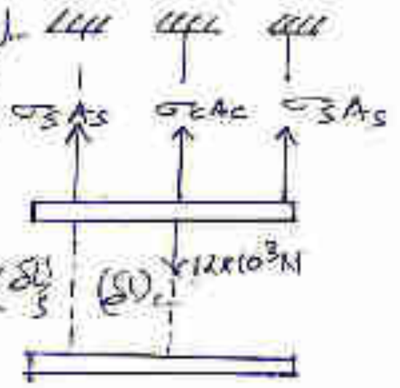
2#

$2\sigma_s' A_s + \sigma_c' A_c = 12 \times 10^3$

$2\sigma_s' . 400 + \sigma_c' \times 600 = 12 \times 10^3$

$\sigma_s' + \frac{3}{4} \sigma_c' = 15$

After the thermal stresses if will be removed no effect of thermal stress will be considered, cancel



$(\delta L)_s = (\delta L)_{cu}$

$\left(\frac{PL}{AE}\right)_s = \left(\frac{PL}{AE}\right)_{cu}$

$\left(\frac{\sigma' L}{E}\right)_s = \left(\frac{\sigma' L}{E}\right)_{cu}$

$\frac{\sigma_s' \times \cancel{L}}{E_s} = \frac{\sigma_c' \times \cancel{L}}{E_c}$

$\frac{\sigma_s'}{2\cancel{00} \times 10^3} = \frac{\sigma_c'}{1\cancel{00} \times 10^3}$

$\sigma_s' = 2\sigma_c'$

$2\sigma_c' + \frac{3}{4}\sigma_c' = 15$

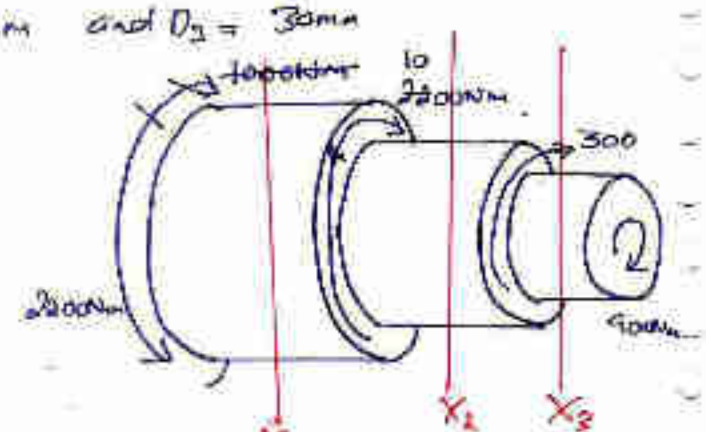
$\sigma_c' = \underline{5.45 \text{ MPa } (T)}$

$\sigma_{st}' = 10.9 \text{ MPa } (T)$

$(\sigma_s)_{net} = \sigma_s + \sigma_{st}' = 16.36 + 10.9 = 27.26 \text{ MPa } (T)$

$(\sigma_c)_{net} = \sigma_c + \sigma_{ct}' = -21.81 + 5.45 = \underline{-16.36 \text{ MPa } (C)}$

Q Determine maximum shear stress in the loaded axle in the figure. $D_1 = 50mm$, $D_2 = 40mm$ and $D_3 = 30mm$



(Always stress calculated at section)

$$(\tau_{max})_1 = \frac{16\,T_{x_1}}{\pi\,d^3_{x_1}}$$

$$(\tau_{max})_1 = \frac{16 \times -2200Nm}{\pi \times (50 \times 10^{-3})^3}$$

$$(\tau_{max})_1 = \left|-89.63\right| MPa$$

$$(\tau_{max})_2 = \frac{-1200 \times 10^3 \times 16}{\pi \times (40)^3}$$

$$(\tau_{max})_2 = \left|-95.49\right| MPa \uparrow$$

$$(\tau_{max})_3 = \frac{16\,T_{x_3}}{\pi\,d^3_{x_3}}$$

$$(\tau_{max})_3 = \frac{16 \times -900 \times 10^3}{\pi \times (30)^3}$$

$$(\tau_{max})_3 = \left|-169.76\right| MPa \uparrow$$ (increases due to J)

Pg. 56.6 Q.0.6

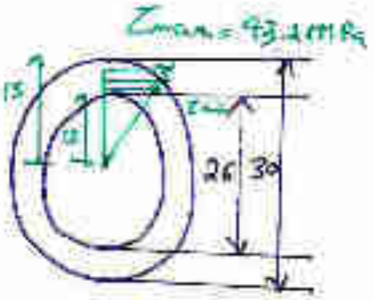
Q $T = 100\,NM$

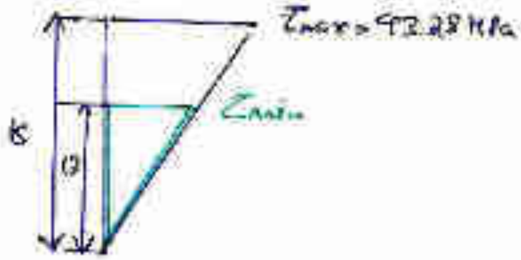
$$\tau_{max} = \frac{16\,T}{\pi d_o^3 (1-\beta^4)}$$

$$\beta = \frac{di}{do} = \frac{26}{30} = 0.8667$$

$$\tau_{max} = \frac{16 \times 100 \times 10^3}{\pi \times 30^3 (1 - 0.8667^4)}$$

$$\tau_{max} = 43.28\ MPa$$





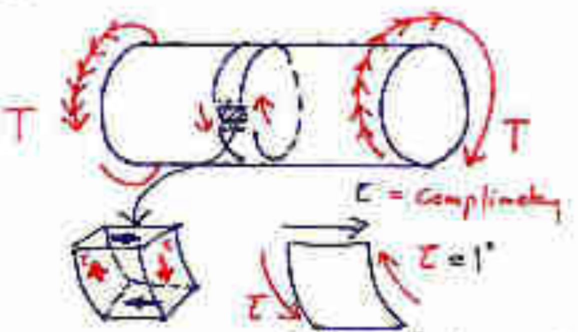
→ Similar Δ ∴ $\frac{\tau_{max}}{15} = \frac{\tau_{min}}{13}$

$$\frac{43.28}{15} = \frac{\tau_{min}}{13}$$

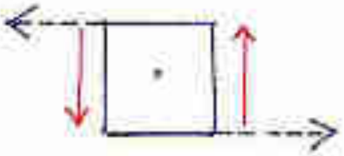
$$\tau_{min} = 37.50 MPa$$

2.3 Hooke's Law for Torsion:

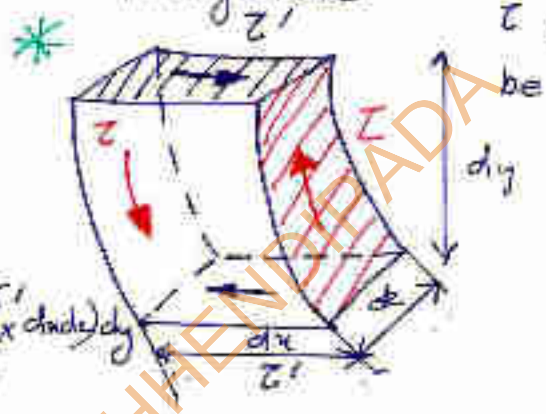
- → $|\tau| = |\tau'|$
- → $\tau = G\left(\frac{\theta}{L}\right)r$; $\gamma = |90-\phi|$; $\tau = G.\gamma$
- → $\frac{\tau}{r} = \frac{T}{J} = G\left(\frac{\theta}{L}\right)$



Forces are balanced, but generates an anticlockwise couple. So moment is not balanced. But everything should be balanced.
τ → coupled caused (anticlockwise)
τ' → coupled caused (clockwise)
↳ Complimentary shear
τ & τ' should be always be together



τ and τ' are equal in magnitude



Moment balancing

↺ = ↻

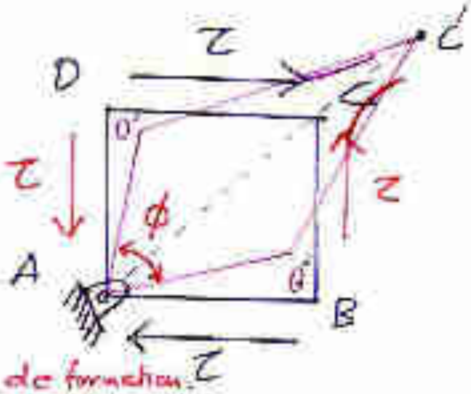
$$(\tau \times dy\,dz)\times dx = (\tau' \times dx\,dz)\,dy$$

$$\tau = \tau'$$

1° shear and Complimentary shear are always equal in equilibrium. We obtained it by "Moment Equilibrium"

Shear strain:

Introducing a hinge support.
Because of 1° and complimentary shear AC increases to AC' and BD becomes B'D'.
So there will be angular deformation.
Total angular change from the ⊥ axis to a point called Shear strain at a point (ϕ)



$$\gamma = |90^\circ - \phi|$$

The angular change undergone by 2 lines passing through a point which were originally perpendicular to each other is defined to be the "Shear Strain" at that point.

$$\gamma = |90^\circ - \phi|$$

Hookes Law:

$\tau \rightarrow$ Shear stress

$\gamma \rightarrow$ Shear strain

$G \rightarrow$ Modulus of Rigidity

$$\tau = G \times \gamma$$

$$\downarrow \gamma = \frac{\tau}{G \uparrow}$$

↳ Resistence of material to angular deformation or distortion.

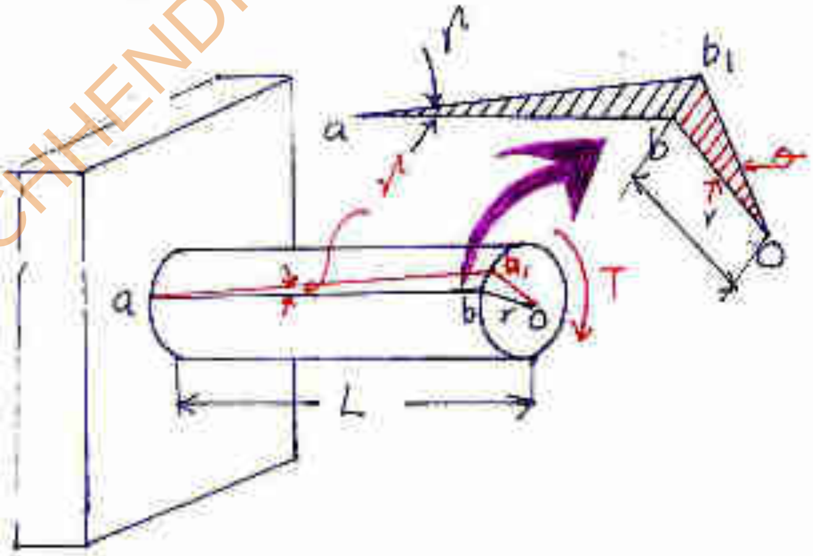
$$\tau = G \times \gamma$$

$$\tau = G \times \frac{bb'}{L}$$

$$\tau = G \times \frac{\theta \times r}{L}$$

$$\tau = G \times \frac{\theta}{L} \times r$$

$$\tau = G\left(\frac{\theta}{L}\right) \times r$$



Eqn: $\tau = \frac{T}{J} \times r$

Hookes law: $\tau = G\left(\frac{\theta}{L}\right) r$

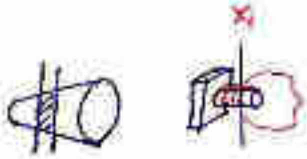
$$\frac{T}{J} = \frac{\tau}{r} = G\left(\frac{\theta}{L}\right)$$

i.e. $\boxed{\frac{\tau}{r} = \frac{T}{J} = G\left(\frac{\theta}{L}\right)}$ ----> $\boxed{\sigma = \frac{P}{A} = E \cdot \left(\frac{\Delta L}{L}\right)}$

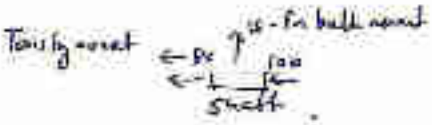
2.4. Angle of Twist:-

$\theta = \frac{T.L}{J.G}$ - - - -→ $SL = \frac{PL}{AE}$ uniform

$SL = \int \frac{P.dx}{AE}$ $\theta = \int \frac{T.dx}{G.J}$ ——→ Variable



$\theta = \theta_1 + \theta_2 + ..$ ——→ Stepped

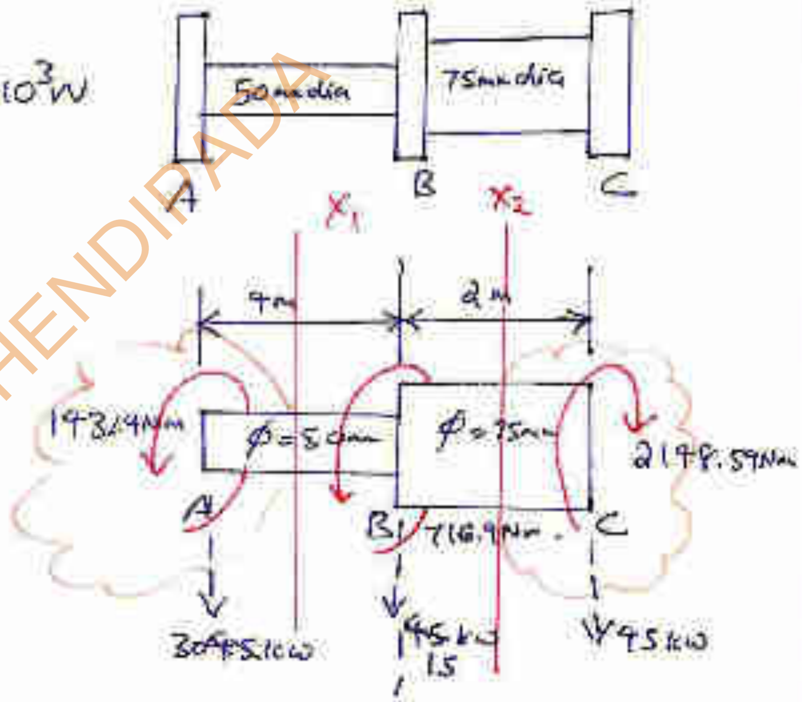


Be No-56 Q3.4

9 N = 200 rpm

$P_A = 30\times10^3 W$; $P_B = \overset{1}{4}5\times10^3 W$

$P_C = 45\times10^3 W$



$P_B = 15\times10^3 W$; $P = \frac{2\pi NT}{60}$

$15\times10^3 = \frac{2\pi\times200\times T_B}{60}$

$T_B = 716.19$ Nm

$P_A = 30\times10^3 W$; $T_A = 2\times T_B$

$T_A = 1432.4$ Nm

$P_C = 45\times10^3 W$; $T_C = 3\times T_B$.

$T_C = 2148.59$ Nm

$$\tau_{x_1} = \frac{16\times T_{x_1}}{\pi\times d_{x_1}^3} = \frac{16\times|1432.4|\times10^3}{\pi\times50^3} = 58.36 \text{ MPa}.$$

$$\tau_{x_1} = \frac{16\times T_{x_2}}{\pi\times d_{x_2}^3} = \frac{16\times|-2148.59|\times10^3}{\pi\times75^3} = 25.96 \text{ MPa}$$

$$\theta_{AC} = \theta_{AB} + \theta_{BC}$$

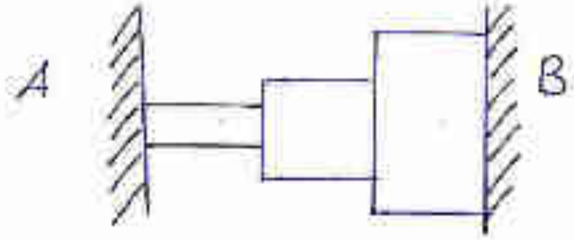
$$\theta_{AC} = \frac{T_1 L_1}{G_1\times\frac{\pi d^4}{32}} + \frac{T_2 L_2}{G_1\times\frac{\pi d^4}{32}}$$

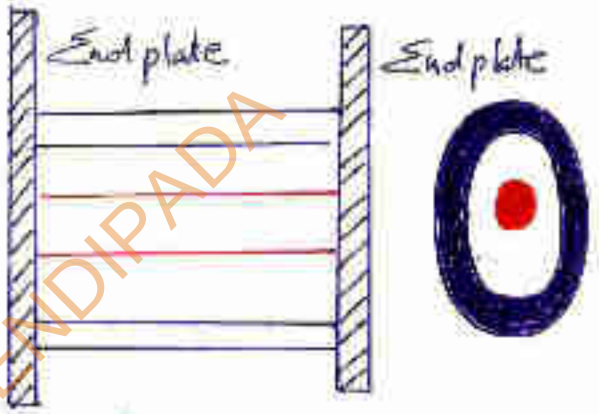
$$\theta_{AC} = \left(\frac{1432.4 \times 4000}{85 \times 10^4 \times \frac{\pi \times 50^4}{32}} + \frac{2148.59 \times 2000}{8.5 \times 10^4 \times \frac{\pi \times 75^4}{32}} \right) \times 10^3$$

$$\theta_{AC} = \left(1.09 \times 10^{-4} + 1.62 \times 10^{-5} \right) \times 10^3 = 0.1252^c$$

i.e. $\theta_{AC} = \underline{\underline{7.17^\circ}}$

2.5. Statically Indeterminate Shafts:-



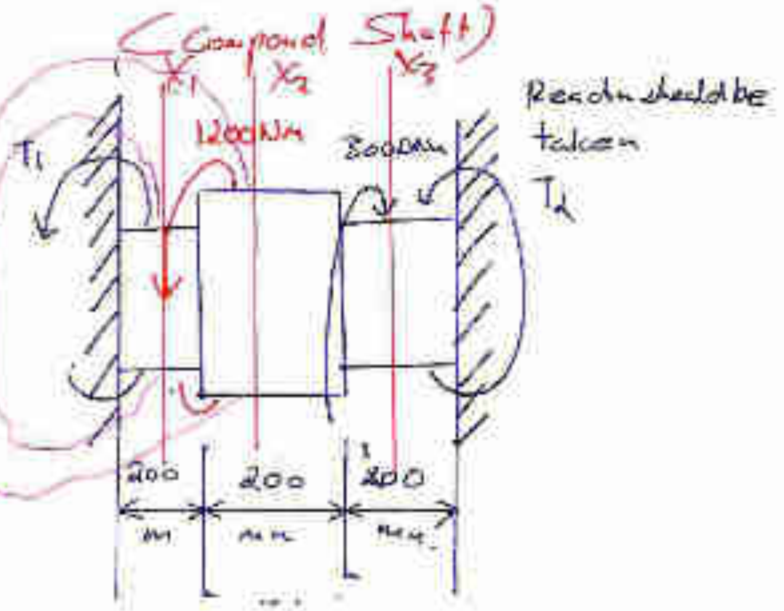


$(\theta)_{overall} = 0$

$(\theta_3) = (\theta_4)$

? Determine the maximum Shear stress? $d_1 = 30$, $d_2 = 50$, $d_3 = 45$

— Reaction T_1 & T_2, any direction can be taken, if you the direction



Reaction should be taken

Σ gm = $\Sigma M = 0$.

$+T_1 + 1200 + T_2 = 3000$.

$T_1 + T_2 = 1800$

$T_{x_1} = -T_1$

$T_{x_1} = -T_1 - 1200$

$T_{x_3} = -T_1 - 1200 + 3000$

$T_{x_3} = -T_1 + 1800$

Comp: $(\delta\theta)_{overall} = 0$.

$$\delta\theta_1 + \theta_2 + \theta_3 = 0$$

$$\frac{T_1 L_1}{GJ_1} + \frac{T_2 L_2}{GJ_2} + \frac{T_3 L_3}{GJ_3} = 0$$

$$\frac{-T_1 \times \cancel{200 \times 10^3}}{G \times \frac{\pi \times (30)^4}{32}} + \frac{(-T_1 - 1200)\, \cancel{100 \times 10^3}}{G \times \frac{\pi \times 50^4}{32}} + \frac{(-T_1 + 1800) \times \cancel{10^3 \times 100}}{G \times \frac{\pi \times 95^4}{32}} = 0$$

$$\frac{-T_1 \times 2}{\cancel{G} \times \frac{\cancel{\pi} \times (30)^4}{\cancel{32}}} + \frac{(-T_1 - 1200) \times 2}{\cancel{G} \times \frac{\cancel{\pi} \times 50^4}{\cancel{32}}} + \frac{(-T_1 + 1800)}{\cancel{G} \times \frac{\cancel{\pi} \times 95^4}{\cancel{32}}} = 0$$

$$\frac{-2T_1}{G \times \frac{\pi \times (30)^4}{32}} + \frac{-2T_1 - 2400}{G \times \frac{\pi \times 50^4}{32}} + \frac{-T_1 + 1800}{G \times \frac{\pi \times 95^4}{32}} = 0$$

$$\frac{-2.51 \times 10^{-5} T_1}{G} + \frac{(-2T_1 - 2400)\, 1.62 \times 10^{-6}}{G} + 2.48 \times 10^{-6} \frac{(-T_1 + 1800)}{G} = 0$$

$$-2.51 \times 10^{-5} \times T_1 - 3.24\, T_1 \times 10^{-6} - 3.88 \times 10^{-6} + -2.48 \times 10^{-6} T_1 + 4.464 \times 10^{-3} = 0$$

$$-3.08 \times 10^{-5} T_1 = -4.46012 \times 10^{-3}$$

$$T_1 =$$

$$\frac{-2T_1}{(30)^4} + \frac{(-2T_1 - 2400)}{50^4} + \frac{(-T_1 + 1800)}{95^4} = 0$$

$$-2.469 \times 10^{-6} T_1 + 1.6 \times 10^{-7}(-2T_1 - 2400) + 2.44 \times 10^{-7}(-T_1 + 1800) = 0$$

$$-2.469 \times 10^{-6} T_1 - 3.2 \times 10^{-7} T_1 - 2.44 \times 10^{-7} T_1 = + 3.84 \times 10^{-5}$$

$$-3.033 \times 10^{-6} T_1 = -3.84 \times 10^{-5}$$

$$\frac{-T_1 \times \cancel{200 \times 10^3}}{G \times \frac{\pi \times 30^{3+1}}{32}} + \frac{(-T_1 - 1200) \times \cancel{100} \times 10^3}{G \times \frac{\pi \times 50^{3+1}}{32}} + \frac{(-T_1 - 1200 + 3000) \times 100 \times 10^3}{G \times \frac{\pi \times 95^{3+1}}{32}} = 0$$

$$\frac{-2T_1}{30^4} + \frac{(-T_1 - 1200) \times 2}{50^4} + \frac{(-T - 1800)}{95^4} = 0.$$

$$-2.469 \times 10^{-6} T_1 + 3.2 \times 10^{-7}(-T_1 - 1200) + 2.44 \times 10^{-7}(-T_1 + 1800) = 0.$$

$$(-2.469 \times 10^{-6} - 3.2 \times 10^{-7} - 2.44 \times 10^{-7}) T_1 = 3.84 \times 10^{-4} - 4.392 \times 10^{-4}$$

$T_1 = 18.19$ Nm ; $T_2 = 1781.81$ Nm

$$\tau_1 = \frac{16 \times -18.19 \times 10^3}{\pi \times 30^3} = |-3.43| = 3.43\,MPa$$

$$\tau_2 = \frac{16\,|-18.19-1200| \times 10^3}{\pi \times 50^2} = |-49.63| = 49.63\,MPa$$

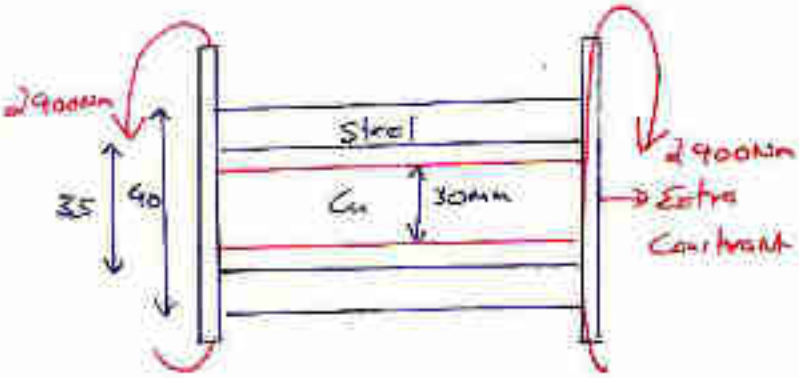
$$\tau_3 = \frac{16\,|-1781.81)| \times 10^3}{\pi \times 45^3} = |-99.58| = 99.58\,MPa$$

Q A solid copper shaft of diameter 30mm is enclosed with in a hollow steel shaft of inner dia 35mm and outer 40mm. respectively. The assembly is then closed at the ends and is subjected to a twisting moment of 2900Nm. If G_{steel} is $2 \times G_{Cu}$ determine

i) Max. stress in Cu shaft?

ii) Max and min stress in steel shaft?

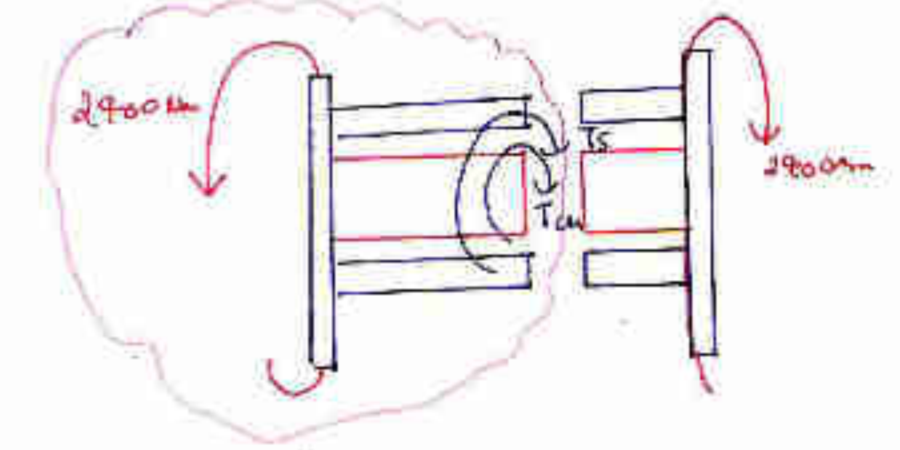
—



i) $(\tau_{Cu})_{max} = \frac{16\,T_{Cu}}{\pi d^3}$.

$(\tau_s)_{max} = \frac{16\,T_s}{\pi d_o^3 (1-\beta^4)}$

Eqm: $T_s + T_{Cu} = 2900$ Nm



Comp: $\theta_S = \theta_H$

$$\frac{T_S \times \cancel{L_S}}{G_S \times J_S} = \frac{T_{cu} \cdot \cancel{L_{cu}}}{G_{cu} \times J_{cu}}.$$

$$\frac{T_S}{\frac{\pi}{32}(40^4-35^4)2\times \cancel{G_{cu}} \times \cancel{\frac{\pi\times 30^4}{32}}} = \frac{T_{cu}}{\cancel{G_{cu}}\, \cancel{\frac{\pi d 30^4}{32}}}$$

$$\frac{T_S}{2\times(40^4-35^4)} = \frac{T_{cu}}{30^4}$$

$$T_S = 2.615\, T_{cu}$$

$$\therefore\ 2.615\, T_{cu} + T_{cu} = 2400 \times 10^3$$

$$T_{cu} = \underline{\underline{663.90 \text{ Nm}}}$$

$$T_S + 663.90 = 2900$$

$$T_S = \underline{\underline{1736.09 \text{ Nm}}}$$

$$(\tau_{max})_{cu} = \frac{16 \times T_{cu}}{\pi \times 30^3}$$

$$(\tau_{max})_{cu} = \frac{16 \times 663.90 \times 10^3}{\pi \times 30^3}$$

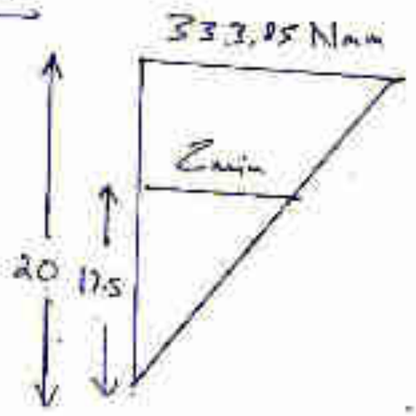
$$(\tau_{max})_{cu} = \underline{\underline{125.23 \text{ N/mm}^2}}$$

$$(\tau_{max})_S = \frac{16 \times T_{S}}{\pi d_o^3 (1-\beta^4)} \qquad \beta = \cancel{\frac{40}{}}\ \frac{35}{40} = 7/8$$

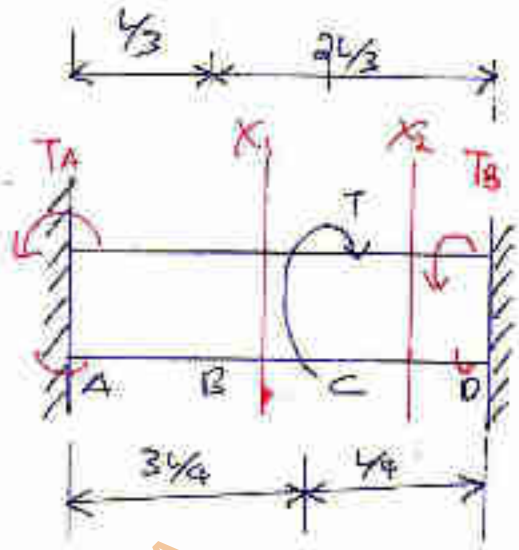
$$(\tau_{max})_S = \frac{16 \times 1736.09}{\pi \times 40^3 (1-(7/8)^4)} = \underline{\underline{333.85 \text{ N/mm}^2}}$$

$$\frac{333.85}{20} = \frac{\tau_{min}}{17.5}$$

$$(\tau_{min})_S = \underline{\underline{292.136 \text{ N/mm}^2}}$$



#1. Q If the angle of twist b/w B and D are ϕ, the angle of twist at section C w.r.t to D is ——



Eqn: $T = T_A + T_B$

Comp: $(\theta_{overall}) = 0$

$$\theta_{AC} + \theta_{CD} = 0$$

$$\frac{T_{X_1} \cdot L_{AC}}{(GJ)_{AC}} + \frac{T_{X_2} \cdot L_{CD}}{(GJ)_{CD}} = 0$$

$$\frac{-T_A \times 3L/4}{(GJ)_{AC}} + \frac{(-T_A + T) \times L/4}{(GJ)_{CD}} = 0$$

$-T_A = T_{X_1}$

$-T_A + T = T_{X_2}$.

$$-T_A \times \cancel{3L/4} - T_A \times \cancel{L/4} + T\,\cancel{L/4} = 0$$

$$-3T_A - T_A + T = 0.$$

$$+4T_A = T$$

$$T = 4T_A.$$

$$\underline{\underline{T_A = T/4}}$$

$$T = T_A + T_B$$

$$\therefore T = T/4 + T_B$$

$$T_B = T - T/4 = \underline{\underline{\frac{3T}{4}}}$$

$$\theta_{CD} = \frac{(-T_A + T) \times L/4}{GJ}$$

$$\theta_{CD} = \frac{(-T/4 + T) \times L/4}{GJ}$$

$$\theta_{CD} = \frac{\frac{3T}{4} \times L/4}{GJ} = \underline{\underline{\frac{3TL}{16GJ}}}$$

$$\theta_{CD} = \frac{3TL}{16GJ}.$$

$$\theta_{BD} = \theta_{BC} + \theta_{CD}$$

$$\theta_{BD} = \frac{-T_A \cdot L_{BC}}{GJ} + \frac{(-T/4 + T) \times L_{CD}}{GJ}$$

$$\theta_{BD} = \frac{-T/4 \times 5L/12}{GJ} + \frac{3/4 \cdot T \times \frac{L}{4}}{GJ}$$

$$\theta_{BD} = \frac{-5TL}{48GJ} + \frac{9TL}{48GJ}$$

$$\theta_{BD} = \frac{4TL}{48GJ}$$

Since $\theta_{BD} = \phi$

$$\phi = \frac{3}{3} \times \frac{4TL}{48GJ}.$$

$$\phi = \overset{\theta_C}{\frac{3TL}{16GJ}} \times \frac{4}{9}$$

$$\phi = \theta_C \times 4/9$$

$$\underline{\underline{\theta_C = 9\phi/4}}$$

20/7/2017

2.6 Shaft Strength:-

The maximum twisting moment that a shaft of given material and dimensions can resist safely is known as the strength of that shaft.

Safe: $\tau_{max} \le \hat{\tau} \longrightarrow$ Allowable shear stress / Material /Yield value } By lab testing

$$\frac{T}{J} \cdot r_{max} \le \hat{\tau}$$

$$T \le \frac{\hat{\tau} . J}{r}$$

$$T \le \hat{\tau} . \frac{J}{r_{max}} \rightarrow Z_p$$

T → Allowable load

$\hat{\tau}$ → Material

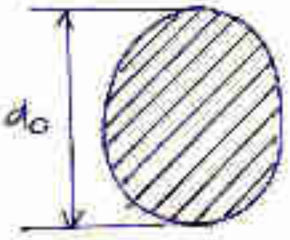
$\frac{J}{r_{max}}$ → Cross-sectional dimensions. /called as "Section Modulus" or "Polar sections modulus"

$$\boxed{Z_p = J/r_{max}}$$

a) Same material - Same cross-sectional area - Same length → Solid shaft & Hollow shaft

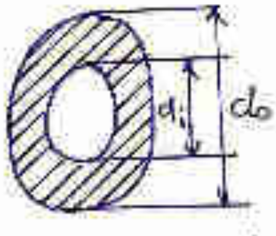
*

Same cross sectional area
\+
Same length
= Same weight



$$J_S = \frac{\pi d^4}{32}$$

$$r_{max} = \frac{d}{2}$$



$$J_H = \frac{\pi}{32}(d_o^4 - d_i^4)$$

$$J_H = \frac{\pi}{32} d_o^4 \left(1 - \frac{d_i^4}{d_o^4}\right)$$

$$J_H = \pi/_{32} \, d_o^4 (1 - \beta^4)$$

$$r_{max} = \frac{d_o}{2}$$

$$\frac{\hat{T}_H}{\hat{T}_S} = \frac{\cancel{\hat{\tau}_H} \times \frac{J_H}{(r_{max})_H}}{\cancel{\hat{\tau}_S} \times \frac{J_S}{(r_{max})_S}}$$

Same material.

$$\frac{\hat{T}_H}{\hat{T}_S} = \frac{\frac{\frac{\pi}{32} d_o^4 (1-\beta^4)}{d_o/2}}{\frac{\frac{\pi d^4 C}{32}}{d/2}}$$

$$\boxed{\frac{\hat{T}_H}{\hat{T}_S} = \frac{d_o^3 \times (1-\beta^4)}{d^3}}$$

Same cross sectional area

$$A_H = A_S$$

$$\frac{\pi d_o^2}{4}(1-\beta^2) = \frac{\pi}{4} d^2$$

$$d_o^2(1-\beta^2) = d^2$$

$$\frac{d_o^2}{d^2} = \frac{1}{(1-\beta^2)}.$$

$$\frac{d_o}{d} = \left[\frac{1}{(1-\beta^2)}\right]^{1/2}$$

$$\Rightarrow \frac{\hat{T}_H}{\hat{T}_S} = \left[\frac{1}{(1-\beta^2)}\right]^{1/2 \times 3} \times [1-\beta^4]$$

$$\frac{\hat{T}_H}{\hat{T}_S} = \frac{[1-\beta^4]}{[1-\beta^2]^{3/2}}$$

$$\frac{\hat{T}_H}{\hat{T}_S} = \frac{[1-\beta^4]}{[1-\beta^2]^{3/2}}$$ i.e. always $\hat{T}_H > \hat{T}_S$

$\hookrightarrow \hat{T}_H > \hat{T}_S$ [hollow shaft stronger]

Avg. stress is more in hollow than solid.

Same material from solid ... [illegible]

Hollow ... [illegible]

- Apply the ... [illegible]

[illegible]

b) Same material + Same strength [Same Max. τ] + Same length.

$$\frac{W_H}{W_S} = \frac{M_H \cdot g}{M_S \cdot g} = \frac{\cancel{\rho} \cdot V_H \cdot \cancel{g}}{\cancel{\rho} \cdot V_S \cdot g}$$

$$\frac{W_H}{W_S} = \frac{A_H \cdot \cancel{L}}{A_S \cdot L} = \frac{\frac{\pi}{\cancel{4}} d_o^2 (1-\beta^2)}{\cancel{\pi/4} \cdot d^2}$$

$$\frac{W_H}{W_S} = \left(\frac{d_o}{d}\right)^2 (1-\beta^2)$$

$$\hat{T}_H = \hat{T}_S$$

$$\cancel{\tau_H} \frac{J_H}{(r_{max})_H} = \cancel{\tau_S} \cdot \frac{J_S}{(r_{max})_S}$$

$$\frac{\frac{\cancel{\pi}}{32} d_o^4 (1-\beta^4)}{d_o/2} = \frac{\frac{\cancel{\pi}}{32} d^4}{d/2}$$

$$d_o^3(1-\beta^4) = d^{4-1}$$

$$d_o^3(1-\beta^4) = d^3$$

$$\frac{d_o}{d} = \left[\frac{1}{1-\beta^4}\right]^{1/3}$$

$$\frac{W_H}{W_S} = \left[\frac{1}{1-\beta^4}\right]^{1/3 \times 2}(1-\beta^2)$$

$$\frac{W_H}{W_S} = \left[\frac{1}{1-\beta^4}\right]^{2/3}(1-\beta^2)$$

$$\frac{W_H}{W_S} = \frac{[1-\beta^2]}{[1-\beta^4]^{2/3}}$$

$$\frac{W_H}{W_S} = \frac{[1-\beta^2]}{[1-\beta^4]^{2/3}}$$

So weight of hollow will be less than solid shaft

c) Same twisting moment + Same cross sectional area:

$$\frac{(\tau_{max})_H}{(\tau_{max})_S} = \frac{\frac{16T}{\pi d_o^3(1-\beta^4)}}{\frac{16T}{\pi d^3}}$$

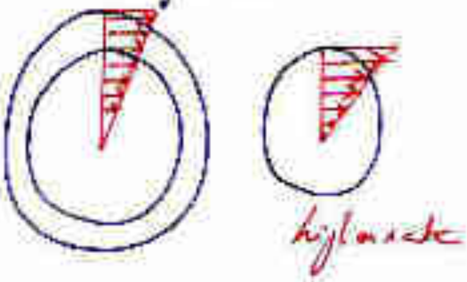
$$\frac{(\tau_{max})_H}{(\tau_{max})_S} = \frac{d^3}{d_o^3(1-\beta^4)}$$

$$\frac{(\tau_{max})_H}{(\tau_{max})_S} = \frac{1}{\left[\frac{1}{(1-\beta^2)}\right]^{3/2}} \cdot \frac{1}{(1-\beta^4)}$$

$$\frac{(\tau_{max})_H}{(\tau_{max})_S} = \frac{(1-\beta^2)^{3/2}}{(1-\beta^4)}$$

$$\frac{(\tau_{max})_H}{(\tau_{max})_S} = \frac{(1-\beta^2)^{3/2}}{(1-\beta^4)}$$

τ increase linearly so that and increase ate higher side in smaller shaft. higher rate the τ will increase.



∴ Hollow shaft will be more safe.

d) Same material + Same torque + Same length + Same Cross Section.

Keep in mind:- As the area of cross section so that the outside diameter of hollow will be more than the solid so that the area increases thus rises the polar moment of inertia. i.e Area increases J also increases.

$$\frac{\theta_H}{\theta_S} = \frac{\frac{TL}{\cancel{G} J_H}}{\frac{TL}{\cancel{G} J_S}} = J_S/J_H$$

So that the angle of twist is lesser or the, deformation will be lesser.

→ "Why hollow shaft not considered as normal case?"

More friction so that due to more outer velocity than the hollow shaft the friction is always there.

So that there will more mass moment of inertia away from the centre. So that it will require more torque carried by the shaft.

Same practical torque used to rotate the shaft so it will be used I_{xx} + friction.

e) Same material + Same Outer Diameter:

$$\frac{T_H}{T_S} = \frac{d_o^3}{d^3}(1-\beta^4)$$

Here $d_o = d$

$$\frac{T_H}{T_S} = (1-\beta^4)$$

So that the solid shaft will transfer more value.

3-16-55 Q2/9

$\tau = \frac{16T}{\pi d^3}$

$T = \frac{\pi d^2}{\tau 16}$

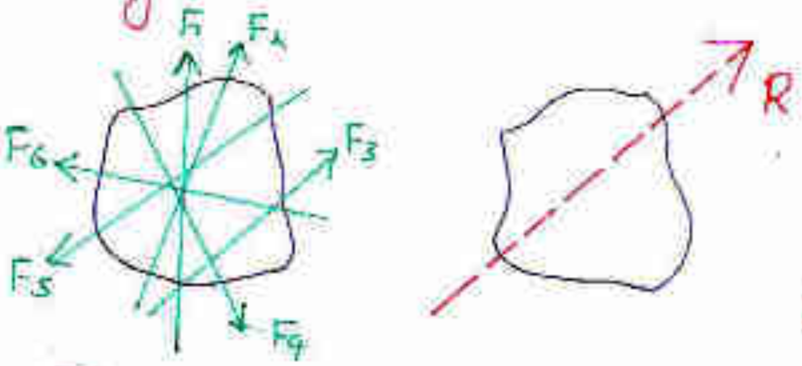
$\frac{T_{hol}}{T_{so}} = (1-\beta^4) = \left(1-\left(\frac{1}{2}\right)^4\right) = 1-1/16 = 15/16$

$\frac{T_s}{T_h} = \frac{16}{15}$

3. SHEAR & BENDING:

3.1 Centroid / Distributed Load:

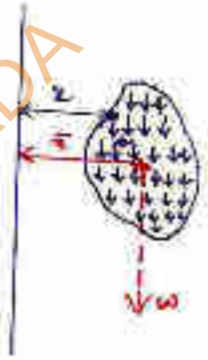
* Varignon's theorem:-



$$F_1 x_1 + F_2 x_2 + \ldots = R.d$$

$$\begin{pmatrix}\text{Moment of}\\ \text{Torque}\end{pmatrix} = \begin{pmatrix}\text{Total of}\\ \text{Moments}\end{pmatrix}$$

a. Centroid



Varig.T

No differance b/w C.G in and C.M because of g ① and ② chge with. g. But with the 'g' only C.G exists. Otherwise C.M. exist.

Outer space - C.M.

C.g not relevent w/o C.M

Centre of gravity:

$$W.\bar{x} = \int x\,dw \longrightarrow \boxed{C.G}$$

$$Mg\bar{x} = \int x.g\,dm$$

Centre of Mass : $M\bar{x} = \int x.dm \longrightarrow \boxed{C.M}$

$$\rho\bar{x}v = \int x.\rho.dv$$

$$v\bar{x} = \int x.dv$$ — depend on Shape and size of body

C of G
C of M
C of V
C of A

} Varignon's theorem

Centroid of volume: $V.\bar{x} = \int x.dv$

Sometime Volume is smaller so they are seen are total area × thickness. There are called plane bodies.

$$A \times t\,\bar{x} = \int x\,t \times dA$$

$$A\bar{x} = \int x\,dA$$

Moment of total area = Moment of individual area.

Centroid of area : $A\bar{x} = \int x.dA$

* V.T → $A.\bar{x} = \int x.dA$ → is representing a "force". Entire moment represents the total force.

* $\begin{pmatrix}\text{Moment of} \\ \text{an area}\end{pmatrix} = \int x.dA$

$$= \frac{1}{\rho t}\int \rho.t.x\,dA$$

$$= \frac{1}{\rho t}\int \rho x\; t\,dA$$

$$= \frac{1}{\rho t g}\int x.\,dm$$

$$= \frac{1}{\rho g t}\int x.\,dm. \xrightarrow{} \text{Represents the force.}$$

Eg: $\text{Moment of an area} = \int y.dyA$

$$= \frac{1}{\rho g}\int \rho g.y\,dyA$$

$$= \frac{1}{\rho g}\int p.dyA$$

$$= \frac{1}{\rho g}\int p\,dA \rightarrow \text{force}$$

* Value of $\bar{x}$ if reference axis passes through centroid

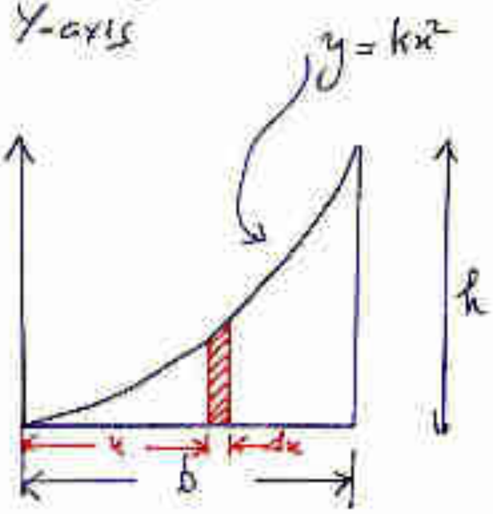
i.e. $\bar{x} = 0$ ∴ $A\overset{0}{\cancel{\bar{x}}} = \int x.dA$

∴ $\int x.dA = 0$ → First moment of area.

⟹ If an axis happens to pass through Centroid then first moment of area about that axis will be "Zero".

Conversly, if first moment of area is "Zero" about an axis then the axis will pass through the "Centroid".

Q The centroid of location of area bounded by parabolic curve $y = kx^2$ shown in figure with respect to Y-axis



According Varignon theorem.

$$A \cdot \bar{x} = \int x \cdot dA$$

No area is known then that the A will be integrated.

$$\bar{x} \cdot \int dA = \int x \cdot dA$$

$$\bar{x} \int y \, dx = \int x \cdot y \, dx.$$

$$\bar{x} \int_0^b \cancel{k}x^2 dx = \int_0^b \cancel{K}x^2 \cdot x \, dx$$

$$\bar{x} \int_0^b x^2 dx = \int_0^b x^3 dx.$$

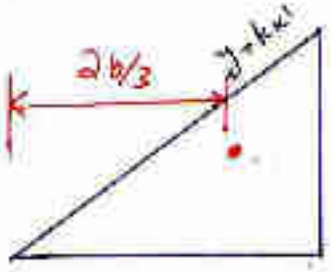
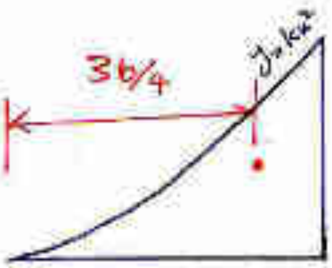
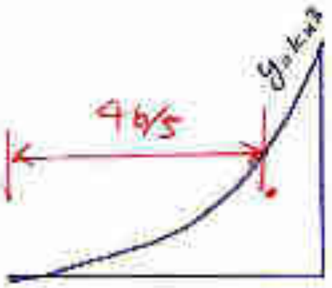
$$\bar{x} \cdot \left[\frac{x^3}{3}\right]_0^b = \left[\frac{x^4}{4}\right]_0^b.$$

$$\bar{x} \cdot \frac{b^3}{3} = \frac{b^4}{4}.$$

$$\cancel{\bar{x}} \times \frac{\cancel{b^4}}{\cancel{4}} = \frac{\cancel{b^3}}{\cancel{3}}$$

$$\cancel{\bar{x}} = \frac{b^4}{4} - \frac{b^3}{3}$$

$$\bar{x} = \frac{3b}{4}$$

	Area	$\bar{x}$
	$\frac{1}{2}bh$	$\frac{2b}{3}$
	$\frac{1}{3}bh$	$\frac{3b}{4}$
	$\frac{1}{4}bh$	$\frac{4b}{5}$

? Locate of centroid of figure about base.

— $A\bar{x} = \int x\,dA$

Total area = (20×120) + (20×100) + (80×20)

A = 2400 + 2000 + 1600

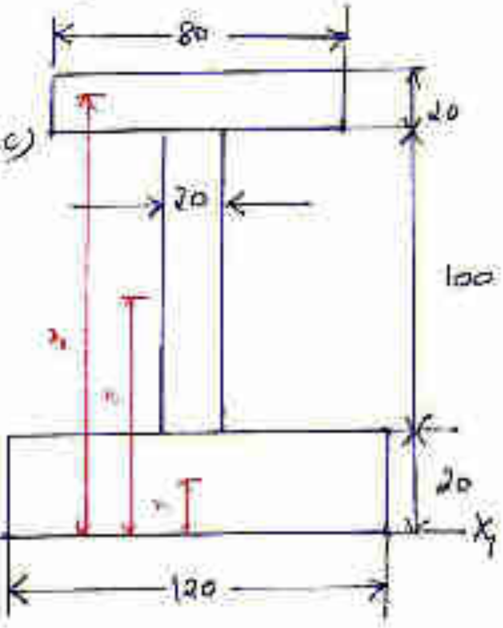
A = 6000

$6000.\bar{x} = \int x_1 dA_1 + \int x_2 dA_2 + \int x_3 dA_3$

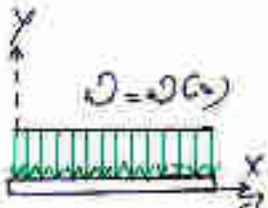
$6000.\bar{x} = \frac{20}{2}\times 2400 + \left(20+\frac{100}{2}\right)\times 2000 + \left(20+100+\frac{20}{2}\right)\times 1600$

6000.$\bar{x}$ = 24000 + 140000 + 208000

$\bar{x}$ = 62 mm from base



b. Distributed load:



Rate of loading

UDL [ω = C] = (Load intensity) = (Height of load intensity curve)

→ To determine the total load and where it is acting? W = ?; $\bar{x}$ = ?

at dx → 0, can be assumed as rectangle

$dA_{LIC} = \omega dx$

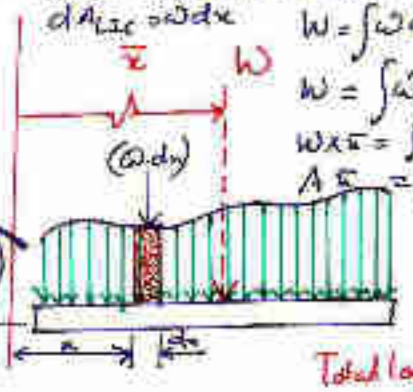
$W = \int \omega dx = \int dA_{LIC}$

$W = \int \omega dx = A_{LIC}$

$W \times \bar{x} = \int x(\omega dx)$

$A\bar{x} = \int x\, dA$

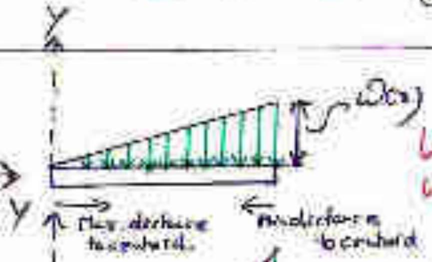
Centroid



Total load W = A_{LIC}

$W = \int \omega dx = A_{LIC}$

Resultant force acts at centroid of LIC area.

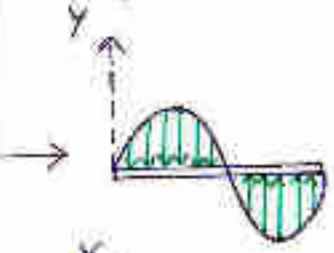


UVL = [ω = Ax]

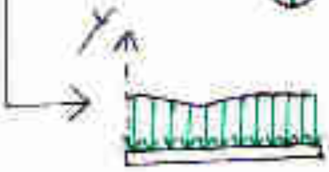
Uniformly varying load

PVL [ω = kx²]

Parabolically varying load



[ω = k sin x]



[ω = f(x)]

Total load $W = \int \omega dx = A_{LIC}$

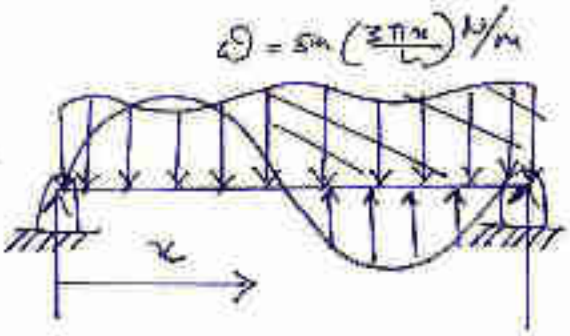
Resultant force acts at the "Centroid" of LIC Area

$\int A\bar{x} = \int x dA.$

Resultant of a distributed load acts at the "centroid" of LIC area

Gate 2014

Q. A simply supported beam of L length is subjected to a distributed load $\omega = \sin\left(\frac{3\pi x}{L}\right)$ N/m, where x is the distance in m measured from left support. Determine the reaction at the right support?



$$\omega = \sin\left(\frac{3\pi x}{L}\right)$$

$$W = \int_0^L \sin\left(\frac{3\pi x}{L}\right) dx$$

$$W = \left[\frac{-\cos\frac{3\pi x}{L}}{3\pi/L}\right]_0^L$$

$$W = \frac{L}{3\pi}\left[-\cos\overset{3\pi}{(\theta)} + \cos\frac{3\pi \times 0}{L}\right]$$

$$W = \frac{2L}{3\pi}$$

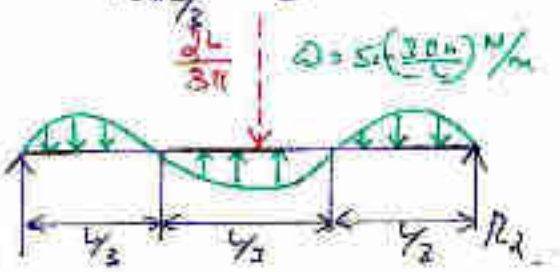
$$\omega = \sin\frac{3\pi x}{L}$$

$$\omega_{x=0} = 0$$

$$\omega_{\sin L}{}_{x=} = 0$$

$$\omega_{x=L/3} = 0$$

$$\omega_{x=2L/3} = 0$$



C.G will be at the centre of the beam.

So $R_1 + R_2 = \frac{2L}{3\pi}$

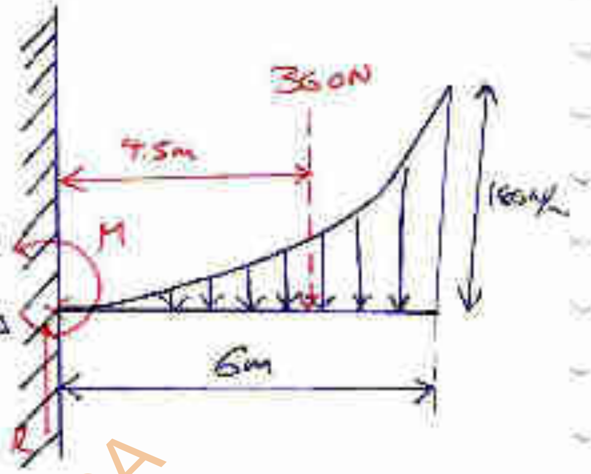
i.e. $R_1 = R_2$

$$2R_1 = \frac{2L}{3\pi}$$

$$R_1 = \frac{2L}{6\pi} = \frac{L}{3\pi}$$

$$R_2 = \frac{2L}{3\times 2\times \pi} = \frac{L}{3\pi}$$

Q A Cantilever beam a is subjected to a distributed load of $\omega = 5x^2$ over a length of 6m where x is the distance measured from the fixed support at the left end. determine the reaction force and the reaction moment at the fixed support?



$\omega = 5x^2$.

$W = \int_0^6 5x^2 \, dx$.

Area $= \frac{1}{3} \times 6 \times 180$

Area $= \underline{\underline{360N}}$

$W = \left[\frac{5x^3}{3}\right]_0^6$

$W = \frac{5 \times 6 \times 6 \times 6}{3}$

$W = \underline{\underline{360 N}}$

at $x = 0$: $\omega = 0$ N/m

at $x = 6$: $\omega = 180$ N/m.

hinged support there will be θ. Rotation can be there.

$A.\bar{x} = \int x \, dA$

$\bar{x} = \frac{3}{4} \times 6$.

$\bar{x} = \frac{3}{4} \times 6 = 4.5m$ from A

ΣF_y ; $\underline{\underline{360 N = R}}$

ΣM_A ; $M = 360 \times 4.5 = \underline{\underline{1620 Nm}}$ (cw)

21/7/2017

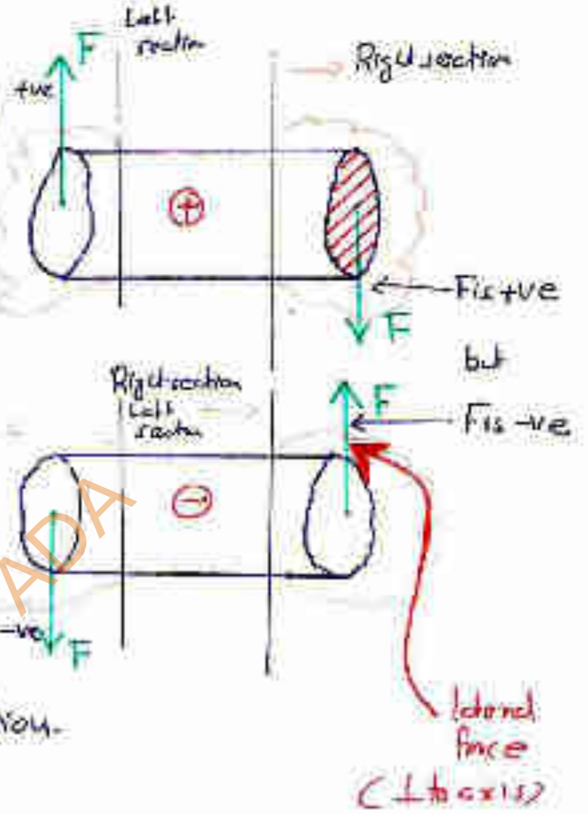
3.2 SHEAR FORCE & BENDING MOMENT:-

a. SHEAR FORCE:-

Left section : +ve ↑ -ve ↓
Right section : +ve ↓ -ve ↑

Equal and opposite force acting "parallel" to the cross sectional area can make shear loading or shear Force

Shear is Force at a section is the algebraic sum of all the lateral forces acting on either side of that section.



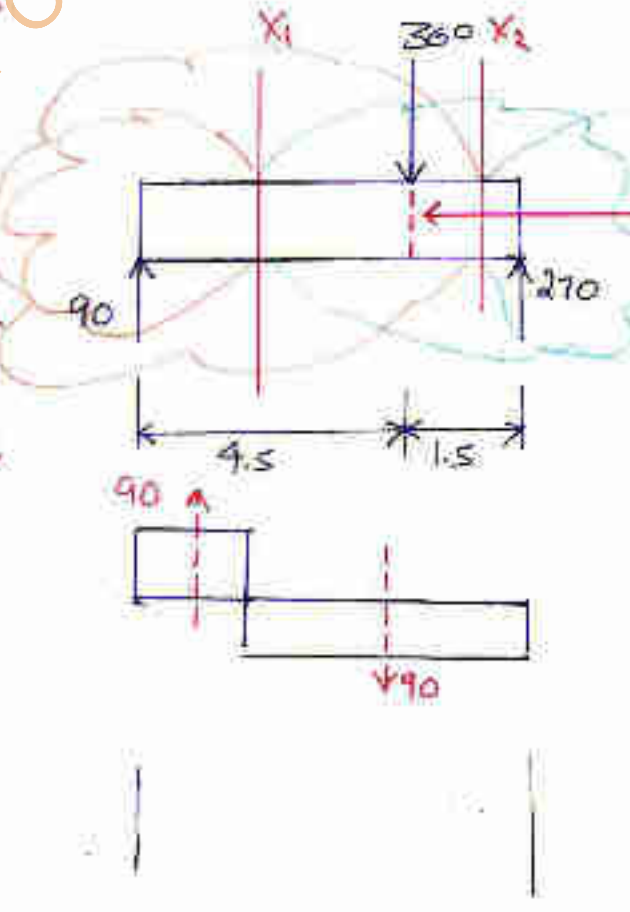
Eg: $SF_{x_1} = +90$ (left)↑

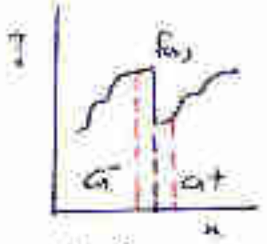
$SF_{x_{2-}} = +360 - 270$

$SF_{x_{2+}} = +90$ (Right)↓

$SF_{x_2} = +90 - 360 = -270$↑ (left)

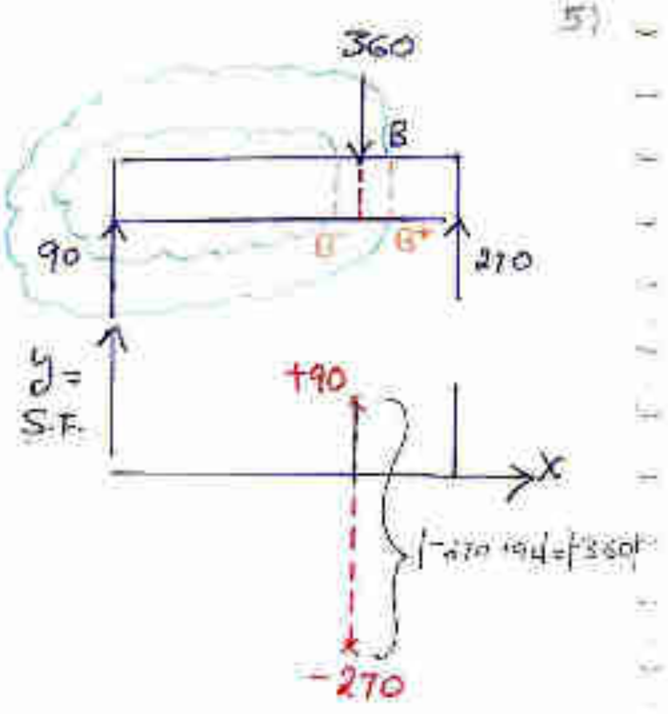
$SF_{x_2} = -270$ (Right)↓

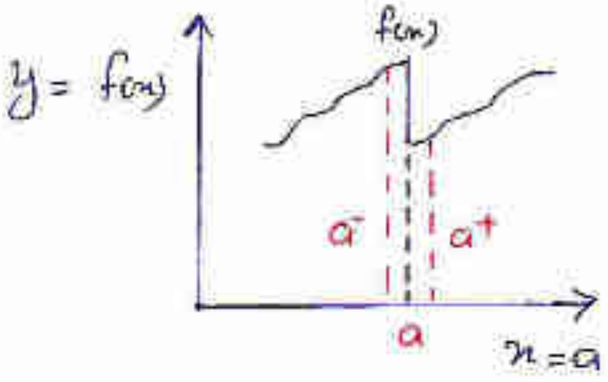




at $x = a$
the function is discontinuous.
f can't be defined,
f can be defined
as LHL $x \to a^-$
RHL $x \to a^+$

At B shear force can't be calculated because consider the following case of calculus.





at x = a ; f(x) is discontinuous. at x = a^+ and x = a^- it can be defined. f has a sudden change at x = a.

∴ There is a sudden change in shear force

Similarly calculate SF at B^-

$F_{B^-} = +90$ (left)

$F_{B^+} = +90 - 360 = -270$ (left)

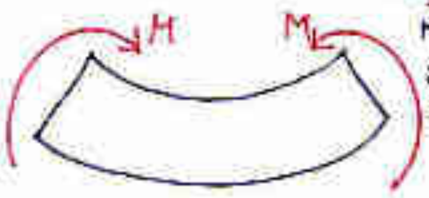
→ At a section where a concentrated load is acting, Shear Force is discontinuous and it undergoes a sudden change in its value.

→ The magnitude of total sudden change of shear force is always equal to the magnitude of "Concentrated load" acting there.

→ At such a section, the shear force value would be calculated "just before" and "just after" the section.

b. BENDING MOMENT:

Statically determinate beams.
I - does not play any role in SFD & BMD of.



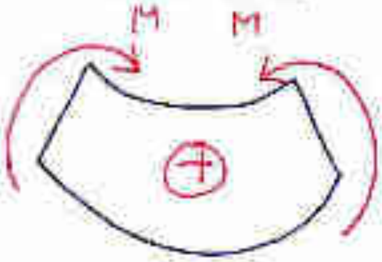
→ D.O.F.
Mechanism >1
Structure = 0
Indeterminate < 0

Equal and opposite moment acting about transverse axis of a member constitute its bending.

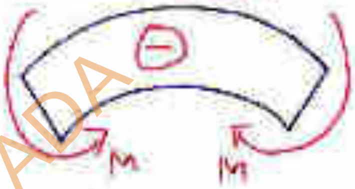
✓ - possible
✗ - impossible configuration.

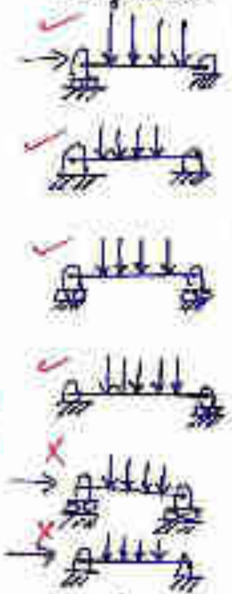
left: CW +ve	ACW -ve
Right: ACW +ve	CW -ve

SAGGING



HOGGING



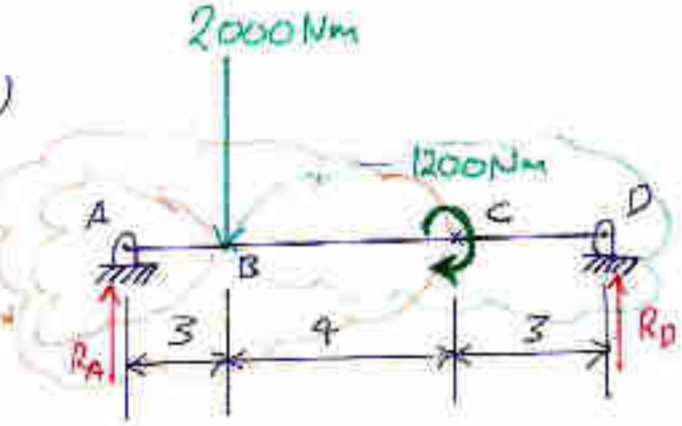


Bending moment at a section is the algebraic sum of all the transverse moments transmitted to that section from either side.

Eg: Using Equilibrium

$R_A + R_D = 2000$ $(\Sigma F_y = 0)$

→ (Never count couple while balance forces)



$\Sigma M_A = 0$.

(should cancel forces be multiplied by distance)

$2000 \times 3 + 1200\cancel{0} = R_D \times 10$

~~15000 = RD×10~~

$R_D =$ ~~1500~~ ~~1280N~~ 720

$R_A = 2000 - 7280 =$ ~~720N~~ 1280N

B.M at B ⇒

$R_A \times 3 = M_B$

$M_B = 1280 \times 3 = 3840\text{ Nm}$ (left)

$M_B = -1200 + 720 \times 7 = 3840\text{ Nm}$ (Right)

Dig calculation of B.M at B C:

$M_C = 1280 \times 7 - 2000 \times 4 = +960$ (left).

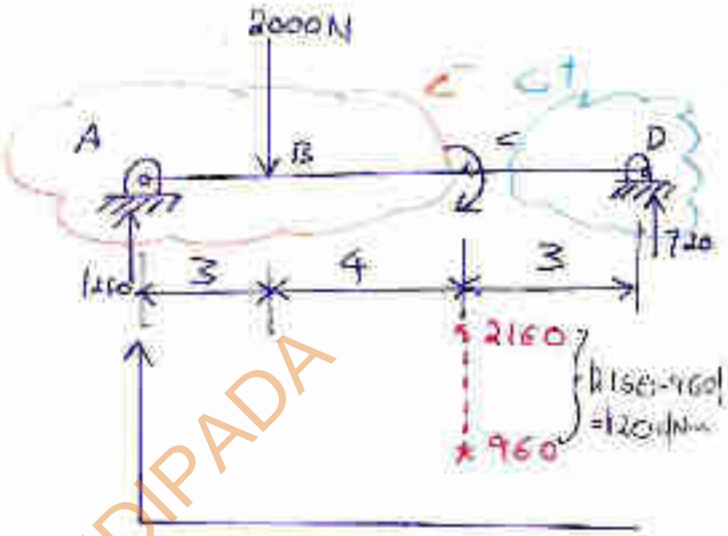
$M_C = 720 \times 3 = +2160$ (Right).

Similarly to that Shear force B.M difference at a point where moment act. So calculate B.M twice. Once before and one after.

Calculate BM about C^-

$M_{C^-} = 1280 \times 7 - 2000 \times 4 = +960$ (C^-)

$M_{C^+} = 720 \times 3 = +2160$ (C^+)



→ Bending moment is discontinous and hence undergoes a sudden change at a section where a concentrated moment is acting

→ The total sudden change in the bending moment value is always equal to the "magnitude" of the couple acting there.

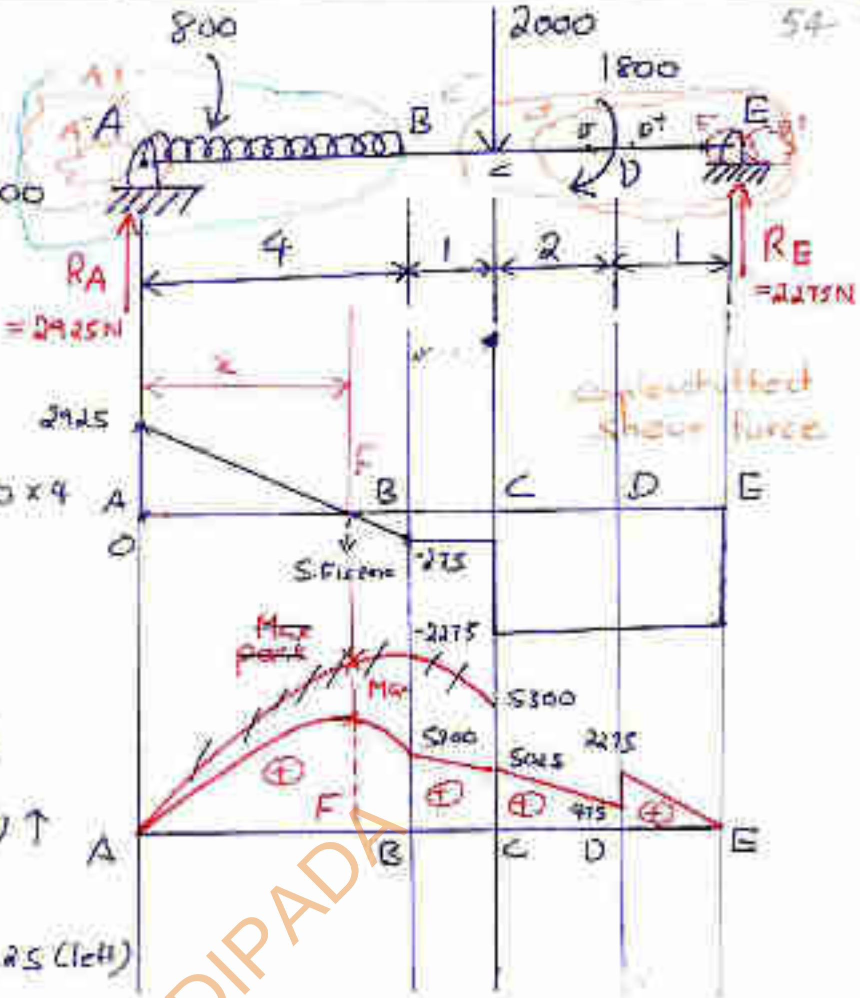
→ B.M. value will be calculated just before and just after such a section.

TYPES OF LOAD	SHEAR FORCE	BENDING MOMENT
$(\omega=0)$ NO LOAD	(x^0) CONSTANT	(x^1) LINEAR
UDL $(\omega = x^0)$	(x^1) LINEARLY	(x^2) PARABOLIC
U.V.L $(\omega = x)$	(x^2) PARABOLIC	(x^3) CUBIC
$\omega = x^n$	$SF \propto x^{n+1}$	$B.M \propto x^{n+2}$

When Shear Force is zero B.M will get "Maximum Value"

Shear Force is zero $\left(\frac{dM}{dx} = 0\right)$

Q $\Sigma M_A = 0$



$800 \times \frac{4}{2} \times 4 + 2000 \times 5 + 1800$

$= R_E \times 8$

$R_E = \overset{22}{1875}$ N

$R_A + R_B = 2000 + 800 \times 4$

$R_A = 2925$ N

Shear Force Diagram:

At A^-; $SF_A = 0$ (left)

At A^+; $SF_A = {}^+2925$ (Right) ↑

Consider B;

At B^+; $SF_B = -(800 \times 4) + 2925$ (left)

$SF_B = -275$ ↓

Consider C;

At C^-; $SF_C = -275$ (left).

$SF_C = -2275 + 2000 = -275$ (right)

At C^+; $SF_C = -2275$ (Right)

Consider E;

At E^-; $SF_{E^-} = -2275$

At E^+; $SF_{E^+} = 0$

Bending Moment Diagram:

At A ; $M_A = 0$

At B ; $M_B = -(2000 \times 1) - 1800 + (2275 \times 4)$

$M_B = {}^+5300$

At C ; $M_C = -1800 + 2275 \times 3 = {}^+5025$ (1)

At D^-, $M_{D^-} = -1800 + 2275 \times 1 = {}^+475$ (Right)

At D^+, $M_{D^+} = 2275 \times 1 = 2275$ (Right)

At E, $M_E = 0$

SFD & BMD starts with zero and end with zero.

In simply supported beam w/o couple always be "+ve" (BMD is +ve)

Assume distance x be A to F where the B.M maximum.

BM

$~~SF~~_F = 2925\times x + {}^{-}800x \cdot x/2.$

at the point SF is zero. $SF = 2925x - 800x = 0$

$0 = 2925\times x + {}^{-}800 \cdot \frac{x^2}{2x}.$ $x = \underline{\underline{3.65m}}$

$800 \cdot \frac{x^{\cancel{2}}}{2\cancel{x}} = 2925\times\cancel{x}$

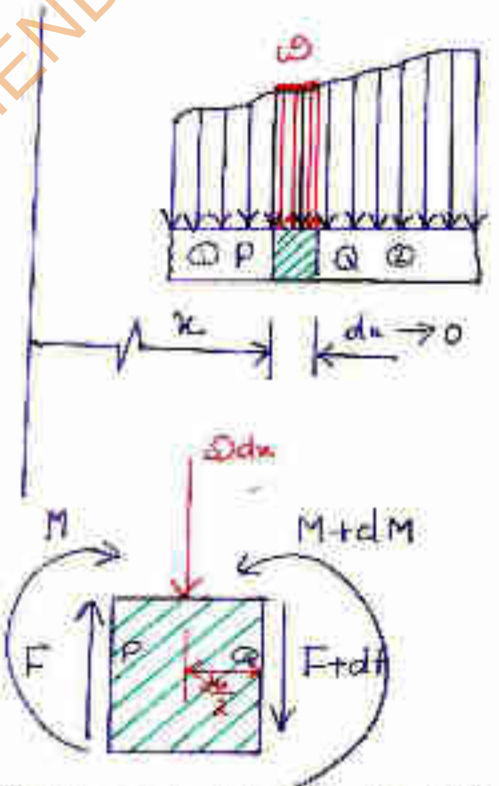
$x = \underline{\underline{3.65m}}$ from A

$M_F = 2925\times x - 800\cdot x\times x/2.$

$M_F = 2925\times 3.65 - 800\times 3.65\times 3.65/2$

$M_F = \underline{\underline{5327.25 Nm}}$

3.3 Relation of SF & B.M:-



On balance Force, $\Sigma F_y = 0$

$\cancel{F} = \omega dx + \cancel{F} + dF$

$\frac{dF}{dx} = -\omega$ ----- (i)

On balance of Moments, $\Sigma M_Q = 0$

$(F dx + \cancel{M}) = (\omega \underbrace{dx \cdot \frac{dx}{2}}_{0} + \cancel{M} + dM)$

$F dx = dM$

$\frac{dM}{dx} = F$ ----- (ii)

Sub. F in (i) $\longrightarrow$

$\frac{d^2M}{dx^2} = -\omega$ ----- (iii)

F- Force applied beam ① on beam PQ

F+dF- Force applied beam ② on beam PQ

M - Moment applied on beam ① on beam PQ

M+dM - Moment applied on beam ② on beam PQ

$\left.\begin{array}{l} \frac{dF}{dx} = -\omega \\ \frac{dM}{dx} = F \\ \frac{d^2M}{dx^2} = -\omega \end{array}\right\}$ For a distributed load acting "along ω" 

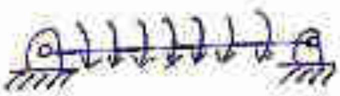
	
$\frac{dF}{dx} = -\omega$	$\frac{dF}{dx} = \omega$
$\frac{dM}{dx} = F$	$\frac{dM}{dx} = F$
$\frac{d^2M}{dx^2} = -\omega$	$\frac{d^2M}{dx^2} = \omega$

(i) $\frac{dM}{dx} = F$; $dM = Fdx$ [F=0 at everywhere]

IF F=0 everywhere M = Constant is "Pure Bending"

Beams can have bending without shear but

Beams don't have Shear force without beamnding



→ Exceptional case where can have don't shear force without bending.

(ii) $\frac{dM}{dx} = F$; $dM = F.dx$ [F=0 at a point]

IF F=0 at a point

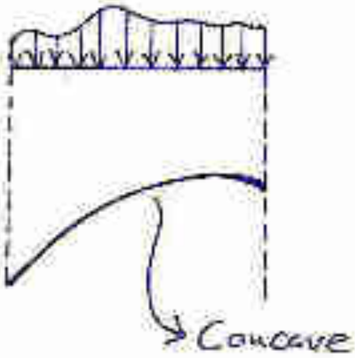
M is "maxima" or "minima"

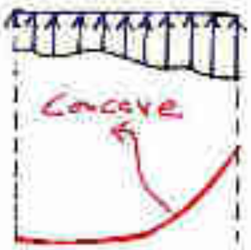
$\frac{d^2M}{dx^2} = -Ve$ ↓ load along g' almost cases

$\frac{d^2M}{dx^2} = +ve$ ↓ Inverted dist. load

(iii)

(iii)





$M'' = -ve$

$\frac{d(M')}{dx} = -ve$

$M' = -ve$ ↓

M' is decreasing so the tangent is decreasing

$M'' = +ve$

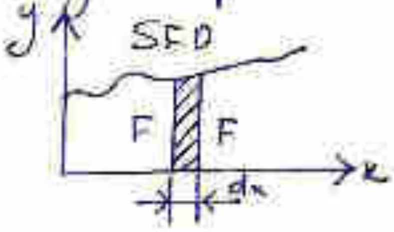
$\frac{d(M')}{dx} = +ve$

$M' = +ve$ ↑

M' is increasing so the tangent is increasing

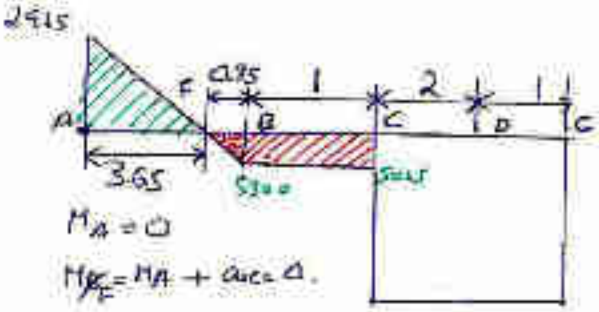
(iv) $\frac{dM}{dx} = F$, On integration from any two points 1 & 2.

$$\int_1^2 dM = \int_1^2 F.dx.$$



$$M_2 - M_1 = A[SFD]_1^2$$

$$\boxed{M_2 = M_1 + A\,[SFD]_1^2}$$



$M_A = 0$

$M_{\cancel{B}F} = M_A + \text{area } \Delta.$

$M_{\cancel{B}F} = 0 + \frac{1}{2} \times 3.65 \times 2925 = 5396.9\ Nm$

$M_B = M_F + \text{area } \nabla$

$M_B = 5396.9 - \frac{1}{2} \times 0.35 \times 275$

$M_B = 5300\ Nm$

$M_C = 5300 - 275 \times 1 = 5025\ Nm$

(V) UDL → $\omega = A$

$dF = -\omega dx = -A dx$

$F = -Ax + B$. ; $F = Ax + B \longrightarrow$ Linear (⟋)

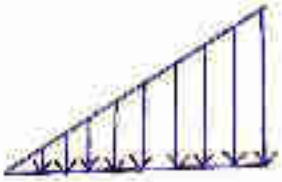
$\frac{dM}{dx} = F$

$dM = F dx$

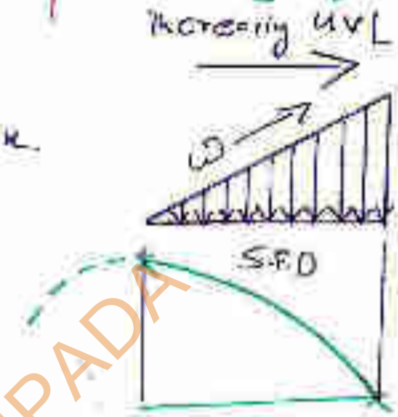
$\int dM \int (-Ax+B) dx$

$M = -\frac{Ax^2}{2} + \frac{Bx}{2} + C \longrightarrow 2^{nd}$ Order parabolic (⌒)

→ UVL :-



$\omega = Ax$.



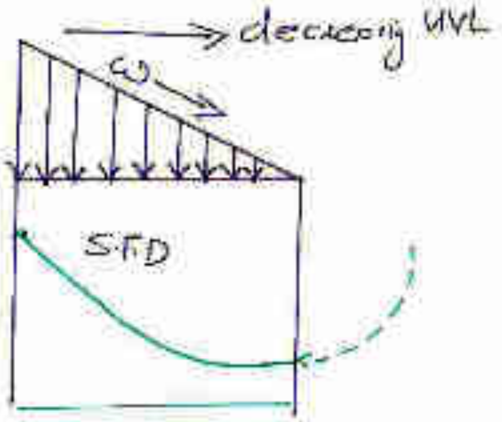
$\omega = Ax$

$dF = -\omega dx = Ax dx$.

$\int dF = \int (-Ax dx)$

$F = -\frac{Ax^2}{2} + B$; $F = \frac{Ax^2}{2} + B \longrightarrow$ parabola (⌒)

$\omega = -Ax + \omega_0$ ω_0



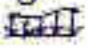
$dF = -\omega dx$

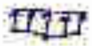
$F_d = \int -(-Ax + \omega_0) dx$.

$F = \frac{Ax^2}{2} + \omega_0 x + B$

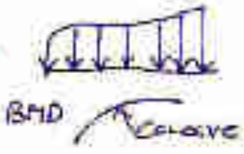
↳ $F = 0$ everywhere → $M = C$ → Pure bending.

↳ $F = 0$ at a point → $\frac{dM}{dx} = 0$ → Maxima / Minima





↳



$M'' = -ve$

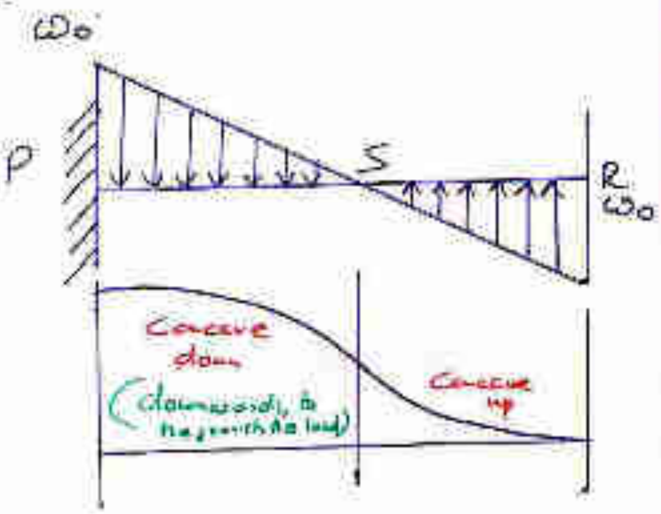
$M'' = +ve$

↳ UDL → SF – linear.
BMD – parabola

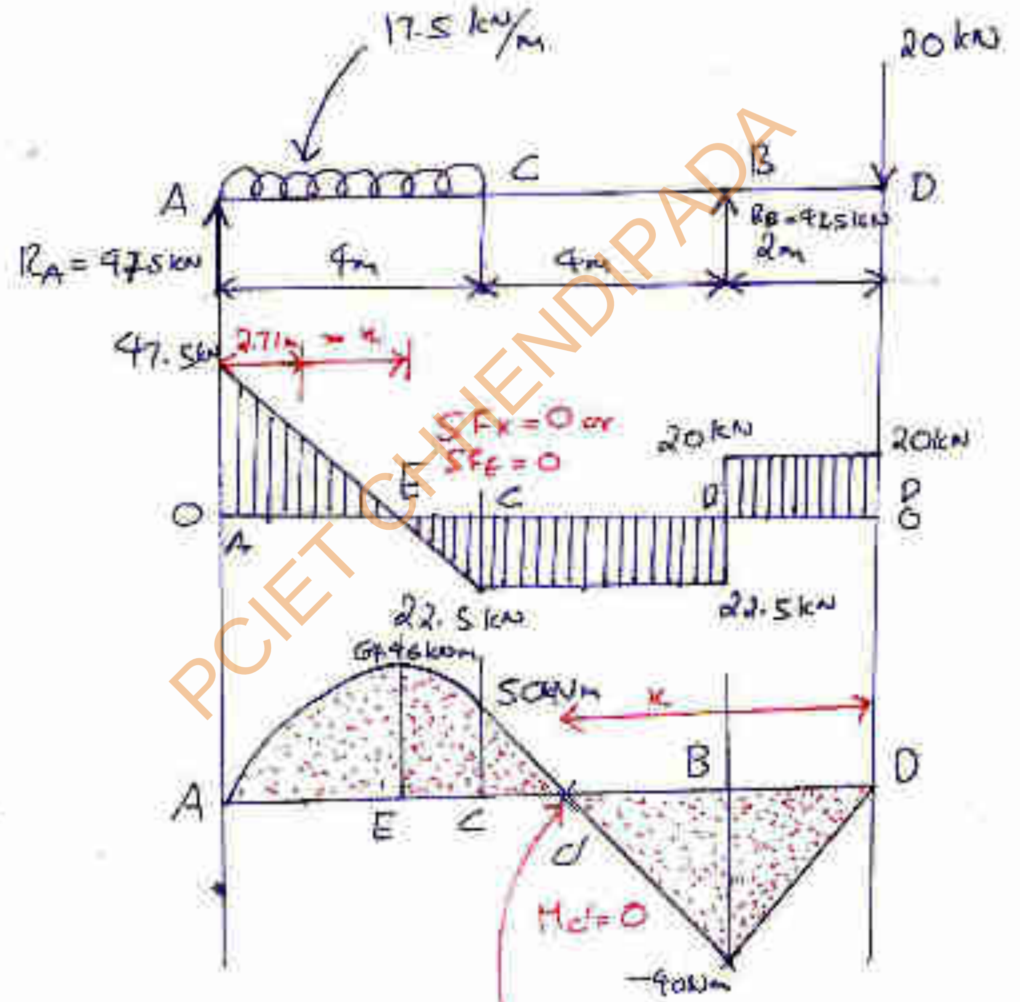
↳ UVL →

$M_2 = M_1 + A_{[SFD]_1}^{2}$

GATE-2005



9.



Point of Contra flexure because B.M will be zero

$\Sigma M_A = 0$

$17.5\times 4\times\frac{4}{2} - R_B\times 8 = -20\times 10$

$70\times 2 + 20\cancel{0}\times 10 = R_B\times 8$

$R_B = 42.5$ kN

$42.5 + R_A = 70 + 20$

$R_A = 47.5$ kN

SFD:

at A: $SF_{A^-} = 0$; $SF_{A+} = +47.5$

at $\overset{C}{B}$: $SF_C = -22.5$.

at B: $SF_{B^-} = 47.5 - (17.5\times4) = -22.5$; $SF_{B^+} = (47.5 - 70) + 42.5 = 20$

at D: $SF_{D^-} = 47.5 - (17.5\times4) + 42.5 = 20$; $SF_{D^+} = 47.5 - (17.5\times4) + 42.5 - 20 = 0$

Assume x distance from point A.

$\therefore -17.5 \times x \times \frac{x}{2} + 47.5 \times x = B.M_{max}$.

at Max. BM shear Force will be zero.

$0 = -17.5 \times \frac{x^2}{2} + 47.5 \times \cancel{x}$

$17.5 \times \frac{x}{2} = 47.5$

$\underline{\underline{x = 2.71\text{ m}}}$ (OE)

Max. B.M will

$BM_{max} = 47.5 \times 2.71 - 17.5 \times 2.71 \times \left(\frac{2.71}{2}\right)$

$B.M_{max} = \underline{\underline{64.46\text{ kNm}}}$ (at E)

Calculate BMD:-

At A: $BM_A = 0$. ; $BM_E = 64.46$ kNm (Max)

At C: $BM_C = 47.5 \times 4 - (17.5 \times 4 \times \frac{4}{2}) = 50$ kNm

At B: $BM_B = 47.5 \times 8 - (17.5 \times 4 \times (\frac{4}{2} + 4)) = -40$ kNm.

At D: $BM_D = 0$

At C' $BM_{C'} = 0$ in section BC

Consider x from D;

$M_{C'} = 42.5 \times (x - 2) + -20 \times x$.

at C' $M_{C'} = 0$

$0 = 42.5x - 85 - 20x$.

$85 = 42.5x - 20x$.

$x = \underline{\underline{3.78\text{ m from D}}}$

Q. If y = f(x) represents the equation of beam curvature which of the following is true at the point of contraflexure?

a) M = 0   b) y'' = 0   [✓ c) Both a and b]   d) cant define.

At "point of contraflexure" M'' = 0 or "point of inflexion" (POI or POC)

M'' = −ve (+)
M'' = +ve (−)
M'' = 0
−ve → 0 → +ve

27/07/2017

Q. $M_A = 0$,

$600\times 4\times \frac{4}{2} + 1600\times 3 + (\frac{1}{2}\times 3\times 600)\times$ ~~28/3~~ $= R_E \times 8$

$R_E =$ ~~1312.5~~ ~~2712.5 N~~ 2175N

$R_A + R_E = (600\times 4) + 1800 + \frac{1}{2}(3\times 600)$

$R_A =$ ~~2725 N~~ 2925N

At A:-

SF at A^- ; $SF_{A^-} = 0$.

SF at A^+; $SF_{A^+} = R_A =$ ~~2725N~~ 2925N

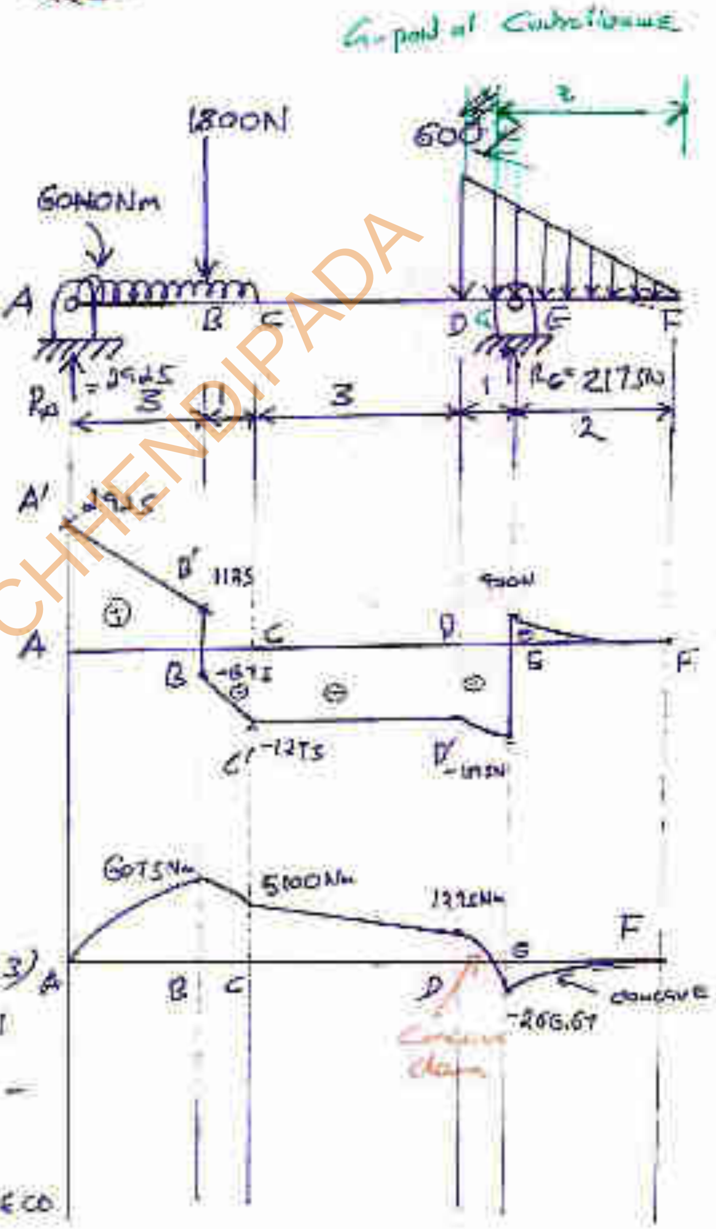
SF at B^-, $SF_{B^-} = 2925 - (600\times 3)_A$

$SF_{B^-} =$ ~~375N~~ 1125N

SF at B^+, $SF_{B^+} = R_A - (600\times 3) - 1800$

$SF_{B^+} =$ ~~2175~~ 2925 − 1600 − 1600

$SF_{B^+} =$ ~~−1425N~~ 675NA



At C^-; $SF_{C'} = R_A - 600\times 4 - 1800 =$ ~~2175~~ 2925 − 2400 − 1800 = ~~−2025N~~ −1275NN

At D^-; $SF_{D^+} = (\frac{1}{2}\times 600\times 3) - 2175 = -1275$ N

At E^{+-}; $SF_{E^-} = -2175 + \frac{1}{2}\times(2\times 400) = -1775$ N

At E^+; $SF_{E^+} = \frac{1}{2}\times 2\times 400 = \underline{400\ N}$ [R]

Bendig moment:

BM at A ; $BM_A = 0$

BM at B ; $BM_B = (2925 \times 3) - (600 \times 3 \times 3/2) = 6075$ Nm. [L]

BM at C ; $BM_C = (2925 \times 4) - (600 \times 4 \times \frac{4}{2}) - (1600 \times 1) = 5180$ Nm [L]

BM at D ; $BM_D = -(\frac{1}{2} \times 3 \times 600 \times 3/3) + 2175 = 1275$ Nm. [R]

BM at E ; $BM_E = -(\frac{1}{2} \times 2 \times 400)\frac{2}{3} - \frac{400 Nm \times 2}{3} = -266.67$ Nm [R]

BM at F ; $BM_F = 0$ Nm.

B/w A & B shear force doesnot cou'd parabola also peak.

~~BM at D ; $+\frac{1}{2} \times 600 \times 3 \times 3/2$~~

~~$-(\frac{1}{2} 600 \times x \times \frac{x}{2}) + 2175 (x-2) = 0$~~

~~$150x^2 - 2175x - 4350 = 0$~~

~~$x^2 + -14.5x + 29 = 0$~~

~~$x = \frac{14.5 \pm \sqrt{14.5 \times 14.5 - 4 \times 1 \times 29}}{2 \times 1}$~~

~~$x = \frac{14.5 \pm 9.708}{2}$~~

~~BM at B & G ;~~

~~$-(\underbrace{\frac{1}{2} \times 200x \times x}_{area} \times \frac{x}{3}) + 2175 \times (x-2) = 0$~~

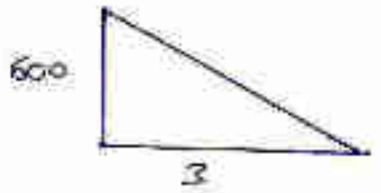
~~$-(100 \times \frac{x^3}{3}) + 2175x - 2175 = 0$~~

~~$33.3x^2 - 2175x + 2175 = 0$~~

~~$33.3x^2 - 65.25x + 130.5 = 0$~~

~~$x = \frac{65.25 \pm \sqrt{65.25^2 - 4 \times 33.3 \times 130.5}}{2 \times 33.33}$~~

~~$x = 2.15$m from F~~



$\frac{600}{3} = \frac{y}{x}$

$y = 200x$

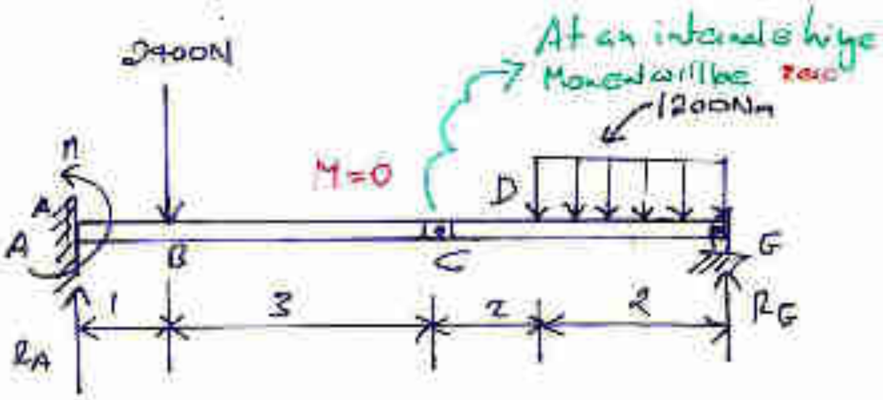
area $= \frac{1}{2} \times 200x \times x = 100x^2$

BM @ G :-

$100x^2 \times \frac{x}{3} + 2175 (x-2) = 0$

$x = 2.15$m from F

9



Moments about A;

$2400 \times 1 + (2400 \times (1+3+2+2/_2)) =$

$R_E \times 8 + M$

$19200 = R_E \times 8 + M.$

$R_A + R_{B_E} = 2400 + (1200 \times 2)$

$R_A + R_{\cancel{B}_G} = 4800$

$1900 = R_E \times 8 + M$

$R_A + R_G = 4800$

→ At point C internal hinge,
Moments can be zero.

An extra moment will not help. Disturbe it.

In case internal hinge is there we can have it moment can be exist.

B.M at internal hinge is Zero.

$BM_C = 0$

$-1200 \times 2 \times (2 + 2/_2) + R_E \times 4 = 0$

$R_E = \underline{\underline{1800\,N}}$

$R_A = 4800 - 1800 = 3000\,N$

$19200 = 1800 \times 8 + M$

$M = 4800\,N$

SF Diagram:

at point A: $SF_{A^-} = 0$; $SF_{A+} = 3000N$; At point B : $SF_{B^-} = 3000N$: $SF_{B^+} = 3000 - 2400$ $= 600N$

At point C: $SF_C = 600N$ [left]

At point D: $SF_D = 3000 - 2400 = 600N$. [left]

At point E: $SF_{E^-} = 3000 - 2400 - (1200 \times 2) = -1800N$; $SF_{E^+} = 0$ [left]

BM diagram:

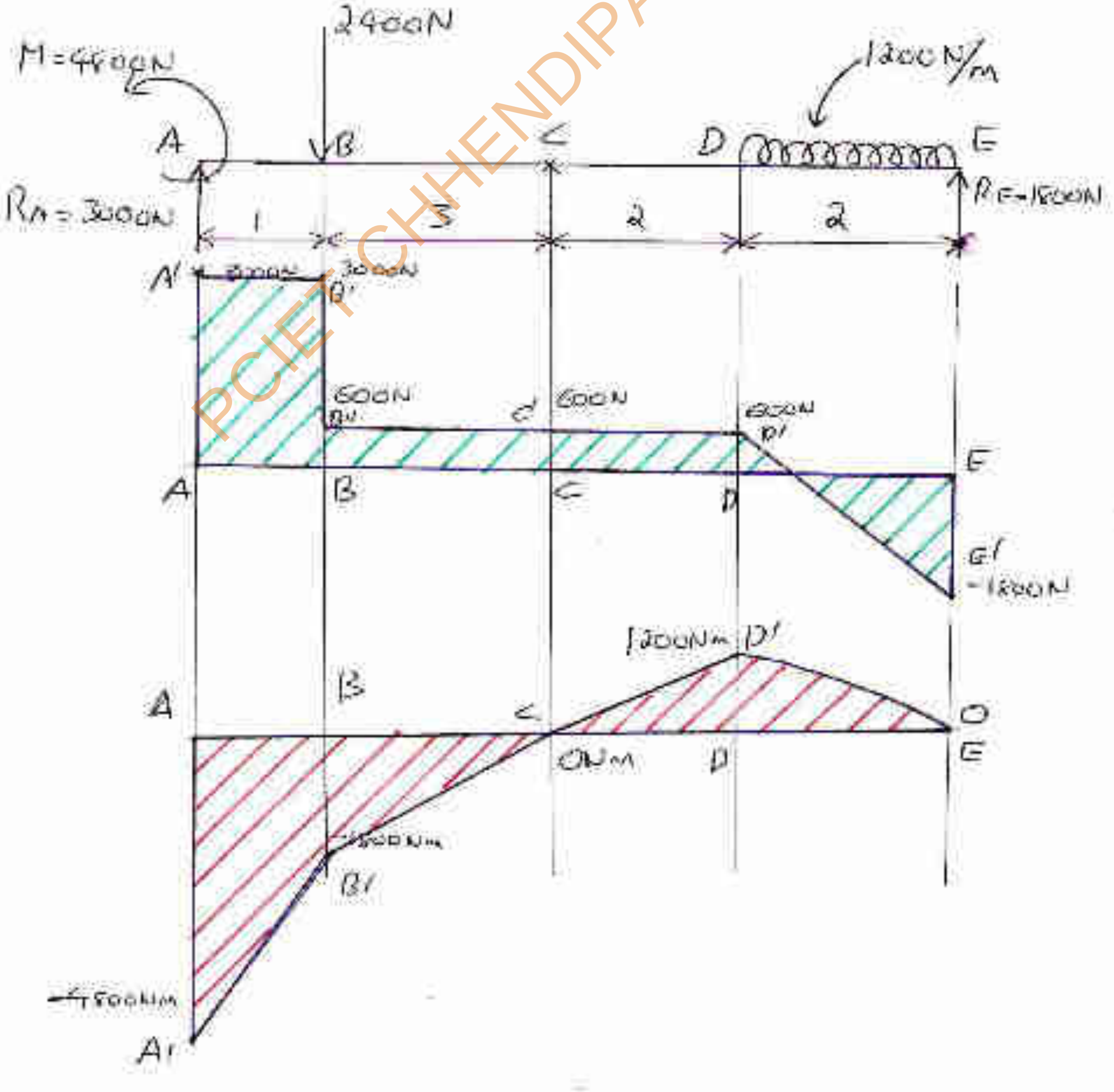
at A: $BM_{A^-} = 0Nm$; $BM_{A+} = -4800Nm$ [left]

at B: $BM_B = (3000 \times 1) - 4800 = -1800Nm$ [left]

at C: $BM_C = (3000 \times 3) - 4800 - (2400 \times 3) = 0Nm$ [left]

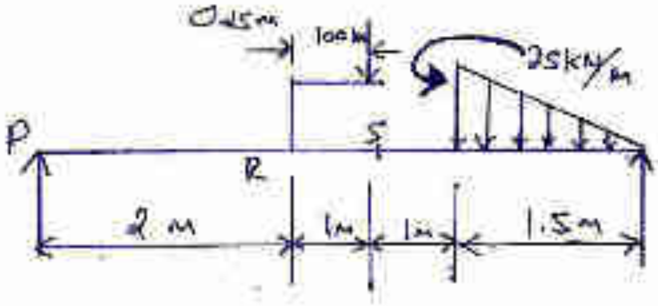
at D: $BM_D = (1800 \times 2) - ((1200 \times 2)(2/2)) = 1200Nm$ [right]

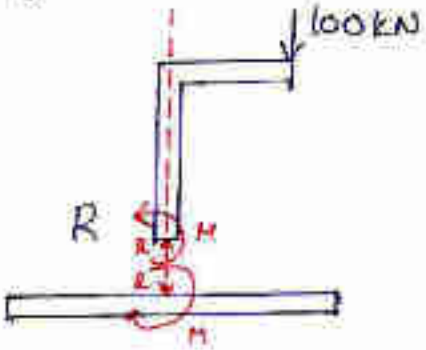
at E: $BM_E = 0Nm$.

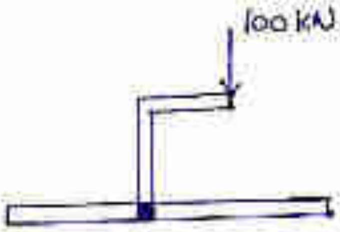


Problem 9

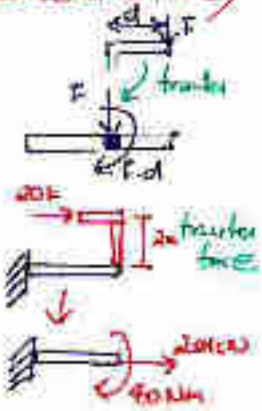
Frame there, so we have to remove/seperate the frame from the beam. To determine SF diagram we need independent free body diagram.







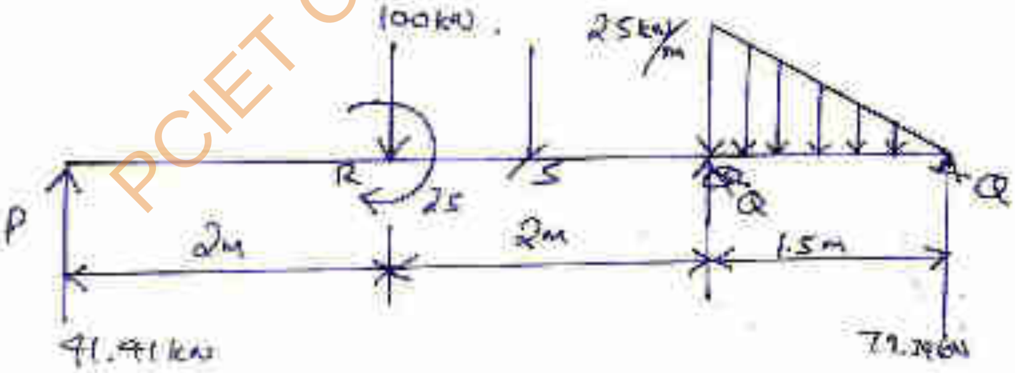
Transfer the force to the support and also the moment (F×⊥)



For the Support { $\Sigma F = 0;\ R = 100\text{ kN}.$

$\Sigma M = 0;\ M = 100 \times 0.25 = 25\text{ kNm (C.C.W)}.$

For the beam R = 100kN; M = 25 kNm (CW)



$M_P = 0$

$$25 + 100 \times 2 + \left(25 \times 1.5 \times \frac{1}{2} \times \left(4 + \left(\frac{1.5}{3}\right)\right)\right) = R_Q \times \cancel{5.5}\,4$$

$R_Q = 77.34\text{ kN}$

$P + Q = 100 + \frac{1}{2} \times 1.5 \times 25$

$R_P = 41.41\text{ kN}$

SF_p+ ; $SFp+ = 41.41$ kN

BM_s ; $41.41 \times 3 + 25 - (100 \times 1) = BM_s$

$BM_s = 41.41 \times 3 + 25 - 100 = \underline{\underline{49.23 \text{ kNm}}}$

Page 29

Q.1

Purely axial load.

→ At the point only change the sign the shear force only the bending moment will be Maximum.

Page 29 Q.10) A)

 → pure bending (Cantilever subjected to couple)

"Mo always being counted" No

B)

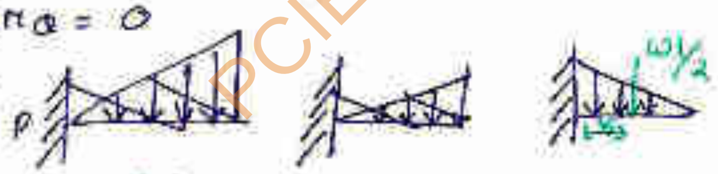
concave down

at Q

$BM_p = \omega l \times l/2 = \frac{-\omega l^2}{2}$

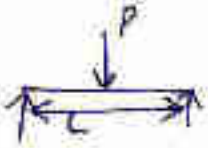
$BM_Q = 0$

B)



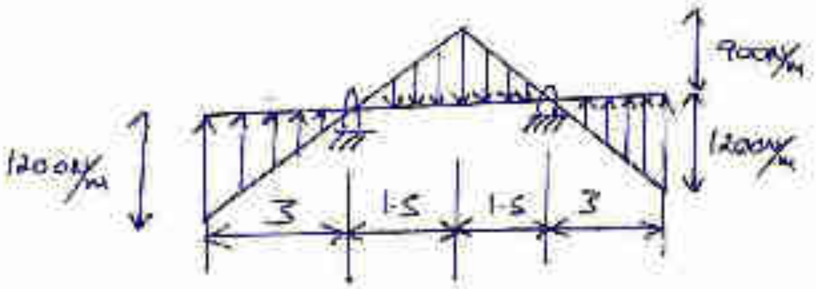
$BM_p = \omega_0 l^2/6$

C)

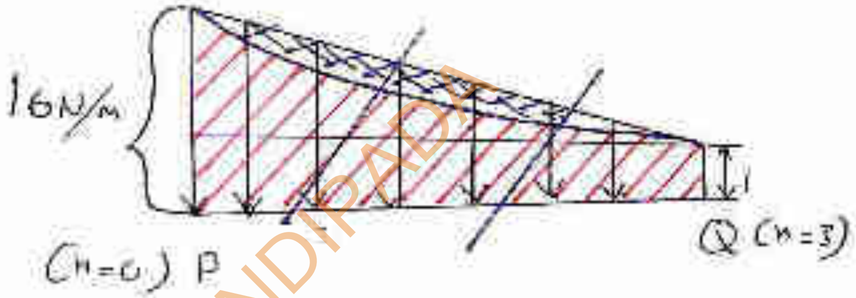


#1. Shear force b/w 2 points P and Q of a beam can be described as $F = 16 - 5x$ where x is zero at P. If B.M at P is 5 kNm and length PQ is 3m then the B.M at Q is ______ kNm.

#2.



#1



~~B.M at Q $\neq$ BM at P + SF [shaded area]~~

~~$B.M_Q$ = 5 kNm + [Area of ~~triangle~~ Parabolic area + Area of rectangle]~~

~~$BM_Q = 5 + [(15 \times 3 \times \frac{1}{3}) + (1 \times 3)]$~~

~~$BM_Q = 5 + [15 + 3] = 5 + 18$~~

~~BM_Q = 23 kNm~~

#1 $\frac{dM}{dx} = F$

$dM = F dx$

$\int_q^p dM = \int F dx$

$M_q = \int_0^3 (16 - 5x)\, dx + M_p$

$M_q = \left[16x - \frac{5x^2}{2}\right]_0^3 + M_p$

$M_q = 16 \times 3 - \frac{5 \times 9}{2} + M_p$

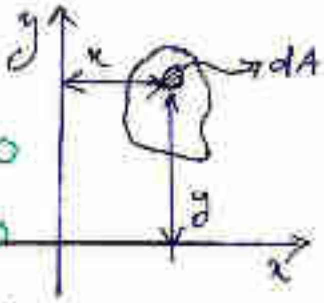
$M_q = 48 - \frac{45}{2} + M_p$

$M_q = 48 - 22.5 + 5$

M_q = 25.5 kNm + 5 = 30.5 kNm

3.4 MOMENT OF INERTIA:

a) Definition:



About Y-axis, $I_y = \int x \times (x dA) = \int x^2 dA > 0$

About X-axis, $I_x = \int y \times (y dA) = \int y^2 \cdot dA > 0$

About X-Y system, $I_{xy} = \int x (y dA) = \int y (x dA) \gtreqless 0$ (combo -ve, +ve, 0)

b) Significance: & Origin:-

Example are shown in torsion $T = \int \underbrace{r \times dr}_{\tau} \times r$

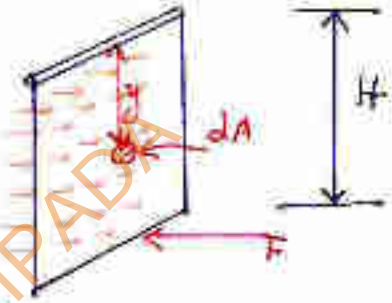
$T = k\int r^2 dr \rightarrow$ MOI

Similarly hinged support the gate

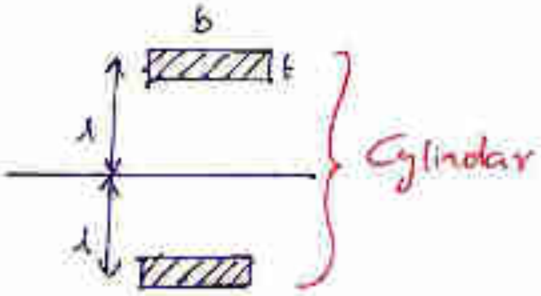
$F \times H = \int (P dA) y$

$F \times H = \int \rho g y \cdot dA y.$

$F \times \overset{H}{\cancel{A}} = \rho g \int y^2 dA \rightarrow$ MOI $I_x = \frac{bd^2}{12}$



→ First Moment does not represent the area. Second moment will reflect the area.



First Moment = $(bt \times \lambda) + bt \times (-\lambda)$

1^{st} Moment $= 0$

2^{nd} Moment $= bt(\lambda)^2 + bt(-\lambda)^2$

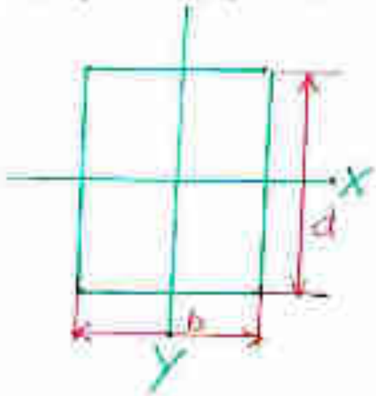
2^{nd} Moment $= 2bt\lambda^2$

* MOI Gives →

"How Area is distributed about an Axis?"

9 STANDARD SECTIONS:

i) Rectangle

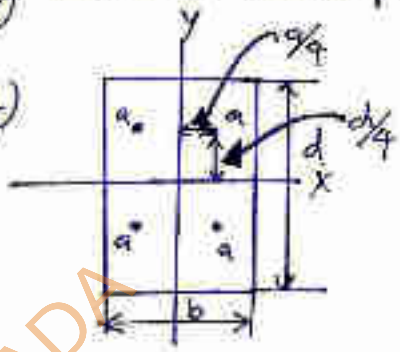


$I_x = \frac{bd^3}{12}$: $I_{xy} = 0$

$I_y = \frac{db^3}{12}$

Entire rectangle as 4 area asuming width 'b' and depth 'd'

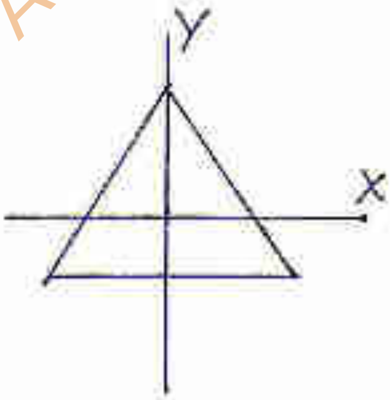
$I_{xy} = a\left(\frac{b}{4}\right)\left(\frac{d}{4}\right) + a\left(\frac{-b}{4}\right)\left(\frac{d}{4}\right) + a\left(\frac{-b}{4}\right)\left(\frac{-d}{4}\right)$
$+ a\left(\frac{b}{4}\right)\left(\frac{-d}{4}\right) = 0$



Similarly triangle.

$I_x = \frac{bh^3}{36}$; $I_{xy} = 0$

[If any area is symmetric about any axis the product of inertia will be zero]

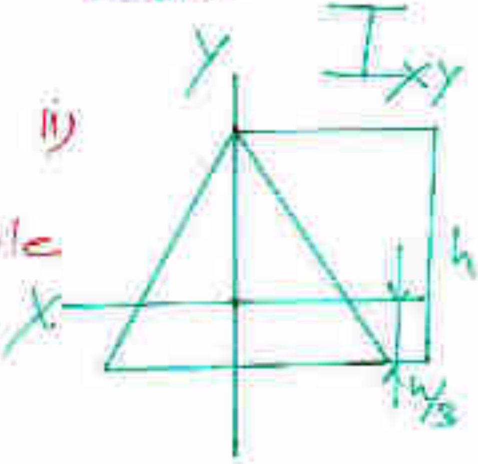


Eg: T-section, I-section, Circle, Elliptical

* "If a section is symmetric about atleast of the centroidal axis. Then product of inertia of that section will be about the symmetric axis and ⊥ one will be always zero."

$I_{xy} = 0$

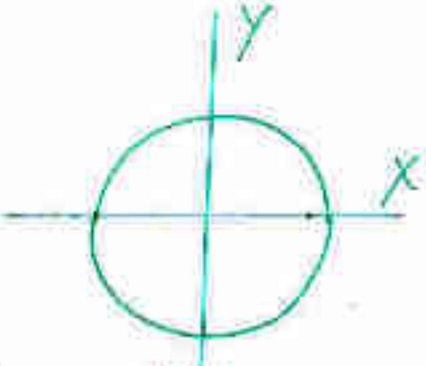
ii) Triangle



$I_x = \frac{bh^3}{36}$

$I_{xy} = 0$

iii) Circle



$I_x = I_y = \frac{\pi d^4}{64}$

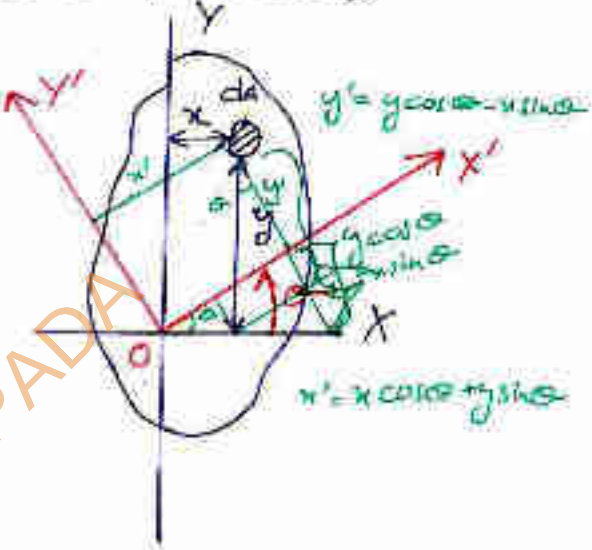
$I_{xy} = 0$

D) Tensor Nature:-

Consider the area where a pole (x-y intersect)

Consider the axis x' and y' axis

→ Same area & Same pole:-



$y' = y\cos\theta - x\sin\theta$

$x' = x\cos\theta + y\sin\theta$

At X-Y axis		At X'-Y' axis
I_x		$I_{x'}$
I_y	⇔ changes	$I_{y'}$
I_{xy}		$I_{x'y'}$

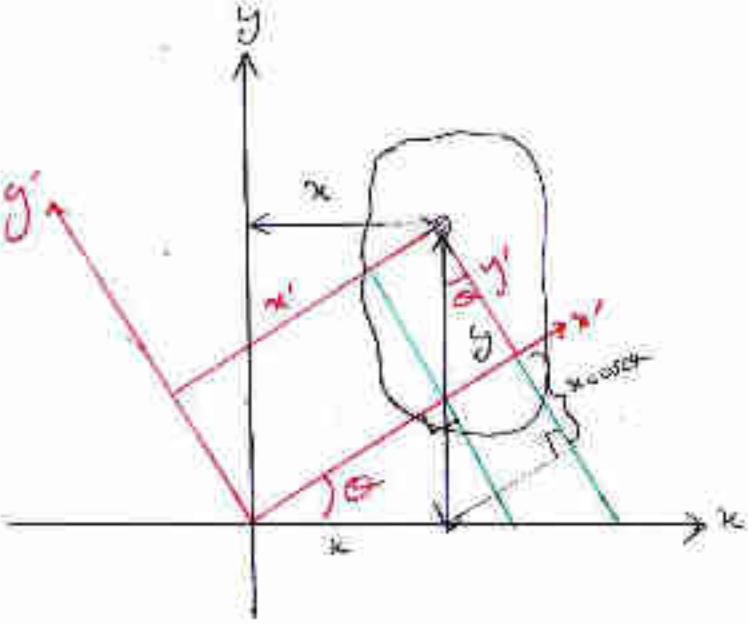
$$\int y'^2 dA = I_{x'} = \frac{I_x + I_y}{2} + \left(\frac{I_x - I_y}{2}\right)\cos 2\theta + I_{xy}\sin 2\theta$$

$$\int x'^2 dA = I_{y'} = \frac{I_x + I_y}{2} \mp \left(\frac{I_x - I_y}{2}\right)\cos 2\theta + I_{xy}\sin 2\theta$$

$$\int x'y' dA = I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

} Transformation Equations

Major principal Axis - Axis of which Max. MOI should be obtained. It occurs at principal angle.



$$x' = x\cos\theta + y\sin\theta$$

$$y' = y\cos\theta - x\sin\theta$$

$$I_{x'} = \int y'^2 dA$$

$$(y')^2 = (y\cos\theta - x\sin\theta)^2 = y^2\cos^2\theta - 2xy\cos\theta\cdot\sin\theta + x^2\sin^2\theta.$$

$$(y')^2 = y^2\cos^2\theta - xy\sin2\theta + x^2\sin^2\theta$$

$$I_{x'} = \int y^2\cos^2\theta - xy\sin2\theta + x^2\sin^2\theta\, d\theta A$$

$$I_{x'} = \int\left[y^2\left(\frac{1+\cos2\theta}{2}\right) - xy\sin2\theta + x^2\left(\frac{1-\cos2\theta}{2}\right)\right]dA.$$

$$I_{x'} = \int\left[\frac{y^2}{2} + \frac{x^2}{2} - xy\sin2\theta + \left(\frac{y^2}{2} - \frac{x^2}{2}\right)\cos2\theta\right]dA$$

$$I_{x'} = \int\left(\frac{y^2+x^2}{2}\right)dA - \int xy\sin2\theta\, dA + \int\left[\frac{y^2}{2} - \frac{x^2}{2}\right]\cos2\theta\, dA$$

$$I_{x'} = \frac{I_x+I_y}{2} - I_{xy}\sin2\theta + \frac{I_x-I_y}{2}\cos2\theta$$

$$I_{x'} = \frac{I_x+I_y}{2} + \frac{I_x-I_y}{2}\cos2\theta - I_{xy}\sin2\theta$$

$$\boxed{\int x^2 dA = I_y \;;\; I_x = \int y^2 dA \;;\; \int xy\, dA = I_{xy}}$$

$$(x')^2 = (x\cos\theta + y\sin\theta)^2$$

$$(x')^2 = x^2\cos^2\theta + 2xy\sin\theta\cos\theta + y^2\sin^2\theta.$$

$$(x')^2 = x^2\left(\frac{1+\cos2\theta}{2}\right) + xy\sin2\theta + y^2\left(\frac{1-\cos2\theta}{2}\right)$$

$$(x')^2 = \frac{x^2}{2} + \frac{y^2}{2} + xy\sin2\theta + \left(-\frac{y^2}{2} + \frac{x^2}{2}\right)\cos2\theta.$$

$$I_y = \int x'^2\,dm.$$

$$I_y = \frac{I_x+I_y}{2} + I_{xy}\sin2\theta + \frac{-I_x+I_y}{2}\cos2\theta.$$

$$I_y = \frac{I_x+I_y}{2} \mp \left(\frac{I_x-I_y}{2}\right)\cos2\theta + I_{xy}\sin2\theta$$

$$x'y' = (x\cos\theta + y\sin\theta)(\cancel{x\cos\theta +} -x\sin\theta + y\cos\theta)$$

$$x'y' = (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)$$

$$x'y' = xy\cos^2\theta - x^2\sin\theta\cos\theta + y^2\sin\theta\cos\theta - xy\sin^2\theta$$

$$x'y' = xy\left(\frac{1+\cos2\theta}{2}\right) - \frac{x^2\sin2\theta}{2} + \frac{y^2\sin2\theta}{2} - xy\left(\frac{1-\cos2\theta}{2}\right)$$

$$x'y' = \cancel{\frac{xy}{2}} + \frac{xy\cos2\theta}{2} + \left(\frac{y^2-x^2}{2}\right)\sin2\theta - \cancel{\frac{xy}{2}} + \frac{xy\cos2\theta}{2}$$

$$x'y' = xy\cos2\theta + \left(\frac{y^2-x^2}{2}\right)\sin2\theta.$$

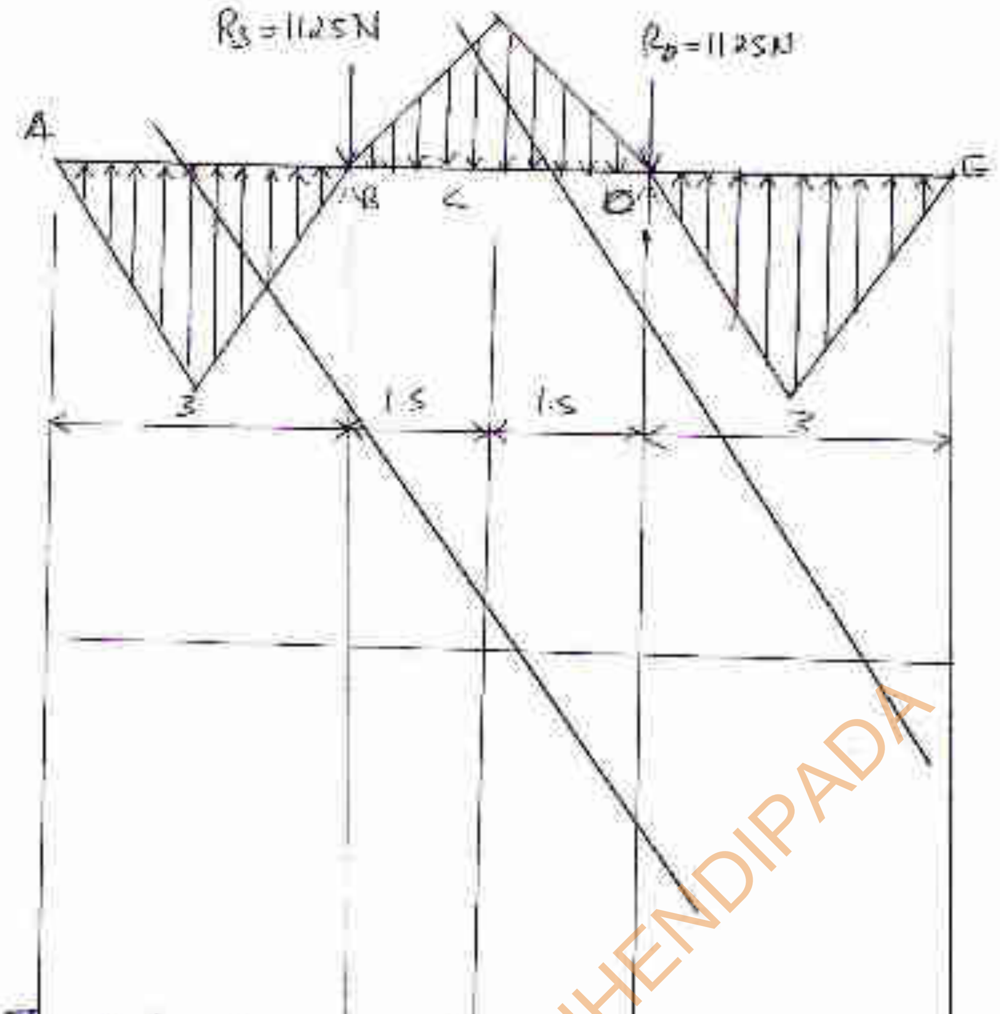
$$\int x'y'\,dA = I_{x'y'}$$

$$I_{x'y'} = \int xy\cos2\theta\,dA + \int \frac{y^2-x^2}{2}\sin2\theta\,dA.$$

$$I_{x'y'} = I_{xy}\cos2\theta + \left(\frac{I_x-I_y}{2}\right)\sin2\theta$$

$$I_{x'y'} = \left(\frac{I_x-I_y}{2}\right)\sin2\theta + I_{xy}\cos2\theta$$

#2.



To determine reactions:-

$M_B = 0$

$$\left(\tfrac{1}{2}\times 3\times 1200\right)\left(2\times \tfrac{3}{3}\right) + \left[\left(\tfrac{1}{2}\times 1.5\times 900\right)\left(2\times\frac{1.5}{3}\right)\right] + \left[\left(\tfrac{1}{2}\times 1.5\times 900\right)\left(1.5 \overset{+}{\times} \frac{1.5}{3}\right)\right]$$

$$= R_D\times 3 + \left[\left(\tfrac{1}{2}\times 3\times 1200\right)\left(3+\frac{2\times 3}{3}\right)\right]$$

$$(1800\times 2) + (675\times 1) + (675\times 2) = 3R_D + (1800\times 5)$$

$$3600 + 675 + 1350 = 3R_D + 9000$$

$R_D = \underline{\underline{-1125\,N}}$ (Assumed upward, original dir. downward)

$R_B + R_D + \left(\tfrac{1}{2}\times 3\times 1200\right) + \left(\tfrac{1}{2}\times 1200\times 3\right) = \tfrac{1}{2}\times 3\times 900$

$R_B = \underline{\underline{-1125\,N}}$ (Assumed upward, original dir. downward)

SF diagram:

At A: $SF_A = 0$ [left]

At B^-: $SF_{B^-} = (\frac{1}{2} \times 3 \times 1200) = 1800N$ [left]

At B^+: $SF_{B^+} = (\frac{1}{2} \times 3 \times 1200) - 1125 = 675N$ [left]

At C: $SF_C = (\frac{1}{2} \times 3 \times 1200) - 1125 - (\frac{1}{2} \times 1.5 \times 900) = 0N$ [left]

At D^-: $SF_{D^-} = -(\frac{1}{2} \times 1200 \times 3) + 1125 = -675N$ [Right]

At D^+: $SF_{D^+} = -(\frac{1}{2} \times 1200 \times 3) = -1800N$ [Right]

B.M. Diagram:

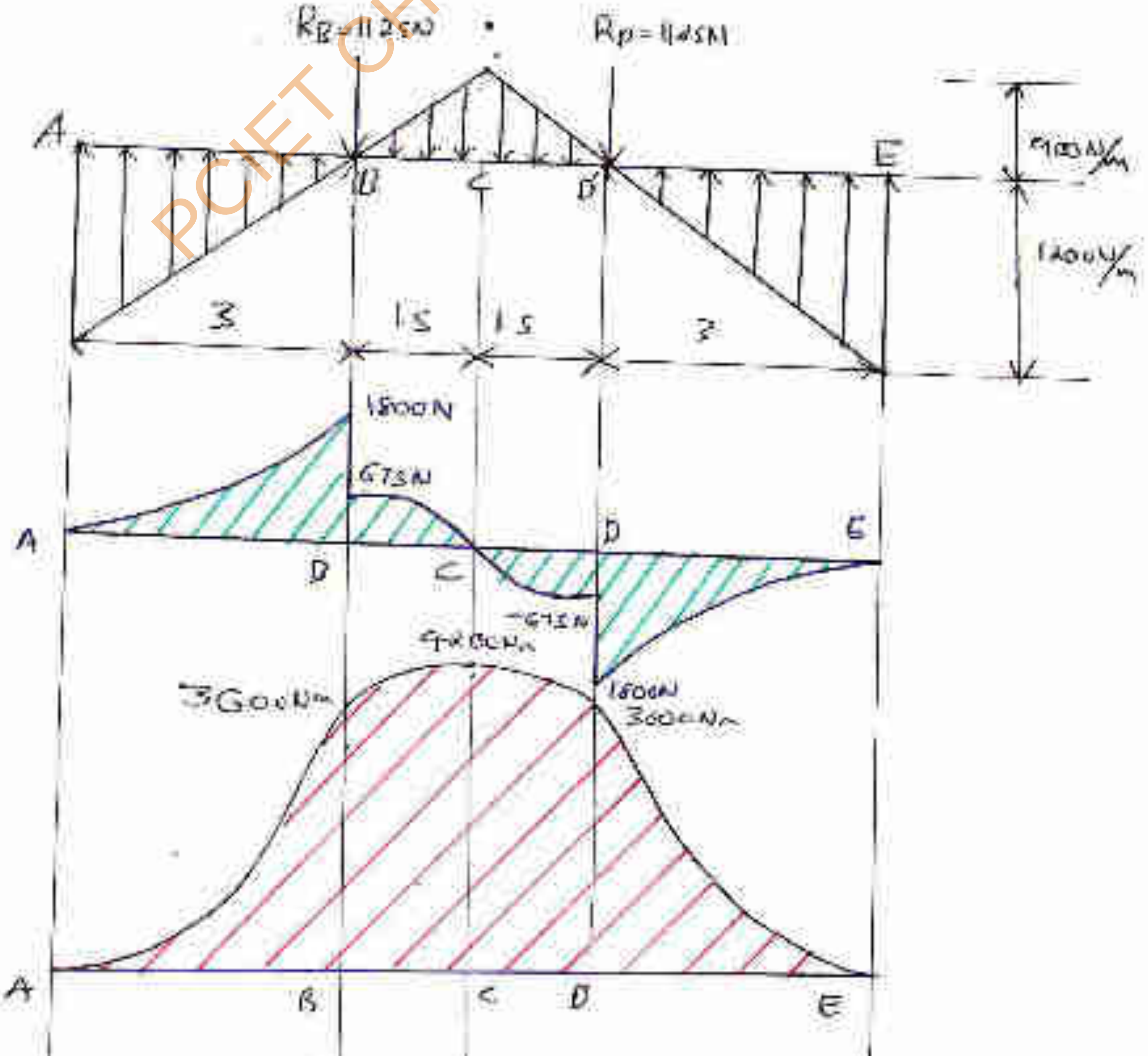
At A: $BM_A = 0$

At B: $BM_B = (\frac{1}{2} \times 3 \times 1200 \times \frac{2 \times 3}{3}) = 3600Nm$ [left]

At C: $BM_C = [(\frac{1}{2} \times 3 \times 1200)(1.5 + \frac{2 \times 3}{3})] - 1125 \times 1.5 - [(\frac{1}{2} \times 1.5 \times 900)\frac{1.5}{3}] = 4275Nm$ [left]

At D: $BM_D = [(\frac{1}{2} \times 3 \times 1200)(\frac{2 \times 3}{3})] = 3600Nm$

At E: $BM_E = 0\,Nm$



Pg No-28
Question 4

9 $\sum M_A = 0$

$3 = R_C \times 3$

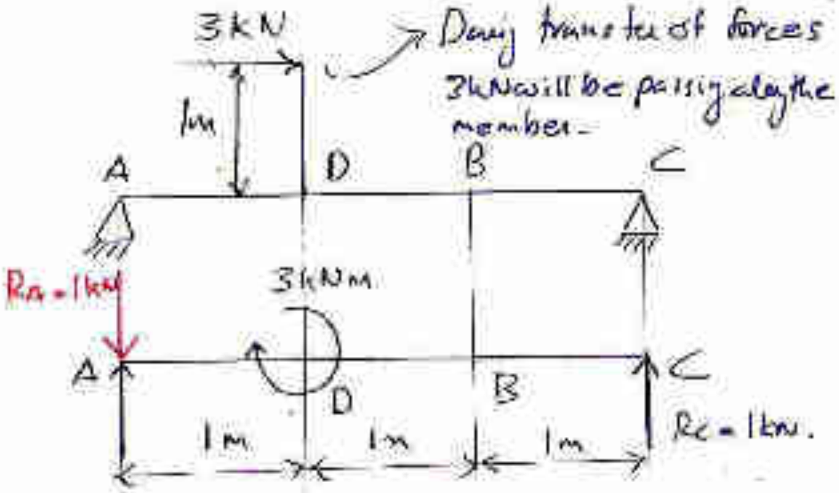
$R_C = \underline{1 kN}$

$R_A + R_C = 0 \quad \sum F_y = 0$

$\therefore R_A = -1 kN$

Assumed direction is wrong new direction will be shown in red line

$BM_B = 1 \times 1 = 1 kNm$ [Right]



Pg No: 28
Q.No 5

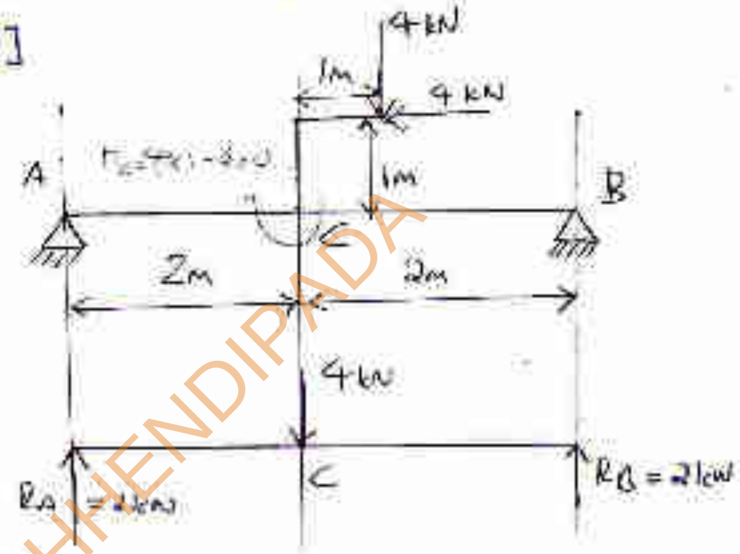
9 $M_A = 0$

$4 \times 2 = R_B \times 4$

$R_B = 2 kN$

$R_A + R_B = 4 kN$

$R_A = \underline{2 kN}$ (up)

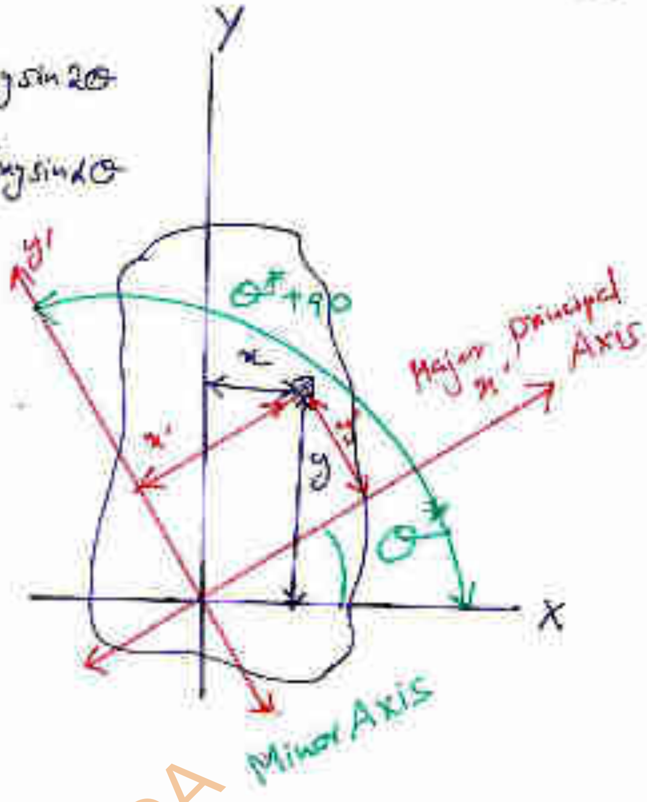


25/7/2017

$$I_{x'} = \left(\frac{I_x+I_y}{2}\right) + \left(\frac{I_x-I_y}{2}\right)\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \left(\frac{I_x+I_y}{2}\right) - \left(\frac{I_x-I_y}{2}\right)\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x'y'} = \left(\frac{I_x-I_y}{2}\right)\sin 2\theta + I_{xy}\cos 2\theta$$



$$I_{x'} + I_{y'} = I_x + I_y$$

$$I_{x''} + I_{y''} = I_x + I_y$$

$$I_{x'''} + I_{y'''} = I_x + I_y$$

$$\text{-----} \quad \text{---------}$$

$$I_{x^n} + I_{y^n} = I_x + I_y$$

At θ^*, we get Max moment of inertia.

Since at θ^*, we get Max. moment of inertia

"We should get Min. moment of inertia $\theta^*+90°$ and the axis is called as Minor principal axis or Minor axis."

$$I_{x'} = \left(\frac{I_x+I_y}{2}\right) + \left(\frac{I_x-I_y}{2}\right)\cos 2\theta - I_{xy}.\sin 2\theta$$ [because $I_{y'}$ axis is major principal axis]

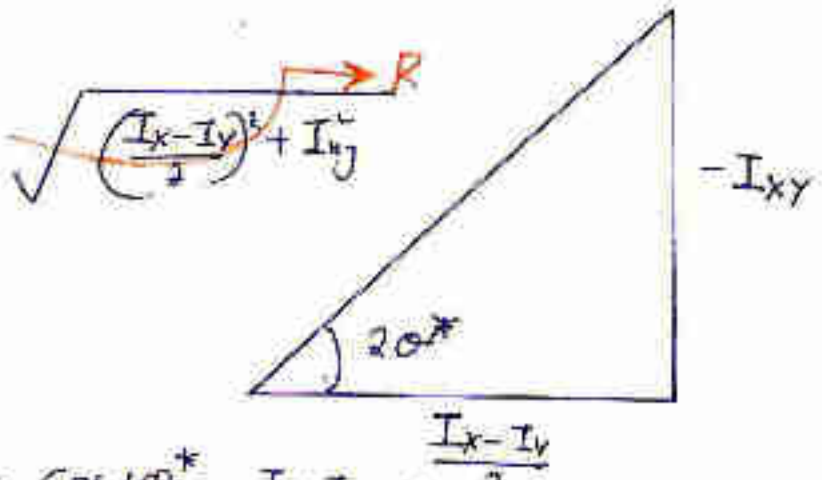
$$\frac{dI_{x'}}{d\theta} = \left(\frac{I_x-I_y}{2}\right)-\sin 2\theta.2 - I_{xy}.\cos 2\theta.2$$

$$\frac{d}{d\theta}I_{x'} = -2\sin 2\theta\left(\frac{I_x-I_y}{2}\right) - 2\cos 2\theta\, I_{xy}$$

$$\frac{d}{d\theta}I_{x'} = 0$$

$$-\cancel{2}\sin 2\theta\left(\frac{I_x-I_y}{2}\right) = \cancel{2}\cos 2\theta.\ I_{xy}$$

$$\operatorname{Tan} 2\theta^* = \frac{-I_{xy}}{\left(\frac{I_x-I_y}{2}\right)}$$



$$\sin 2\theta^* = \frac{-I_{xy}}{R}; \quad \cos 2\theta^* = \frac{\frac{I_x-I_y}{2}}{R}$$

Put $\sin 2\theta^*$ and $\cos 2\theta^*$ in $I_{x'}$

$$I_{x'} = \frac{I_x+I_y}{2} + \left(\frac{I_x-I_y}{2}\right)\cos 2\theta^* - I_{xy} \cdot \sin 2\theta^* \quad [\theta \text{ will be } \theta^*]$$

$$I_{x'} = \frac{I_x+I_y}{2} + \left(\frac{I_x-I_y}{2}\right) \times \frac{I_x-I_y}{2R} - I_{xy} \cdot \frac{-I_{xy}}{R}$$

$$I_{x'} = \frac{I_x+I_y}{2} + \left(\frac{I_x-I_y}{2}\right)^2 \cdot \frac{1}{R} + \frac{I_{xy}^2}{R}$$

$$\therefore I_{x'} = \frac{I_x+I_y}{2} + \frac{1}{R}\left[\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2\right] \rightarrow R^2$$

$$I_{x'} = \underbrace{\frac{I_x+I_y}{2}}_{\rightarrow I_{avg}} + R$$

$$I_{x'} = \frac{I_x+I_y}{2} + R \rightsquigarrow [I_{x'} = I_{max}]$$

$$I_{max} = \frac{I_x+I_y}{2} + R \quad ; \quad R = \left[\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2\right]^{1/2}$$

$$\rightarrow I_{max} = I_{avg} + R$$

$$I_{y'} = I_{avg} - R$$

$$I_{x'} + I_{y'} = I_x + I_y$$

$$I_{x'} + I_{y'} = \left(\frac{I_x + I_y}{2}\right) \times 2$$

$$\text{ESGI} \xrightarrow[\text{Warnings}]{} I_{x'} + I_{y'} = I_{avg} \times 2$$

$$I_{max} = I_{avg} + R$$

$$I_{min} = I_{avg} - R$$

$$I_{max} + I_{min} = 2 I_{avg}$$

$$I_{x'y'} = 0$$

$$I_{x'y'} = \left(\frac{I_x - I_y}{2}\right) \sin 2\theta^* + I_{xy} \cos 2\theta^* \quad [\text{put } \sin 2\theta^*, \cos 2\theta^*]$$

$$I_{x'y'} = \left(\frac{I_x - I_y}{2}\right) \cdot \frac{-I_{xy}}{R} + I_{xy} \cdot \frac{\frac{I_x - I_y}{2}}{R}$$

$$I_{x'y'} = -\left(\frac{I_x - I_y}{2}\right) \cdot \frac{I_{xy}}{R} + \left(\frac{I_x - I_y}{2}\right) \cdot \frac{I_{xy}}{R}$$

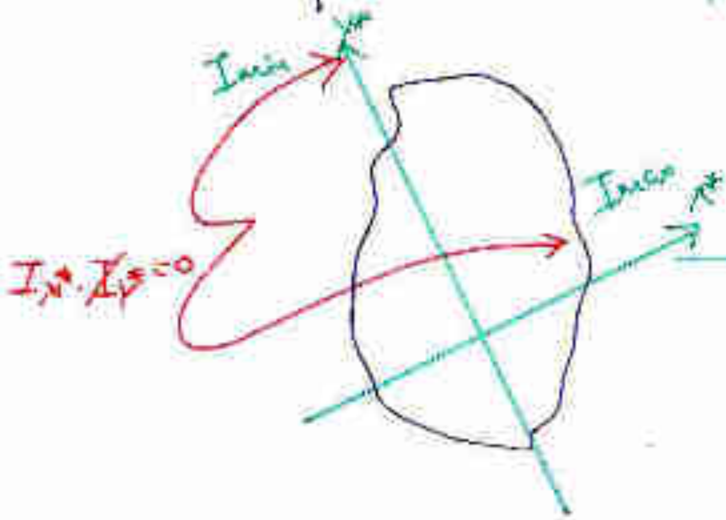
$$I_{x'y'} = 0.$$

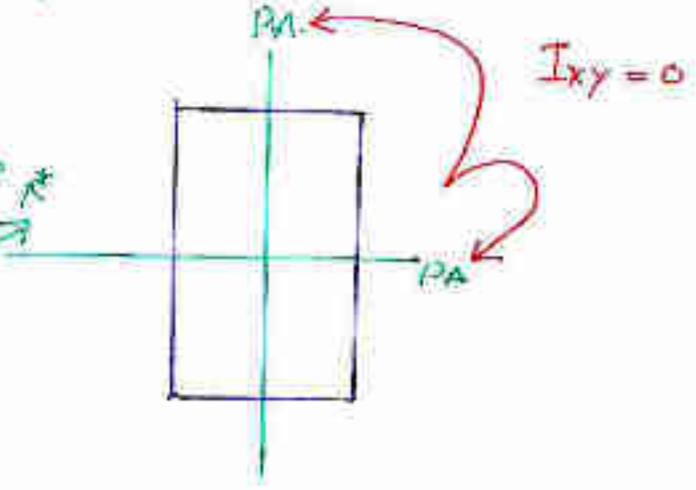
Conclusion: Product of inertia about Principal axis Must always be Zero. Conversly if product of inertia is zero about a pair of axis, then that pair of axes must be the Major and Minor principal axis.

Symmetric Axis will be Principal Axis.

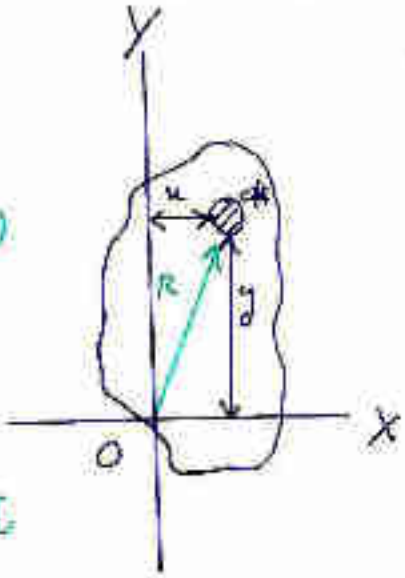
Circle is Symmetric about every diametrical axis.

Every axis is Principal Axis.





E) PERPENDICULAR AXIS THEOREM:-



$I_x = \int y^2 dA$; $I_y = \int x^2 \cdot dA$

To determine M.O.I about 'O' (pole)

So that $I_R = \int R^2 dA$

$$I_o = I_z = I_R = \underbrace{\int R^2 \cdot dA}_{\text{Polar M.O.I}}$$

Here $R^2 = x^2 + y^2$:-

$$I_z = \int (x^2+y^2) dA$$

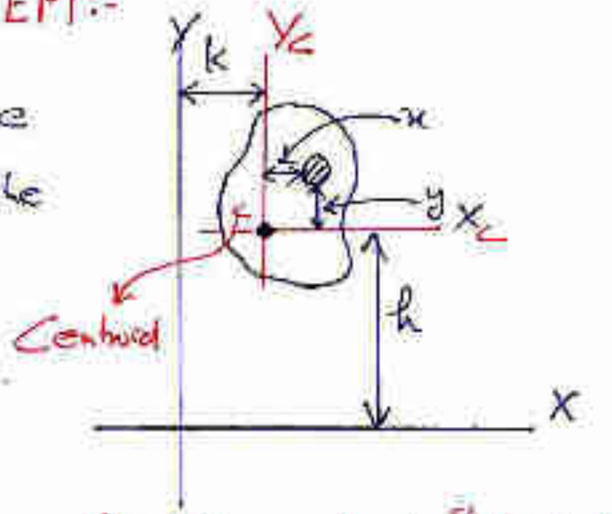
$$I_z = \int x^2 dA + \int y^2 dA$$

$$I_z = I_y + I_x$$

$$I_z = I_x + I_y = I_{x'} + I_{y'} + I_{x''} + I_{y''} + \ldots\ldots$$

"According to this theorem the M.O.I of a given area about a pole is equal to the sum of M.O.I. of that area about any two mutually perpendicular axis passing through the pole."

F) PARALLEL AXIS THEOREM:-



X_c, Y_c are passing through the centroid. Here translation of the axis.

So M.O.I about X-axis;

About the centroid, 1st M.O.I of the about the axis will be Zero

$I_x = \int y^2 dA$

here y will be "y+h"

$I_x = \int (y+h)^2 dA$

$I_x = \int (y^2 + 2hy + h^2) dA$

$\int y\,dA$ is first M.O.A area about a centroidal axis

$\therefore \int y\,dA = 0$

$I_x = \int y^2.dA + 2h\overset{\nearrow 0}{\cancel{\int y\,dA}} + \int h^2 dA$

$I_x = I_{xc} + Ah^2$

$I_x = I_{xc} + Ah^2$

$I_y = \int x^2.dA$

$I_y = \int (x+k)^2 dA$ $\because \int x\,dA = 0$ [because about centroidal axis]

$I_y = I_{yc} + Ak^2$

$I_y = I_{yc} + Ak^2$

$I_{xy} = \int (x+h)(y+k) dA$

$I_{xy} = \int xy + xk + hy + hk \; dA$

$I_{xy} = \int xy\,dA + \int x\overset{\nearrow 0}{\cancel{k}}\,dA + \int y\overset{\nearrow 0}{\cancel{h}}\,dA + \int hk\,dA$

$$\left.\begin{aligned} I_{xy} &= I_{xcyc} + A h . k. \\ I_x &= I_{x^c} + Ah^2 \\ I_y &= I_{y^c} + Ak^2 \end{aligned}\right\}$$ Principal Equation for parallel Axis Theorem

Overview :-

Definition

$\rightarrow I_x = \int y^2 dA$

$\rightarrow I_y = \int x^2 dA$

$\rightarrow I_{xy} = \int xy\,dA$

Principal Axis

$\rightarrow \tan 2\theta^* = \frac{-I_{xy}}{(I_x - I_y)/2}$

$$\left.\begin{aligned} &\rightarrow I_{max} = I_{avg} + R \\ &\rightarrow I_{min} = I_{avg} - R \end{aligned}\right\} (I_{xy})_{\theta^*} = 0$$

⊥-axis theorem

$\rightarrow I_z = I_x + I_y = I_{x'} + I_{y'} = I_{x''} + I_{y''} + \ldots$

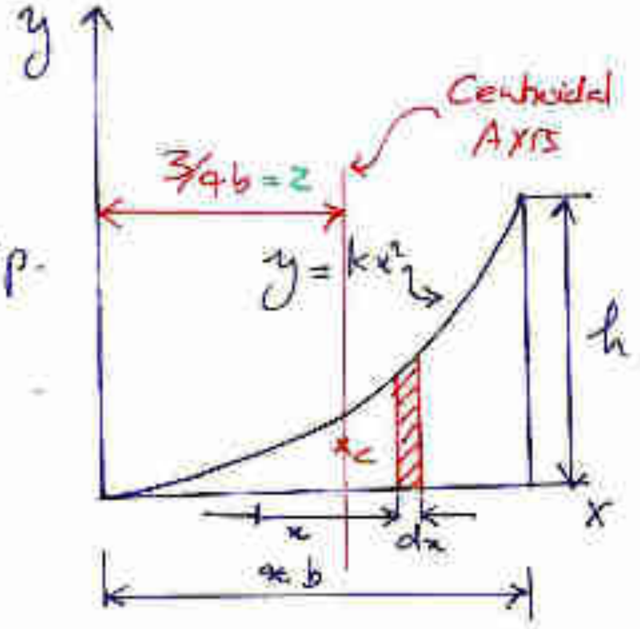
||-axis theorem

$\rightarrow I_x = I_{x^c} + Ah^2$

$\rightarrow I_y = I_{y^c} + Ak^2$

$\rightarrow I_{xy} = I_{x^c y^c} + Ah.k.$

Q Find M.O.I about Y-axis?



$I_y = \int x^2 dA$

Area = dA = area of the strip.

Area = $y\,dx$.

$I_y = \int x^2 . y\,dx$

here $y = kx^2$

Limit will be $0 \to b$

$I_y = \int_0^b x^2 . kx^2 dx$.

$I_y = k \int_0^b x^4 dx$

$I_y = k \left[\frac{x^5}{5}\right]_0^b$

Applying limits →

$I_y = \frac{k \times b^5}{5}$ —— ②

$y = kx^2$

at $x = b$; $y = h$

$h = k . b^2$ —— ①

Substituting ① in ② →

$I_y = \frac{k.b^2.b^3}{5} = \frac{hb^3}{5}$

To evaluate I_{yc} determine I_{yc}

$I_{y\cancel{c}} = I_{y^c} + Az^2$ (New Axis = Centroid + Area)

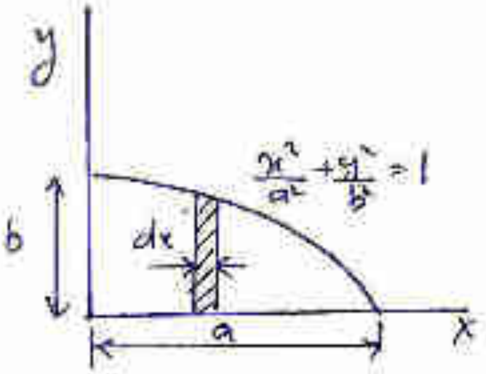
$I_{y\cancel{c}}^{c} = {}^{+}\frac{hb^3}{5} +{}^{-} \frac{1}{3} \times b \times h \times (3/4 . b)^2$

$I_{y\cancel{c}}^{c} = {}^{+}\frac{hb^3}{5} +{}^{-} \frac{1}{\cancel{3}} b^3 h \times \overset{3}{\cancel{9}}/\cancel{16}\ 3/16.$

$I_{yc} = {}^{+}\frac{hb^2}{5} +{}^{-} 3/16\ b^3 h$

$I_y^c = \frac{\cancel{3}1}{80} h b^3$

$$9 \quad I_{yc} = ? \quad ; I_{xc} = ?$$



$$I_y = \int x^2 dA$$

$$I_y = \int x^2 . y \, dA$$

area $= y\,dx = dA$

Here $I_y = \int x^2 . dA$

$$I_y = \int_0^a x^2 . y \, dx . \quad \text{———} ①$$

Here $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2} . \quad \text{———} ②$$

Sub. ② in ①

$$I_y = \int_0^a x^2 . \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$

$$I_y = \frac{b}{a}\int_0^a x^2 . \sqrt{a^2 - x^2}\, dx$$

$x = a\sin\theta$.

$dx = a\cos\theta d\theta$

at $x = 0$; $\theta = 0$

at $x = a$; $\theta = \pi/2$

$$\therefore I_y = \frac{b}{a}\int_0^{\pi/2} a^2\sin^2\theta \sqrt{a^2 - a^2\sin^2\theta} . a\cos\theta d\theta$$

$$I_y = \frac{b}{a}\int_0^{\pi/2} a^4 . \sin^2\theta . \cos\theta . \cos\theta . d\theta .$$

$$I_y = a^3 b \int_0^{\pi/2} \sin^2\theta . \cos^2\theta . d\theta$$

$$I_y = a^3 b \int_0^{\pi/2} \left(\frac{\sin 2\theta}{2}\right)^2 d\theta$$

$$I_y = \frac{a^3 b}{4}\int_0^{\pi/2} \sin^2 2\theta \, d\theta$$

$$I_y = \frac{a^3 b}{4}\int_0^{\pi/2} \left(\frac{1 - \cos 4\theta}{2}\right) d\theta$$

$$I_y = \frac{a^3 b}{8}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$$

$$I_y = \frac{a^3 b}{8}\left[\pi/2 - \frac{\sin 4 \times \pi/2}{4} - 0 + 0\right]$$

$$I_y = \frac{a^3 b}{8} \times \pi/2 = \underline{\underline{\frac{\pi a^3 b}{16}}}$$

$$I_y = I_{yc} + \text{Area} \times \text{gap}^2$$

Here area $= \frac{\pi a b}{4}$

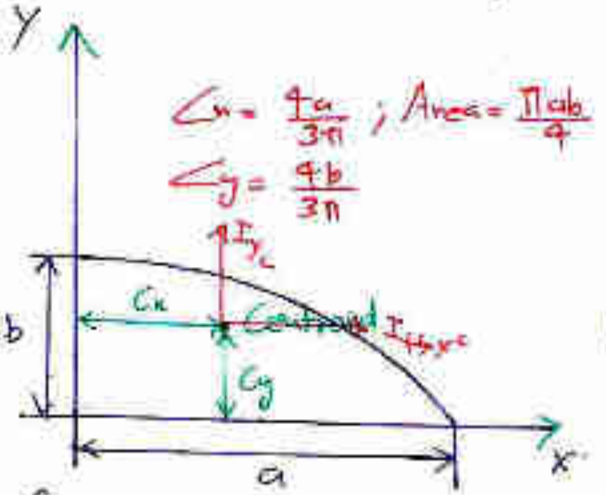
Cn-centroid for x-axis $= \frac{4a}{3\pi}$

$$\therefore \frac{\pi a^3 b}{16} = I_{yc} + \frac{\pi a b}{4} . \left(\frac{4a}{3\pi}\right)^2 .$$

$$I_{yc} = \frac{\pi a^3 b}{16} - \frac{\pi a b}{\cancel{4}} . \frac{\overset{4}{\cancel{16}} a^2}{9\pi^2}$$

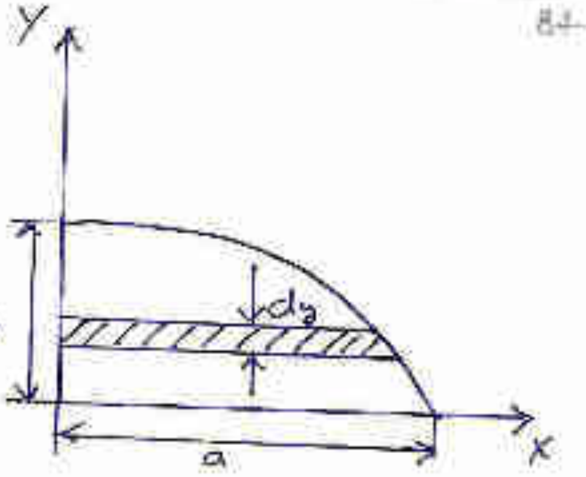
$$I_{yc} = \frac{\pi a^3 b}{16} - \frac{4a^3 b}{9\pi}$$

$$\underline{\underline{I_{yc} = a^3 b\left[\frac{\pi}{16} - \frac{4}{9\pi}\right]}}$$



$C_x = \frac{4a}{3\pi}$; Area $= \frac{\pi ab}{4}$

$C_y = \frac{4b}{3\pi}$



$I_x = \int y^2 \, dA$

$dA = x\,dy$.

$I_x = \int_0^b y^2 \cdot x\,dy$ —— ②

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}$

$x = \frac{a}{b}\sqrt{b^2 - y^2}$ —— ①

Sub. ① in ② $\longrightarrow$.

$I_x = \int_0^b y^2 \cdot \frac{a}{b}\sqrt{b^2-y^2}\,dy$.

$I_x = \frac{a}{b}\int_0^b y^2 \cdot \sqrt{b^2-y^2}\,dy$.

$y = b\sin\theta$.

$dy = b\cos\theta\,d\theta$.

at $y = 0$; $\theta = 0$

$y = a$; $\theta = \pi/2$

$I_x = \int_0^{\pi/2} \frac{a}{b} \cdot b^2\sin^2\theta \sqrt{b^2 - b^2\sin^2\theta} \cdot b\cos\theta\,d\theta$.

$I_x = \frac{a}{b} \cdot b^4 \int_0^{\pi/2} \sin^2\theta \cdot \cos^2\theta\,d\theta$.

$I_x = ab^3 \int_0^{\pi/2} \sin^2\theta \cdot \cos^2\theta\,d\theta$

$I_x = ab^3 \int_0^{\pi/2} \left(\frac{\sin 2\theta}{2}\right)^2 d\theta$.

$I_x = \frac{ab^3}{4} \int_0^{\pi/2} \frac{1 - \cos \overset{4}{2}\theta}{2}\,d\theta$

$I_x = \frac{ab^3}{4} \times \frac{1}{2} \left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$.

$I_x = \frac{ab^3}{8}\left[\pi/2 - \frac{\sin 4 \times \pi/2}{4} - 0 + 0\right]$

$I_x = \frac{ab^3 \cdot \pi}{16} = \frac{\pi ab^3}{16}$

$I_x = I_{x^c} + \text{Area} \times \text{gap}^2$

Here gap $= C_y = \frac{4b}{3\pi}$; area $= \frac{\pi ab}{4}$.

$\frac{\pi ab^3}{16} = I_{x^c} + \frac{\pi ab}{4} \times \left(\frac{4b}{3\pi}\right)^2$

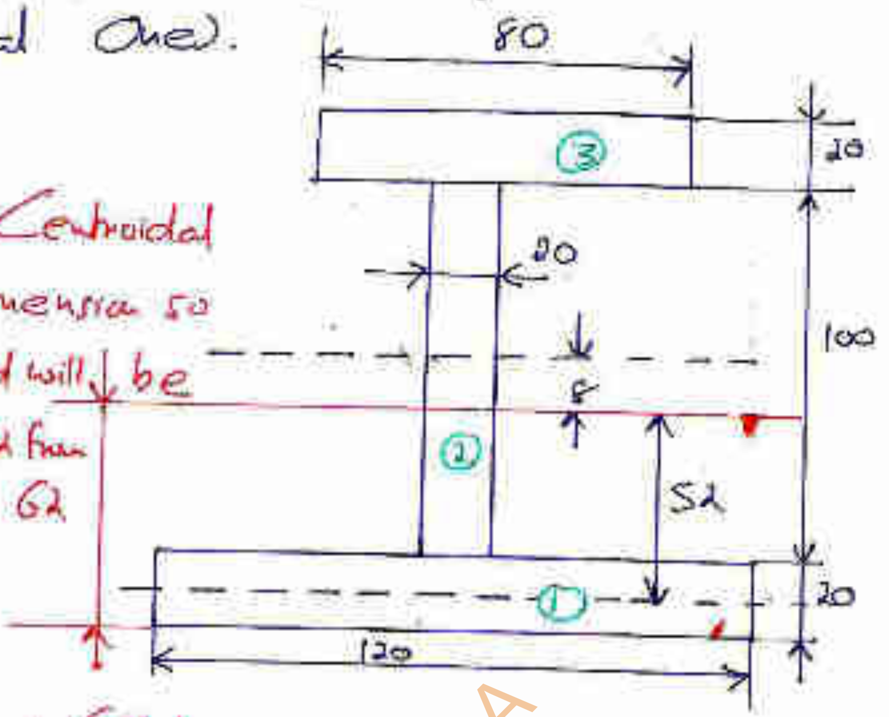
$\frac{\pi ab^3}{16} = I_{x^c} + \frac{\cancel{\pi} ab}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}} b^2}{9\pi^2}$

$\frac{\pi ab^3}{16} = I_{x^c} + \frac{4}{9\pi} ab^3$.

$I_{x^c} = \frac{\pi ab^3}{16} - \frac{4}{9\pi} ab^3$

$I_{x^c} = ab^3 \left[\frac{\pi}{16} - \frac{4}{9\pi}\right]$

9 Determine the M.O.I of Composite figure about Centroidal-Axis (horizontal One).



Similar problem in Centroidal Reigion. Same dimension so that the Centroid will be at a distance of 62 from the base. 62

$\bar{x} = 62$

$I_x = (I_x)_1 + (I_2)_x + (I_3)_x$

Consider Area ①:

Moment of inertia = Centroidal M.O.I + Area×gap².

Here $(I_1)_x = \left[\frac{120\times20^3}{12} + (120\times20)\,(62-10)^2)\right]$

$(I_1)_x = \left[80000 + 2400\times2704\right]$

$(I_{1x}) = 6569600\ mm^4$

Consider area ②:

$(I_2)_x = \left[\left(\frac{20\times100^3}{12}\right) + (20\times100)\times8^2\right]$

$(I_2)\ x = \left[1666666.7 + 128000\right]$

$(I_2)\ x = 1794666.7\ mm^4$

Consider area ③:

$(I_3)\ x = \left[\left(\frac{80\times20^3}{12}\right) + (80\times20)\times\overset{6}{\cancel{7}}8^2\right]$

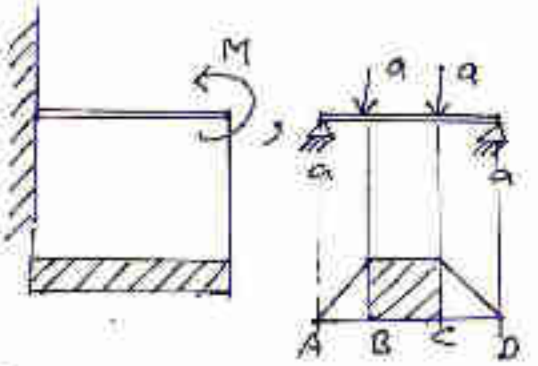
$(I_3)\ x = \left[53333.33 + \overset{4}{\cancel{9734400}}\right]$ 739400

$(I_3)x =$ ~~9784733.33 mm⁴~~ $7451733.33\ mm^4$

$I_x = (I_1)_x + (I_2)_x + (I_3)_x = 1.58\times10^7\ mm^4$

3.5. BENDING:- (only discuss for "Pure bending" $\frac{dM}{dx} = \overset{0}{\cancel{F}}$)

Example of Pure Bending :-
(where all the force will be finally reduced to zero)



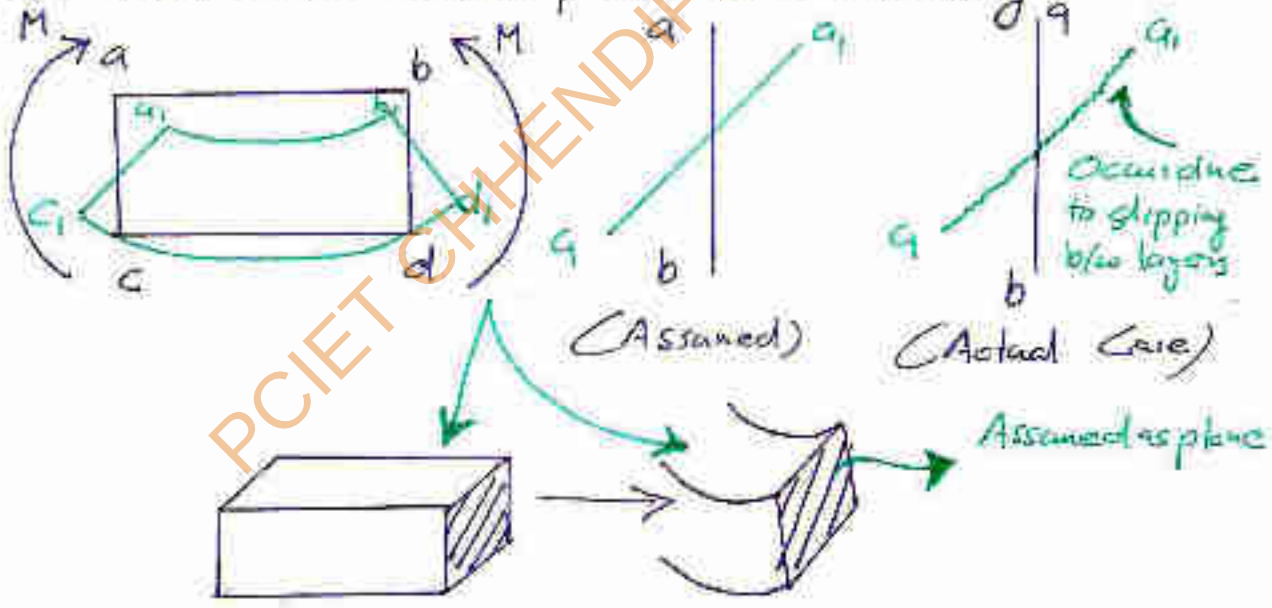
→ 3 cases – Cantilever at a end moment
- Simply supported beams
- Overhanging beam.

ESE*

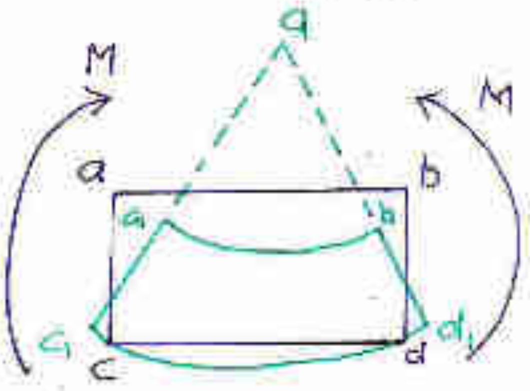
→ ASSUMPTIONS (Pure Bending):-

(i) Shear force is Zero at all points of the beam. Consequently bending moment is constant. (SF=0; BM=c)

(ii) Plane Cross section remain plane after bending



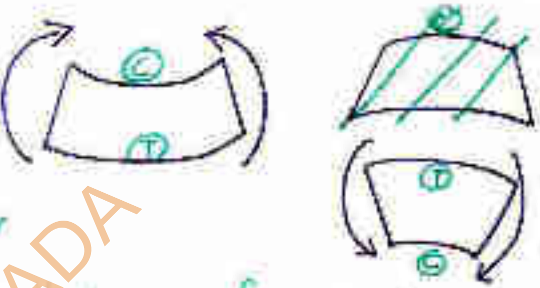
(iii) All longitudinal surfaces of the beam bend into Concentric circular arcs.



(IV) The bending Moment is being applied on one of the principal axis of the section.

b) BENDING STRESS:-

→ Nature : σ

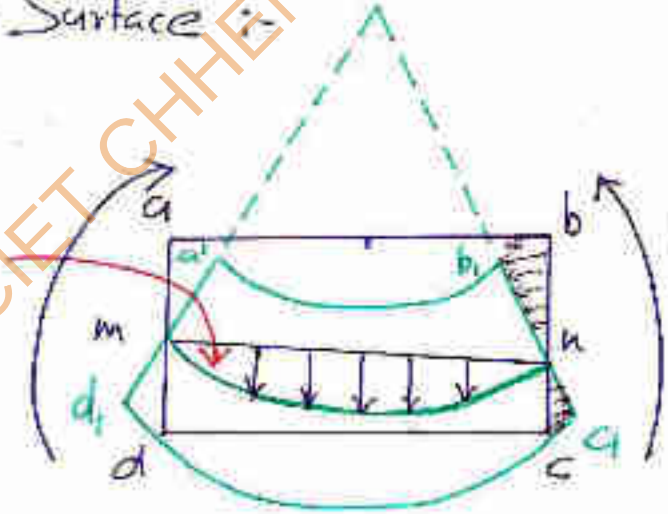


→ Distribution : Non-Uniform, Linear

→ Magnitude : $\sigma = \left(\frac{M}{I_{NA}}\right) y$ [y - distance from N.A]

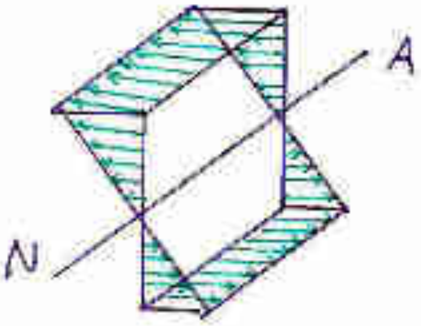
→ Neutral layer / Surface :-

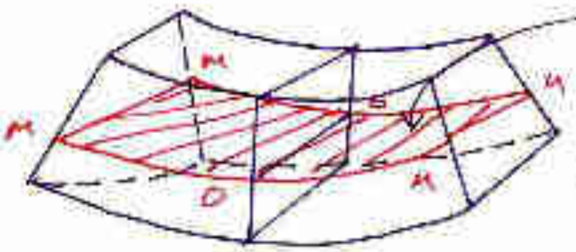
Zero-displacement layer called Neutral layer



bc- b/w +ve and -ve disp. there is zero displacement.

Distribution:-





Neutral Surface

mm
nn
oo } Neutral Axis

Neutral Surface :-

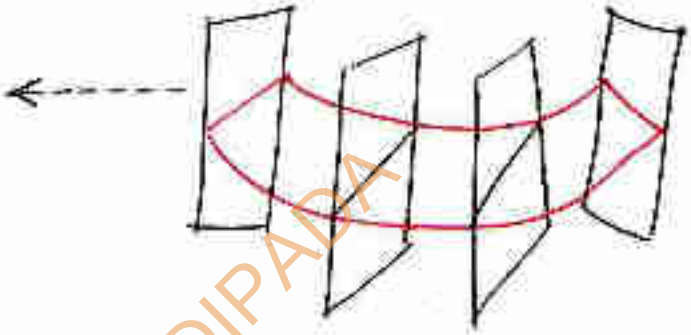
A longitudinal Surface at which axial displacement are zero is known as the Neutral Surface of the beam.

ESE Since axial displacements at neutral Surface are always Zero, the longitudinal Stress and Strain is also be zero.

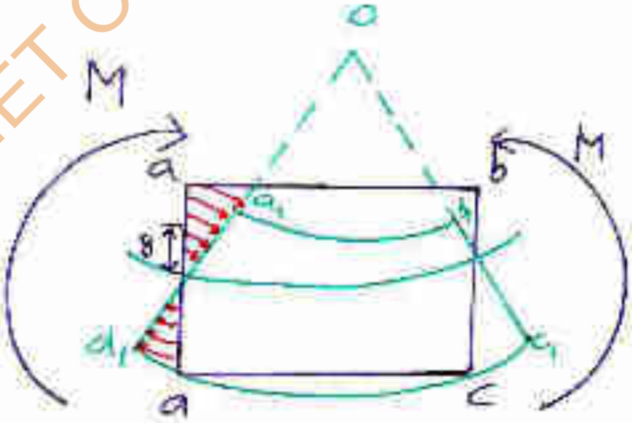
The line Neutral Surface meets particular cross section is known as Neutral Axis of that Cross section.

Neutral layers/Axis

Neutral axes are infinite because of infinite section.



Displacement $\propto$ Distance from the Neutral Axis



From here

Displacement $\propto y$

Stress $\propto y$

Resistance $\propto y$

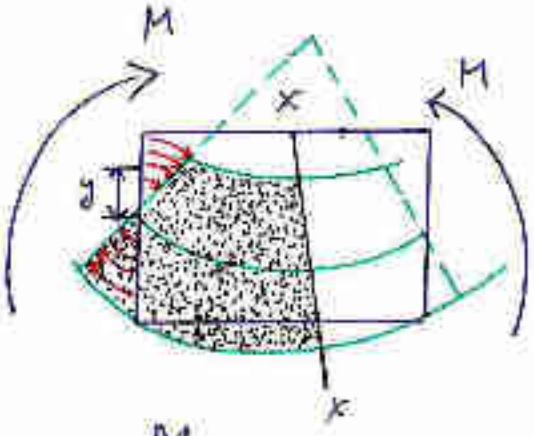
$\sigma \propto y$

$\sigma = ky$

Cutting the Section at X-X

Equilibrium

(Total External Moment) = (Total internal moment.)



$$M = \int \sigma . dA . y$$

$$M = \int k y . y \, dA$$

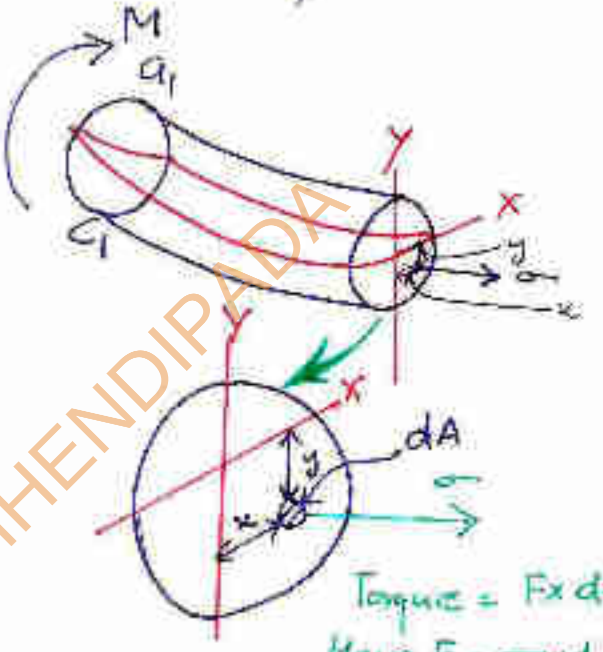
$$M = \int k y^2 . dA$$

$$\frac{M}{k} = \int y^2 dA$$

Since $\int y^2 dA = I$

$$\therefore \; M = K . I_y$$

$$\boxed{K = \frac{M}{I_{N.A}}}$$



Torque = F x disp.

Here $F = \sigma . dA$

disp. = y

Force Equilibrium;

(External Force) = (Total Internal Force)

According the 1st Assumption F=0

$$0 = \int \sigma . dA$$

$$\int \sigma . dA = 0$$

$$\int \frac{M}{I_{NA}} . y \, dA = 0$$

$$\frac{M}{I_{NA}} \int y \, dA = 0$$

Here there are three possibilities → $M = 0$ ✗

→ $I = \infty$ ✗

→ $\int y\, dA$ ✓

$$\therefore \int y\, dA = 0$$

→ Since $\int y\, dA$ is the First moment of inertia. Since first moment of inertia is zero there about Neutral Axis it must pass through the Centroid of the Section.

Taking Moment Equilibrium about Y-axis,

External Moment about Y = 0

External moment = (Total internal moment about Y)

$$\int \sigma . dA \times x = 0$$

$$\int \frac{M}{I} . y . dA . x = 0 .$$

$$\frac{M}{I} \int xy . dA = 0 .$$

$$\downarrow$$

$$I_{xy} = 0$$

→ Since product of inertia about Neutral Axis is Zero, the Neutral Axis must be the principal Axis of the Section.

→ B.M and Moment of inertia are for a unique section.

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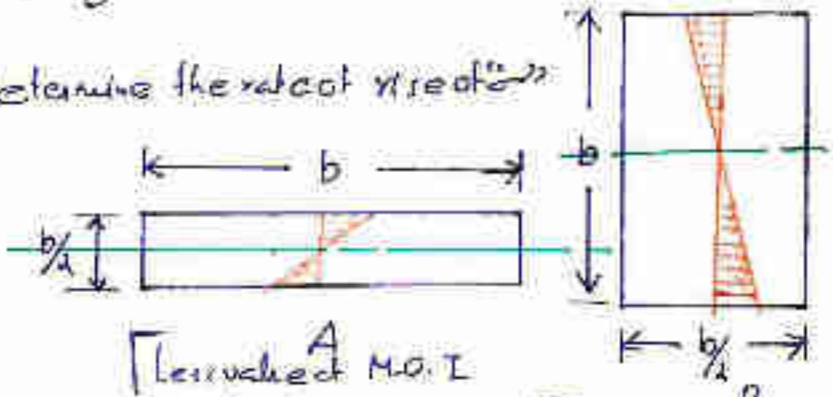
Q B.M applied on the both case are same. Maximum bending stress in A and B related by.

$\frac{M}{I} \rightarrow$ determine the rate of rise of σ



[less value of M.O.I so that $\frac{M}{I}$ will be more so that rate of rise of σ will be high]

[Area farther more the moment of inertia less value of "M/I" Rate of increase of σ will be slow.]

$$\frac{(\sigma_{max})_A}{(\sigma_{max})_B} = \frac{\frac{M}{I_A}\cdot y_A}{\frac{M}{I_B}\times y_B}$$

$$\frac{(\sigma_{max})_A}{(\sigma_{max})_B} = \frac{y_A \cdot I_B}{I_A \cdot y_B}$$

$$\frac{(\sigma_{max})_A}{(\sigma_{max})_B} = \frac{\left(\frac{b/2}{2}\right)\cdot\frac{b}{2}(b)^3\times\frac{1}{\cancel{12}}}{(b/2)\cdot\frac{1}{\cancel{12}}\cdot b\cdot(b/2)^3} = \frac{\cancel{b/4}\times\frac{1}{2}}{\cancel{b/2}\times 1/8} = \frac{16}{8}.$$

$$(\sigma_{max})_A = (\sigma_{max})_B \times 2$$

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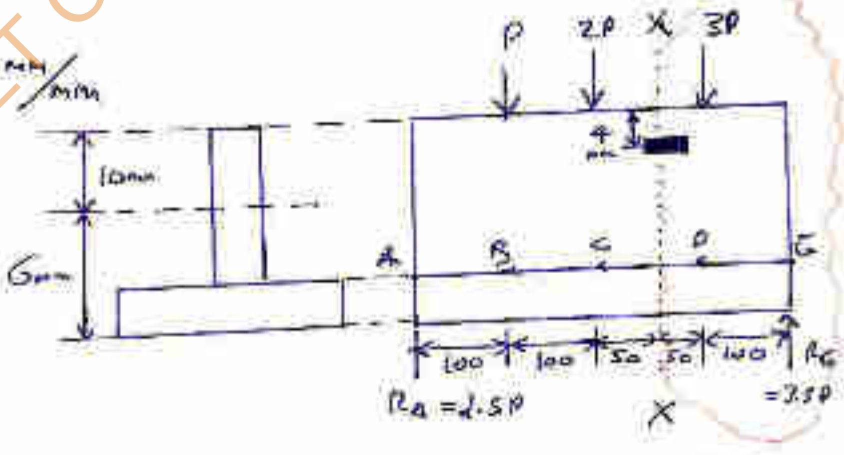
9.2 *

* Compressive strain = 1.5 μ mm/mm

$E = 20\times10^3$ MPa

$I_{NA} = 2176\ mm^4$



$$\delta L = \frac{PL}{AE}$$

$$\frac{\sigma}{\varepsilon} = E$$

μ mm → 10^{-6} mm

Micro mm → N mm → 10^{-6}

$M_A = 0$.

$$P\times100 + 2P\times200 + 3P\times300 = R_E\times400$$

$$P + 4P + 9P = R_E\times4$$

$$R_E = \frac{14P}{4} = \frac{7P}{2}$$

$$R_A + R_E = P + 2P + 3P$$

$$R_A = \frac{10P}{4} = \frac{5P}{2}$$

Calculate the BM at right Section

$BM = -3P \times 50 + 3.5P \times 150$

$BM = \underline{375\ Nmm}$

Force F is at a distance of 6mm from the neutral axis.

$$\frac{M}{I} = \frac{\sigma}{Y}$$

$$\sigma = \left(\frac{M}{I}\right) \cdot Y_{at\ F}$$

At one given section $\frac{M}{I}$ will be constant. Never changes.

$$\sigma = \left(\frac{375P}{2176}\right) \cdot 6$$

$$\sigma = \underline{1.03P}$$

$\varepsilon \cdot E = 1.03P$

$1.5 \times 10^{-6} \times 200 \times 10^{3} = 1.03P$

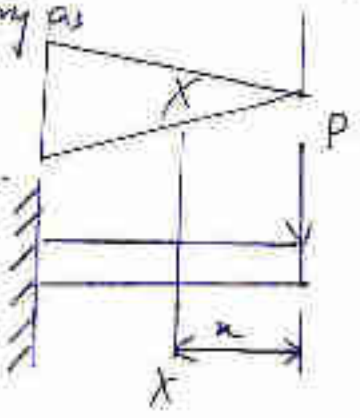
$0.3 = 1.03P$

$P = \underline{0.299}$

A cantilever has a uniform thickness 't' throughout subjected to a point a Point load 'P' at the free end. In order that the max. bending stress remains uniform at σ_0 throughout the beam length the width of the beam section should vary as

[✓ a) x] b) x^2 c) $\sqrt{x}$ d) $\frac{1}{\sqrt{x}}$

where x is measured from free end.



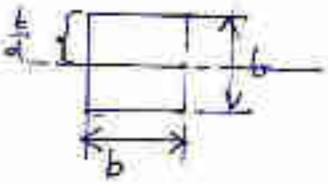
Consider section X–X ; $M = Px$

width of that section be b_x

$$(\sigma_{max}) = \frac{1Px}{\frac{b_x \cdot t^{3}}{\cancel{12}\,6}} \times \frac{\cancel{t}}{\cancel{2}}$$

$$\sigma_{max} = \frac{6Px}{b_x \cdot t^2}$$

$$b_x = \frac{6Px}{\sigma_{max} \cdot t^2} \ ; \ b_x \propto x$$



26/7/2017

Pg No. 93

Q5) * $\sigma = \frac{M}{I} y$

Considering the bottom of the beam.

→ Doit one section

$y_{bottom} = 5mm$

$I = \frac{10 \times 10^3}{12} =$

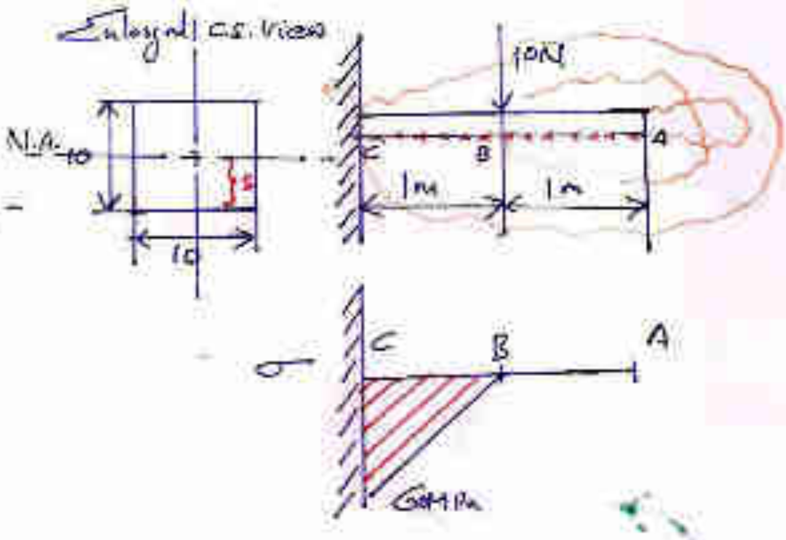
$\sigma = 6 \times 10^{-3} M$

$\sigma_A = 6 \times 10^{-3} M_A$

$M_A = 0$ Because of that section.

$M_B = 0$; Because of that section.

$M_C = 10 \times 1000 = Nmm = \underline{\underline{10^4 Nmm}}$



Here we calculate are used for the bottom midpoint.

Pg No. 94

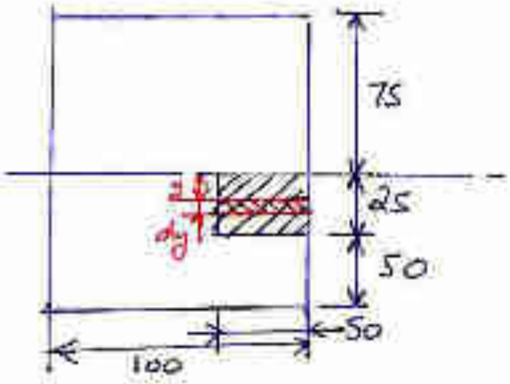
Qu. 6)

$\sigma = 6 \times 10^4 \times 10^{-3} = 60 MPa$

depth = 150mm; width 100mm.

B.M = 16kNm

About the horizontal axis BM = 16kNm.



$\boxed{F = \sigma \times A}$

Integrate

$dA = 50 \times dy$

$$F = \int_{y=0}^{y=25} \sigma \, dA$$

Here σ varies from y = 0 to y = 25

$$F = \int_0^{25} \sigma \times 50 \times dy$$

$$M = \frac{100 \times 150^3}{12} = 2.812 \times 10^7 mm^4$$

$$\sigma = \frac{M}{I} y = \int_0^{25} \frac{16 \times 10^3 \times 10^6}{2.814 \times 10^7} \times y \, dy \times 50$$

For given section one Unique value of B.M. and MoI.

MoI resistant by entire force.

$$F = \int_0^{25} \left(\frac{16 \times 10^6}{\frac{100 \times 150^3}{12}} \right) \times y \times 50\,dy$$

$$F = \int_0^{25} 0.568 \times y \times 50\,dy.$$

$$F = 0.568 \times 50 \times \frac{25^2}{2}$$

$$F = \underline{\underline{8.88\ \text{kN}}}$$

$$\sigma = \frac{F}{A}.$$

$$F = \sigma_{avg} \times A$$

Only valid for rectangular section.

$F = \sigma_{avg} \times A$ [For rectangle only]

$$\sigma = \frac{F}{A}$$

$$F = \sigma_{avg} \cdot A$$

$$\sigma = \frac{M}{I} \cdot y$$

$$\sigma = \frac{16 \times 10^6}{\frac{100 \times 150^3}{12}} \times y$$

at $y = 0$; $\sigma_1 = 0$

at $y = 25$; $\sigma_1 =$ ~~8.89~~ $14.2\ \text{kN/mm}^2$

$$F = \left(\frac{0 + 14.2}{2} \right) \times 50 \times 25 = \underline{\underline{8.875\text{kN}}}$$

Pg No. 49
Q. 1 ?

Calculate Tensile stress?

$M_A = 0$

$M_B = 0 + \bar{0}.3 \times 3 = {}^-0.9\ \text{kN m}$

$M_C = {}^-0.3 \times 3 = {}^-0.9\ \text{kN m}$

$I_{xx} = 3 \times 10^6\ \text{mm}^4$

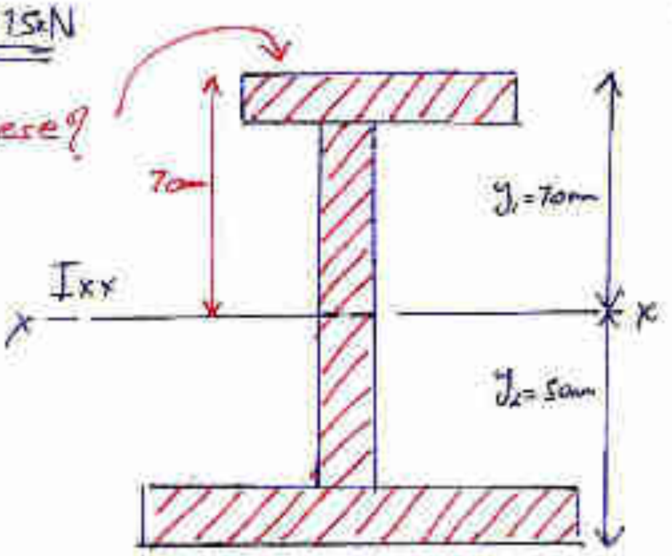
$M_{B_{max}} = 0.9 \times 10^6\ \text{N mm}$

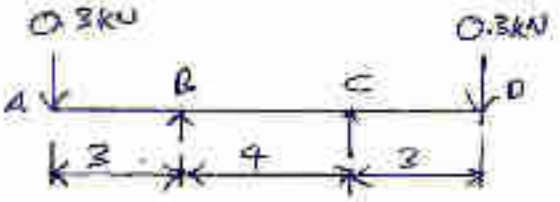
⟶ Tension occur at the top (▨)

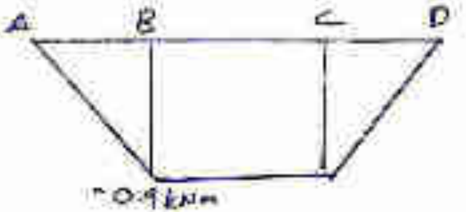
So $y_{max} = 70\text{mm}$.

$$\sigma_T = \frac{M}{I} \times y = \frac{0.9 \times 10^6}{3 \times 10^6} \times 70$$

$$\sigma_T = \frac{3}{10} \times 70 = \underline{\underline{21\ \text{MPa}}}$$







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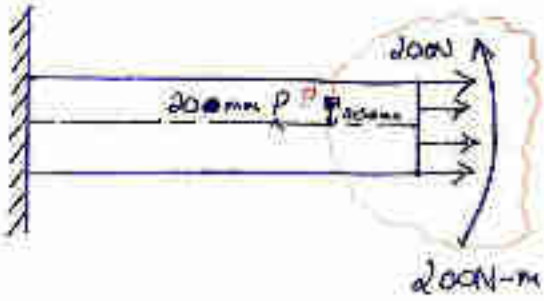
Q. 89 Moment of Inertia = $1.33 \times 10^{-3} m^4$

$F = 200N$; $C.S. = 0.1 m^2$



Point P is at a distance 20mm - from the neutral Axis.

Axial loading = 200N.

Torsional loading = 0N ; So No shear. [No shear - No torsion]

$\tau = 0 N/mm^2$

B.M = 200 N m

Only Axial loading & Bending

$\sigma_a \rightarrow$ axial stress

$\sigma_a = P/A = 200/0.1 = 2000 N/m^2$.

$\sigma_b = \frac{M}{I} \cdot Y$

$\sigma_b = \frac{200}{1.33 \times 10^{-3}} \times 20 \times 10^{-3} = 3007 Pa$

$\sigma_b = \underline{3007 Pa}$

P is at a distance 20mm from Neutral layer

P is subjected to Compression by bending

$\sigma_b > \sigma_a$

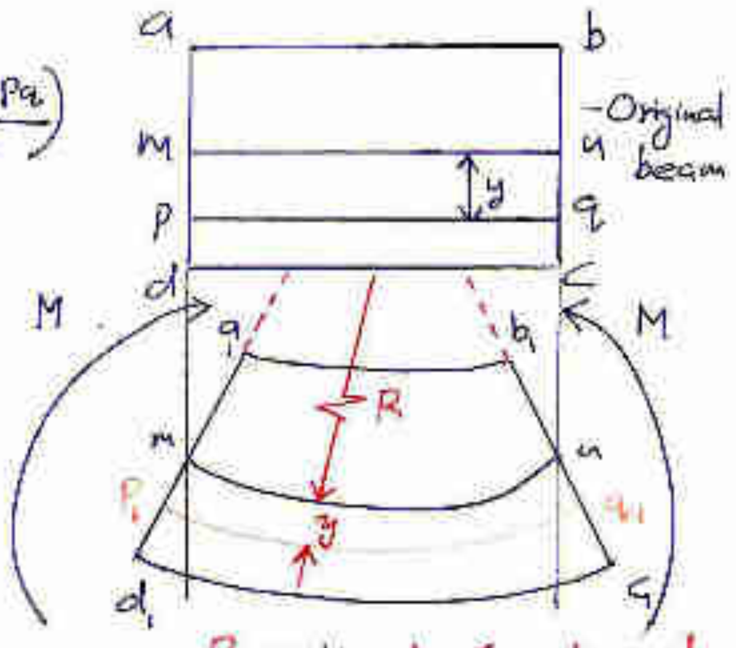
So stress $\sigma_b = 3007 Pa$; $\sigma_a = 2000 N/m^2$

$\sigma = \sigma_b - \sigma_a = 1007 Pa$

So Compression (because of more compression to bending)

So answer will be

c) HOOKE'S LAW:



R-radius of Curvature of Neutral Surface.

Deformed beam.

$$\sigma = \varepsilon . E = E\left(\frac{P_1q_1 - pq}{pq}\right)$$

$$\sigma = E\left(\frac{P_1q_1 - pq}{pq}\right)$$

where $pq = y$.

$P_1q_1 = (R+y)\,\phi$

ϕ is the angle of swept by the arc

Similarly $mn = R\phi$

$$\sigma = E\left(\frac{(R+y)\phi - R\phi}{R\phi}\right)$$

$$\sigma = E\left(\frac{R\phi + y\phi - R\phi}{R\phi}\right)$$

$$\sigma = E\left(\frac{y\phi}{R\phi}\right)$$

$$\sigma = E\left(\frac{y}{R}\right)$$

$$\sigma = E\left(\frac{1}{R}\right)y$$

$$\sigma = E \times \left(\frac{1}{R}\right).y$$

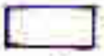
By using Equilibrium, $\sigma = \left(\frac{M}{I}\right)y$.

$\therefore \frac{\sigma}{y} = \left(\frac{M}{I}\right) = \frac{E}{R} \longrightarrow$ Bending Equation / Flexural Eqn.

$R = \frac{L}{\phi} \quad \therefore \frac{1}{R} = \frac{\phi}{L}$

$$\boxed{\frac{\sigma}{y} = \frac{M}{I} = E \times \left(\frac{\phi}{L}\right)}$$

$\frac{1}{R} = \frac{M}{E.I}$



$\rightarrow R = \infty \quad \therefore \frac{1}{R} = 0$

$\rightarrow$ R very small $\quad \frac{1}{R} =$ lge

$\frac{1}{R} \rightarrow$ How the beam deformed ? (More or less)

$\frac{1}{R} = \frac{M}{E.I}$

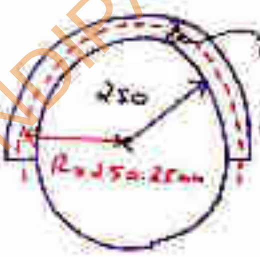
$\frac{1}{R} \rightarrow$ Curvature (Inversly proportional to the radius)

$\frac{1}{R} = \frac{M}{\boxed{EI}} \rightarrow$ If EI is more so that $\frac{1}{R}$ will be less so-that curvature will be almost 0. So 'R' will be come infinite, that is the beam will be perfectly straight.

EI — Flexural Rigidity (Should have higher EI)

Pg. 93 / Q.39?

$\sigma = \left(\overset{M}{\frac{E}{I}}\right) y$



$y = 0.5/2 = 0.25$mm

$\sigma = \left(\frac{E}{R}\right) \cdot y_{max}$

$E_{steel} = 200 \times 10^3$ MPa.

$\sigma_{max} = \left(\frac{200 \times 10^3}{250.25} \times 0.25\right)$

$\sigma_{max} = 199.8 \simeq \underline{\underline{200 \text{ MPa}}}$

d) BEAM STRENGTH:-

The "maximum bending" moment that a beam of given material and dimensions can transmit/Resist safely. is known as that the Strength of the beam.

$$\sigma_{max} \leq \hat{\sigma} \text{ (Allowable Stress)}$$

$$\sigma_{max} \leq \hat{\sigma}$$

$$\frac{M}{I} \cdot y_{max} \leq \hat{\sigma}$$

$$M \leq \hat{\sigma} \cdot \frac{I}{y_{max}}$$

$$\boxed{\hat{M} = \hat{\sigma} \cdot \frac{I}{y_{max}}}$$

$$\hat{M} = \overset{\text{Material}}{\hat{\sigma}} \cdot \underbrace{\frac{I}{y_{max}}}_{\text{Crossectional Dimension.}}$$

$$\frac{I}{y_{max}} = Z \text{ (Section Modulus)}$$

$$\hat{M} = \hat{\sigma} \cdot Z$$

$$\rightarrow Z_{rectangle} = \frac{I}{y} = \frac{bd^3/12}{d/2} = bd^2/6$$

$$\rightarrow Z_{circle} = \frac{I}{y} = \frac{\pi d^4/64}{d/2} = \pi d^3/32$$

Q $\hat{M}_s$ and $\hat{M}_c$ are the strength of a beam of a square cross-section and of circular cross section. having equal area and same material. Which of the following is correct?

a) $\hat{M}_c$ and $> \hat{M}_s$ ~~b)~~ $\hat{M}_s > \hat{M}_c$ c) $\hat{M}_s = \hat{M}_c$ d) None.

— $\hat{M} = \sigma . Z$

$\hat{M}_s = \sigma_s . Z_s$; $\hat{M}_c = \sigma . Z_c$

$$\frac{\hat{M}_s}{\hat{M}_c} = \frac{\sigma_s \; Z_s}{\sigma_c . Z_c}$$

$$\frac{\hat{M}_s}{\hat{M}_c} = \frac{\sigma_s \; d^3/6}{\sigma_c . \pi d^3/32}$$

let side of square be b. [b=d]

$$\frac{\hat{M}_s}{\hat{M}_c} = \frac{\cancel{\sigma_s} . b^3/6}{\cancel{\sigma_s} . \pi d^3/3\cancel{2}}$$ [Same material]

Area of C.S. are equal.

$$\frac{\cancel{\pi} \cancel{d^2}}{\cancel{4}} = b^2$$

$b = \cancel{\pi} \cancel{d}$

$$\frac{\cancel{\hat{M}_s}}{M_c} = \frac{\cancel{b^3/6}}{\pi \cancel{b^3}/32}$$

$$\frac{\hat{M}_s}{\hat{M}_c} = \frac{32}{6\pi}$$

$$\hat{M}_s = \frac{32}{6\pi} \hat{M}_c$$

$$\frac{\pi}{4} . d^2 = b^2$$

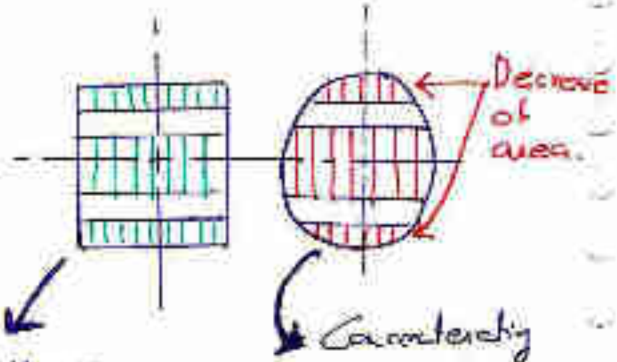
$$b/d = \pi/4 = \sqrt{\pi}/2$$

$$\frac{\hat{M}_s}{M_c} = \frac{b^3/\cancel{6}}{\cancel{\pi} b^3/32} = \frac{b^3/6}{\pi d^3/32}$$

$$\frac{\hat{M}_s}{\hat{M}_c} = \frac{32}{6\pi} . \left(\frac{b}{d}\right)^3$$

$$\frac{\hat{M}_s}{\hat{M}_c} = \frac{32}{6\pi} \left(\frac{\sqrt{\pi}}{2}\right)^3$$

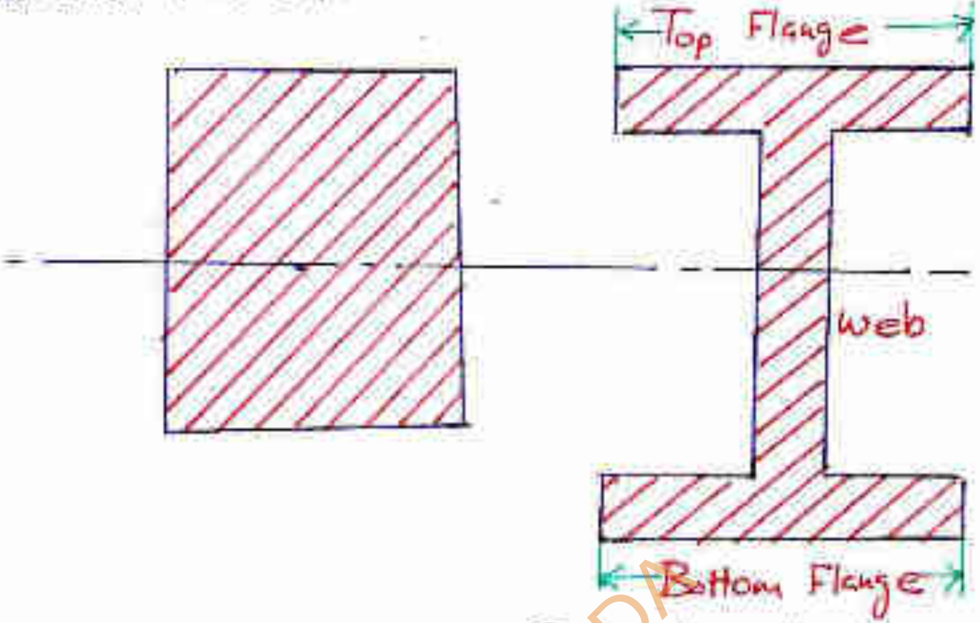
$$\frac{\hat{M}_s}{\hat{M}_s} = \underline{\underline{1.18 > 1}}$$



More M.O.I. More Section Modulus.

Concentrating material near the neutral axis. Poorer M.O.I. Poorer section Modulus.

→ By taking the material away from the neutral axis and just-placing at the periphery. More moment of inertia is placed outside cut outsider. Thus birth of I section came.



<Material Outside - away from Neutral Axis>

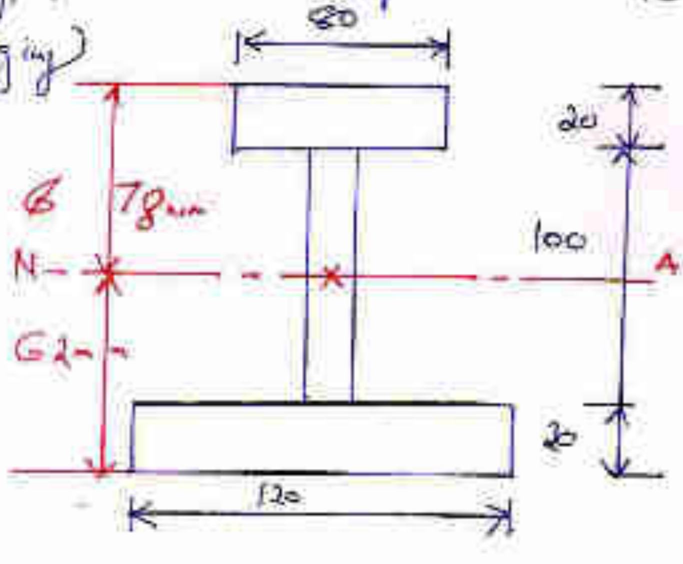
→ Beams are not favourable for tension for which is mainly at the bottom side of section. Eg: For Concrete beam steel reinforcement are given. Every material have more compressive strength than that of tension. Compressive strength > Tensile strength. So material will be more needed at the tension side.

* Q Determine the strength of the beam section shown in figure. If allowable tensile and compressive stress or beam material are 120MPa and 160MPa respectively. If the same section is subjected to 80% of its strength, Determine the Compressive & Tensile (max) developed? (Consider Sagging)

—

$\hat{\sigma}_T = 120$ MPa

$\hat{\sigma}_c = 160$ MPa



Tension $\hat{M}_t \leq \hat{\sigma}_t \frac{I}{y_{max}^t}$ (Tension side)

$M \leq \hat{M}$ (Safe)

min of both. (So the minimum Stress will be at tension)

$\therefore \hat{\sigma} = 120$ MPa.

$M_t = 120 \times \frac{15816000}{62}$.

$M_t = \underline{\underline{30611612.89 \text{ Nmm}}}$

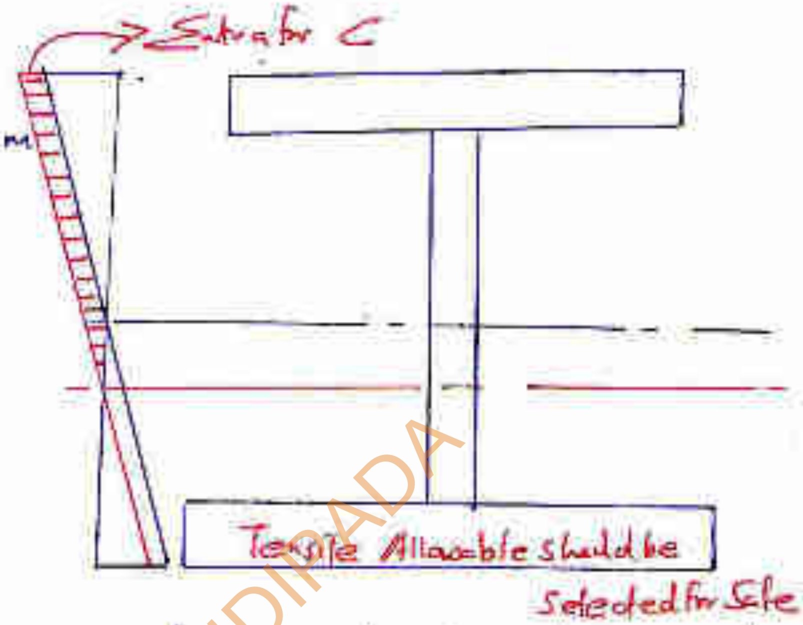
$M_t = \underline{\underline{30.61 \text{ kNm}}}$

Compression

$M_c = \sigma_c \times \frac{I}{y^c}$

$M_c = 160 \times \frac{1581600}{78}$

$M_c = \underline{\underline{32.44 \text{ kNm}}}$



$\mathfrak{g}: M < 10$
$M < 5$
$\therefore \boxed{M < 5}$

Since $M_c > M_t$, because lesser strength will be greater, So leser of M will be selected for the calculation. So that the greater value will be included.

$\therefore M_t = 30.61$ kNm

$M_{actual} = 80\%$ of M_t

$M_{actual} = 24.49$ kNm

$\sigma^c_{max} = \frac{M}{I} \cdot y^c_{max}$

$\sigma^c_{max} = \frac{24.49 \times 10^6 \times 78}{1.58 \times 10^7} = \underline{\underline{120.89}}$ MPa. < 160 MPa (Safe)

$\sigma^t_{max} = \frac{M}{I} y^t_{max}$

$\sigma^t_{max} = \frac{24.49 \times 10^6 \times 62}{15816000} = 96$ MPa < 120 MPa (Safe)

Q Determine the ratio of width to depth of the strongest rectangular section that can be cut out of a circular block of diameter 'd'.

~~Strength of Circular Section = Strength of Rectangular strength.~~

$\sigma = Mz$

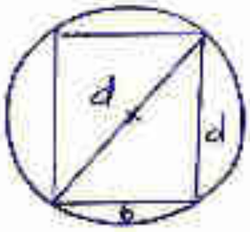
$\cancel{\sigma} \cdot Z_c = \cancel{\sigma} \cdot Z_b$

$Z_c = Z_R$

$\left(\frac{I}{y}\right)_c = \left(\frac{I}{y}\right)_R$

$\frac{\pi d^3}{32} = \frac{bd^{\cancel{3}\,2}}{6}$

$\frac{\pi d^3}{32} = \frac{bd^2}{6}$



$b^2 + d^2 = D^2$

$d^2 = D^2 - b^2$

$\frac{dz}{db} = 0$

Let diameter of Circle will be x.

$\frac{\pi x^3}{32} = \frac{bd^2}{6}$

$M = \sigma \cdot Z_R$

Here σ – same.

$Z_{max} = Z_{Rectangle}$

$Z = \frac{bd^2}{6}$

$Z = \frac{b(D^2 - b^2)}{6}$

$\frac{dz}{db} = 0$

$\frac{1}{6}\frac{d}{db} b(D^2 - b^2) = 0$

$\frac{d}{db} D^2 b - b^3 = 0$

$D^2 - 3b^2 = 0$

$D/b = \sqrt{3}$

$\frac{D}{b} = \sqrt{3}$

$b = \frac{D}{\sqrt{3}}$

$b^2 = \frac{D^2}{3}$

$b^2 = \frac{b^2 + d^2}{3}$

$b^2 - \frac{b^2}{3} = \frac{d^2}{3}$

$\frac{2b^2}{3} = \frac{d^2}{3}$

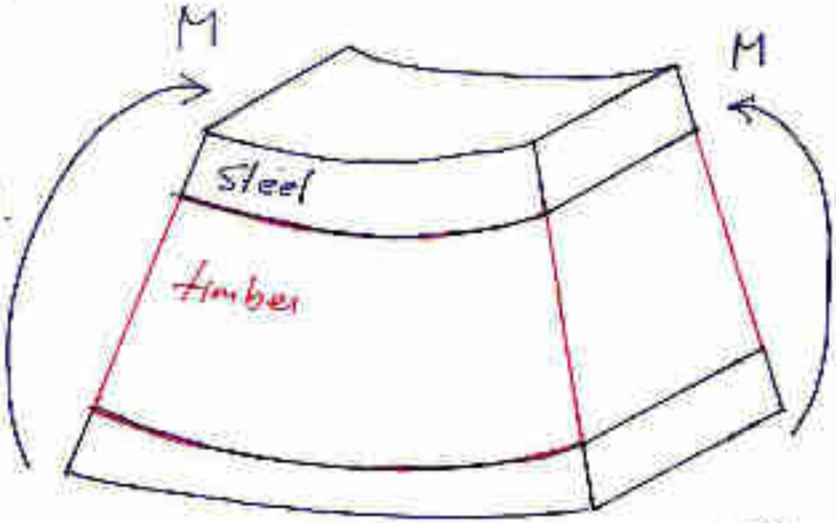
$\frac{b}{d} = \frac{1}{\sqrt{2}}$

→ FLiTcHeD BeAmS:- (E.S.E* Po.V)

Connection b/w them are rigid.

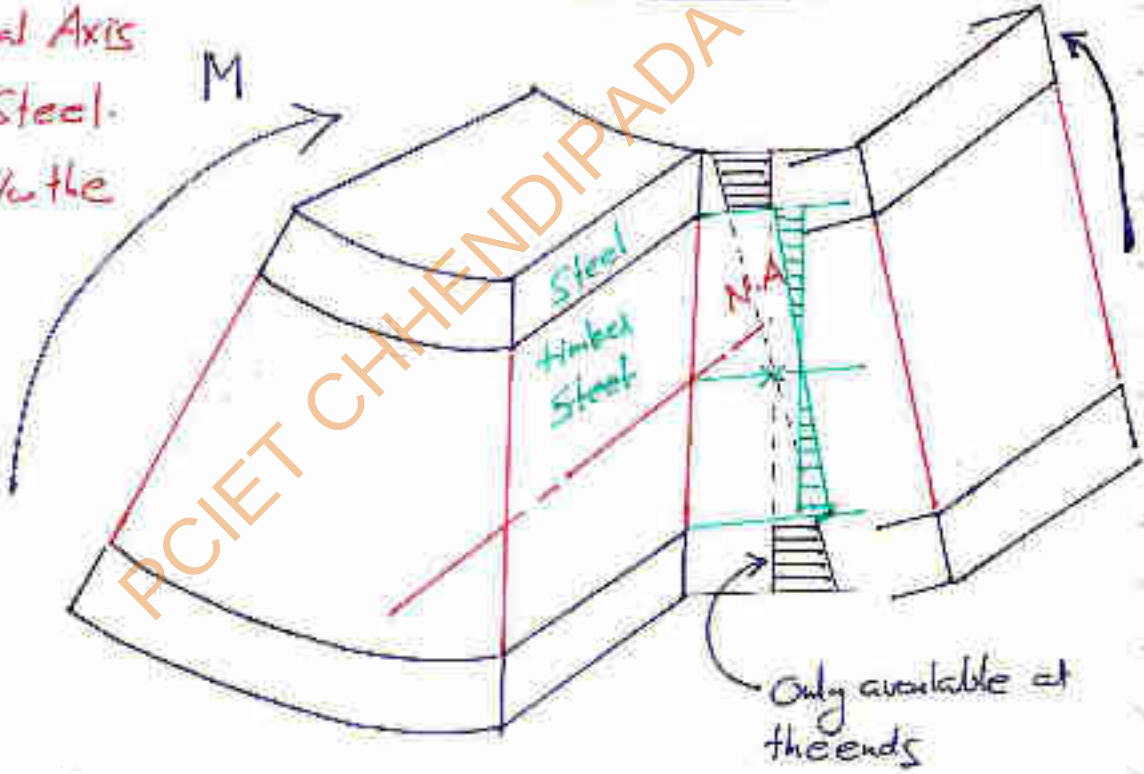
Same Curvature after Bending.

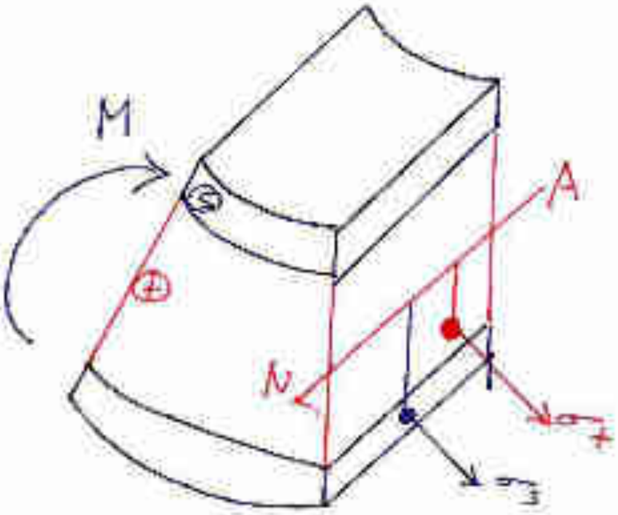
No relative slipping b/w timber & steel material.

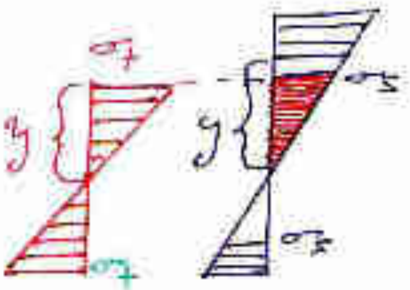


→ Common Neutral Axis for Timber & Steel.

No slipping b/w the layers.







y = y

Equilibrium : $\left(\begin{array}{c}\text{External} \\ \text{Moment}\end{array}\right) = \left(\begin{array}{c}\text{Total Moment of resistance} \\ \text{internally}\end{array}\right)$

$$M = M_T + M_B.$$

$$M = \frac{\sigma_T}{y_T} \cdot I_T + \frac{\sigma_s}{y_s} \cdot I_s$$

Note:- σ/y value will be same because of similar triangles. But $\frac{\sigma}{y}$ value will be taken at the constant surfaces.

In this equation Y is the distance of common distance from the Neutral Axis. and σ_s and σ_T are The stress in steel and timber at the common surface respectively.

Length of Common Surface is Same.

At Common Surface $\text{Strain}_{\text{timber}} = \text{Strain}_{\text{Steel}}$

$$\delta_t = \delta_s$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_t}{E_t}$$

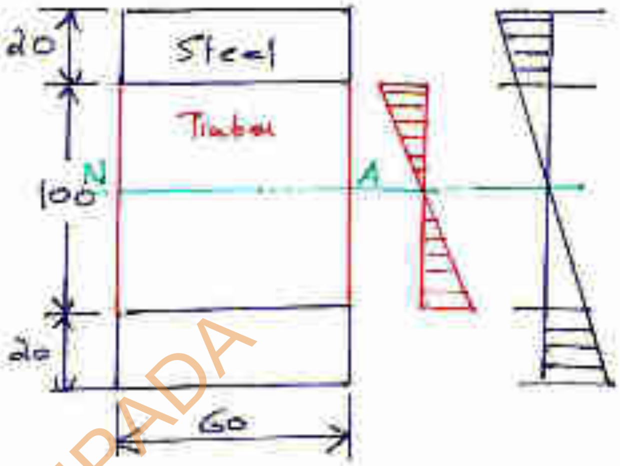
$$\frac{\sigma_s}{\sigma_t} = \frac{E_s}{E_t} \text{ ——— (ii)}$$

$\frac{E_s}{E_t} \rightarrow$ Modular Ratio $\rightarrow$ m

$$M = \frac{\sigma_T \cdot I_T + \sigma_s \cdot I_s}{y} \text{ ——— (i)}$$

$$\left\{\begin{array}{c} \frac{\sigma_s}{\sigma_T} = \frac{E_s}{E_T} \\ M = \frac{\sigma_T \cdot I_T + \sigma_s \cdot I_s}{y} \end{array}\right\}^{*}$$

Q. A timber beam of rectangular cross section 60 mm x 100 mm is having identical two steel plates each 60 mm x 20 mm in cross section flitched symmetrically at top and bottom. The beam is subjected to a bending moment of 10 kNm. If the modular ratio of steel to timber is 20. Determine max. stress developed in both the materials?



$\sigma_S = ?$; $\sigma_T = ?$

$I_S = (I_{Total}) - (I_{Timber})$

$I_S = \left(\frac{140^3 \times 60}{12}\right) - \left(\frac{100^3 \times 60}{12}\right)$

$I_S = 13720000 - 5000000$

$I_S = 8.72 \times 10^6 \text{ mm}^4$

$I_T = \frac{100^3 \times 60}{12}$

$I_T = 5 \times 10^6 \text{ mm}^4$

Here $y = 50 \text{mm}$.

$M = (\sigma_T \cdot I_T + \sigma_S I_S)/y$

$10 \times \cancel{10^6} = \sigma_T \times 5 \times \cancel{10^6} + 8.72 \times \cancel{10^6} \sigma_S / 50$

$500 = 5\sigma_T + 8.72\sigma_S$

$\frac{\sigma_S}{\sigma_T} = \frac{E_S}{E_T}$

$\frac{\sigma_S}{\sigma_T} = 20$

$\sigma_S = 20\sigma_T.$

$\therefore 500 = 5\sigma_T + 8.72 \times 20\sigma_T$ $\qquad \sigma_S = 20\sigma_T$

$500 = 5\sigma_T + 175\sigma_T$ $\qquad \sigma_S = 20 \times 2.78$

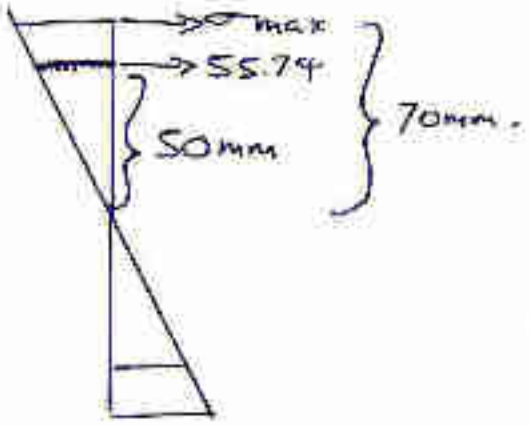
$\sigma_T = \underline{2.78 \text{ MPa}}$ $\qquad \sigma_S = \underline{\underline{55.55 \text{MPa}}}$

They are the Stress values at the Common surface.

At the Surface, σ_T is Max itself ; $\sigma_T = 2.78$ MPa

(Common Surface have to be extreme Surface of timber)

Using Similar triangles,



$$\frac{\sigma_{max}}{70} = \frac{55.74}{50}$$

$$\sigma_{max_S} = \underline{\underline{78.036 \text{ MPa}}}$$

$$\sigma_{T_{max}} = 2.78 \text{MPa}$$

$$\sigma_{S_{max}} = 78.036 \text{ MPa}$$

Greater distance will resist greater resistant to moment

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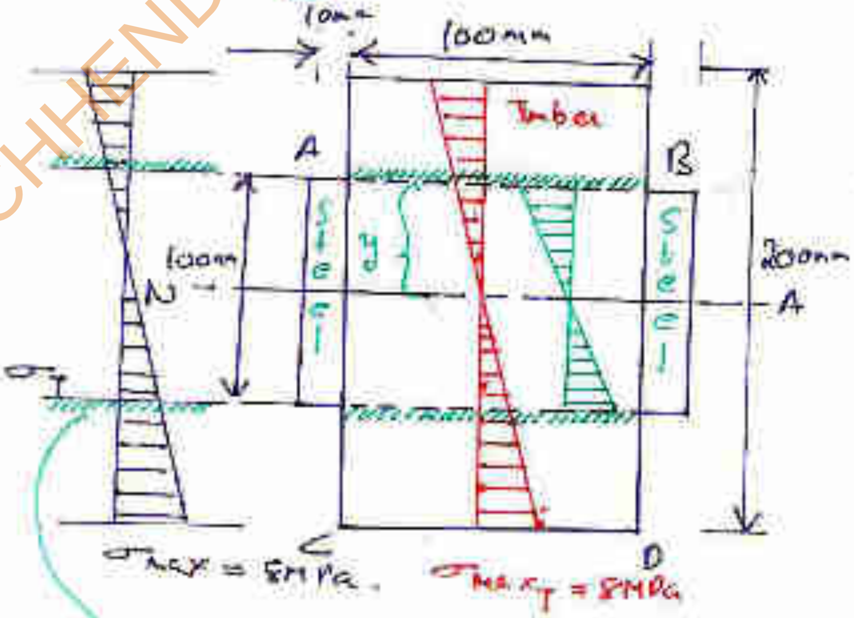
Q.9/10
9

Here the common point of application is along the line AB. Here y will be $\frac{100}{2} = 50$mm.



Also max. stress for timber at CD

∴ By Similar triangle $\frac{\sigma_{max}}{100} = \frac{\sigma_T}{50}$

Here $\sigma_{max} = 8$

$$\frac{8}{100} \times 50 = \sigma_T.$$

$$\sigma_T = \underline{4 \text{MPa}}$$

$$\frac{\sigma_S}{\sigma_T} = \frac{E_S}{E_T} = 20$$

$$\sigma_S = 20 \times 4 = \underline{\underline{80\,MPa}}$$

$$M = \left(\sigma_T I_T + \sigma_S I_S\right) / y$$

→ Here y = distance to common surface $= 100/2 = 50$ (at AB)

$$I_T = \frac{100 \times 200^3}{12} = 66.66 \times 10^{4+2}\,mm^4 = 66.66 \times 10^6\,mm^4$$

$I_S = 2\left(\frac{100^3 \times 10}{12}\right) = 16.66 \times 10^5\,mm^4$ (Because I_{xx} alg the centroid)

$$\therefore M = \left(4 \times 66.66 \times 10^{4+2} + 80 \times 16.66 \times 10^{6-1}\right) / 50$$

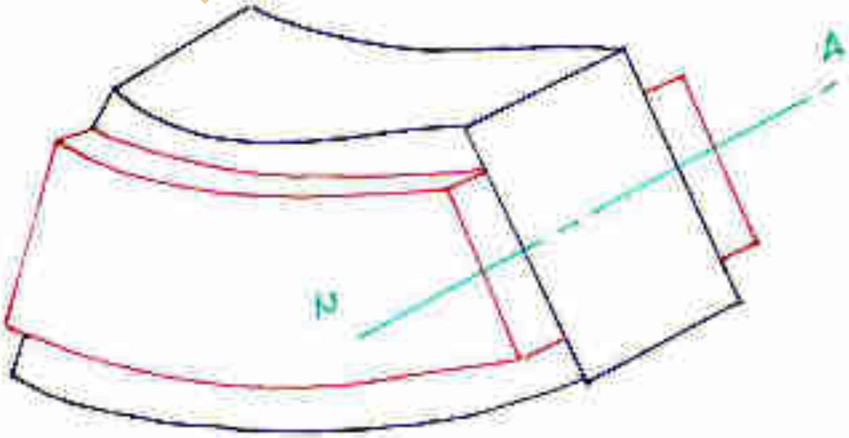
$$M = \left(4 \times 66.66 \times 10^6 + 80 \times 16.66 \times 10^5\right) / 50$$

$$M = 7999466.667\,Nmm$$

i.e. $M = 7.99 \times 10^6$ kNm $\underline{\underline{\approx 8MN}}$

N.B.

→ Draw Stress distribution diagram on the problem first to determine the "Contact Surfaces" and "y".



22/07/2017 *

Q Figure shows a compound section of a beam made from Copper and steel. The modular ratio of steel to copper is 2. Determine the moment of resistance of the section if allowable stress for a steel and copper material are 160 MPa and 100 MPa respectively? Assume rigid connections.

Figure is not symmetric and not made by two material a new procedure approached. So we have to locate the Neutral axis First. So we made the whole material section by any one of given material.

→ To locate the Neutral Axis :- For rectangle section the depth directly proportional to Z. So keep the depth intact. Change the width.

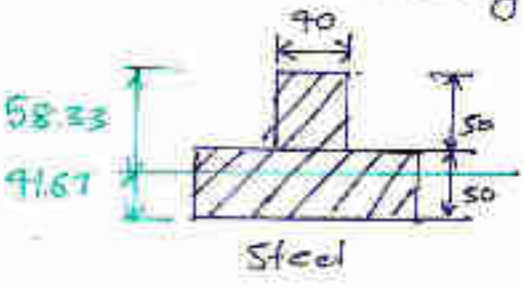
$(A_1+A_2)\bar{x} = A_1x_1 + A_2x_2$.

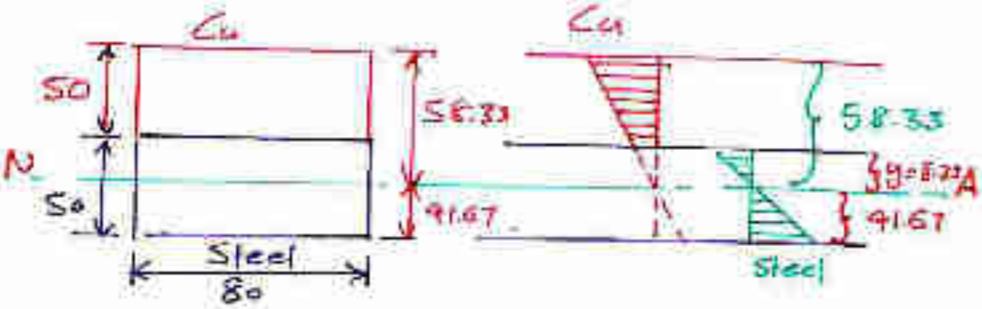
$\bar{x}((40\times50)+(50\times80)) = (40\times50)25+(50\times80)(50+25)$

$\bar{x} = \underline{\underline{58.33}}$ mm (From top)

and 41.6 from bottom

→ Transforming the section in Steel, because steel will become stronger and m = 2 so that that the width will be reduced by half. [m = 2] only here





Moment of Inertia of Cu = $I_{yc} + Ah^2$

$$I_{Cu} = \frac{(50)^3 \times 80}{12} + (50 \times 80) \times (8.33 + 25)^2$$

$$I_{Cu} = 833333.33 + 4443555.6$$

$$\underline{\underline{I_{Cu} = 5.27 \times 10^6 \text{ mm}^4}}$$

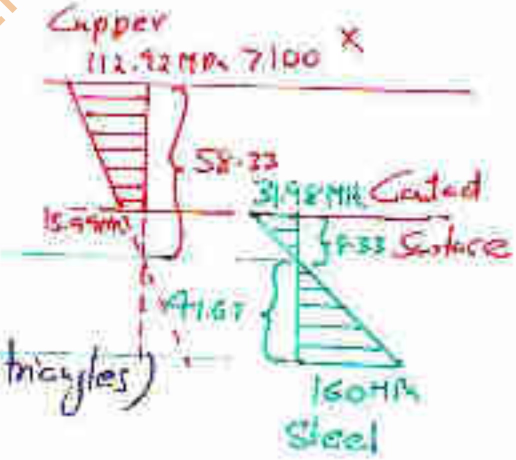
$$I_s = \frac{50^3 \times 80}{12} \cancel{\times} + (50 \times 80)(41.67 - 25)^2$$

$$I_s = \underline{\underline{1944555.73 \text{ mm}^4}}$$

$$(\sigma_{max})_s \leq \sigma_{st}$$

$$(\sigma_{max})_s = 160 \text{ MPa}$$

$$\frac{(\sigma_{cs})}{8.33} = \frac{160}{41.67} \quad (\text{Using Similar triangles})$$



$$(\sigma_{cs})_{st} = \underline{\underline{31.98 \text{ MPa}}}$$

At common Surface $\frac{(\sigma_{cs})_{st}}{(\sigma_{cs})_{Cu}} = 2$ (Modular ratio)

$$(\sigma_{cs})_{Cu} = \frac{31.98}{2} = 15.99 \text{ MPa}$$

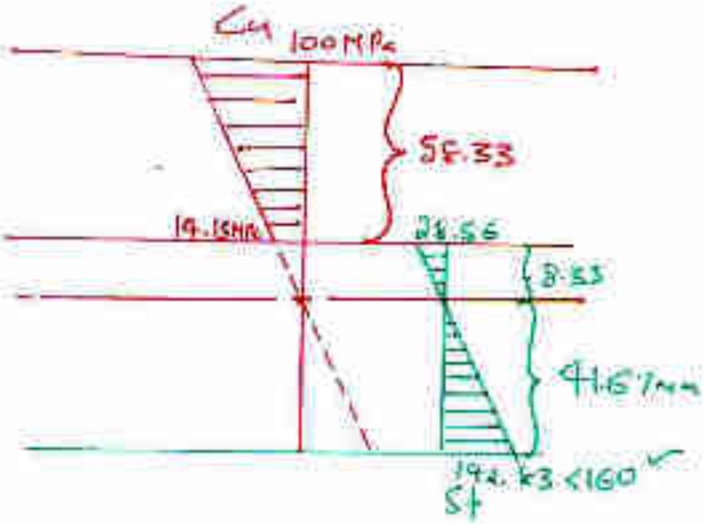
$$\frac{15.99}{8.33} = \frac{(\sigma_{max})_{Cu}}{51.83}$$

Allowable stress

× $(\sigma_{max})_{Cu} = \underline{\underline{112.92 \text{ MPa}}} > 100 \text{ MPa}$ (Not acceptable) let take Cu to Allowable value

$(\sigma_{max})_{cu} \le \sigma^{a}_{cu}$

$(\sigma_{max})_{cu} = 100$



$$\frac{(\sigma_{max})_{cu}}{58.33} = \frac{(\sigma_{cu})_{c.s}}{8.33.}$$

$$\frac{100}{58.33} = \frac{(\sigma_{c.s})_{cu}}{8.33}$$

$$(\sigma_{c.s})_{cu} = \underline{\underline{14.15 MPa}}$$

$$\frac{(\sigma_{c.s})_s}{(\sigma_{c.s})_{cu}} = 2$$

$$(\sigma_{c.s})_s = 2\times 14.15 = 28.56 MPa.$$

Using Similar triangles,

$$\frac{28.56}{8.33} = \frac{(\sigma_{max})_s}{41.67.}$$

$$(\sigma_{max})_s = \underline{\underline{142.83 MPa}} < 160 MPa.$$

$$M = (\sigma_{cu} \cdot I_{cu} + \sigma_s . I_s)/y$$

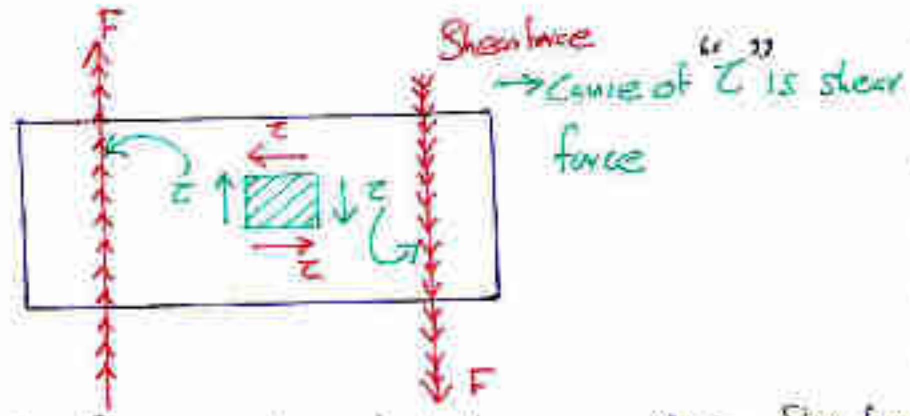
$$M = (14.15 \times 5276858.93) + (28.56 \times 1478555.73) / 8.33$$

$$M = \underline{\underline{15.62 \text{ kNm}}}$$

3.6: SHEAR STRESS IN BEAMS :- a) Introduction:

τ - Vertical shear/ Transverse shear.

τ' - Longitudinal shear/ horizontal shear.



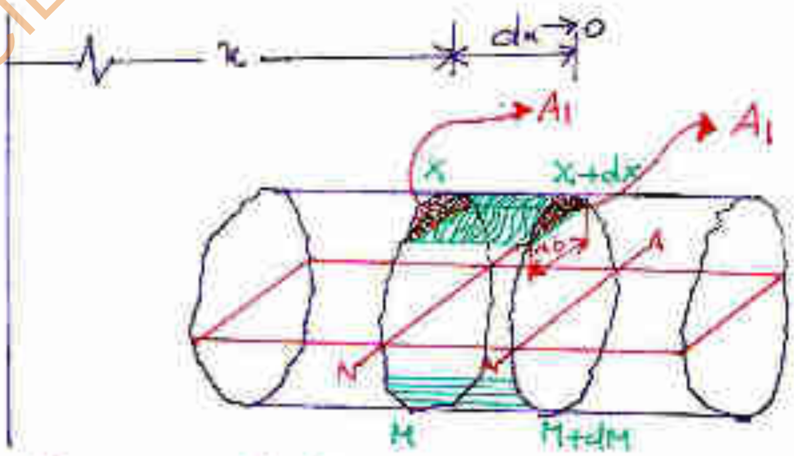
Equal and opposite force acting about transverse Axis - Shear force.

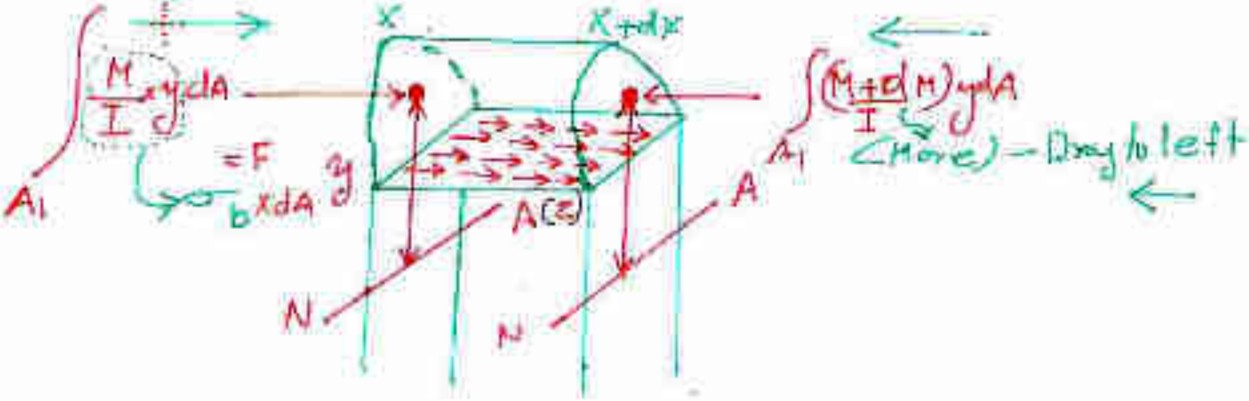
If transverse shear creates a clockwise moment then there will be anticlockwise moments called as longitudinal shear/ horizontal shear.

$$\text{Cause} \quad |\tau| = |\tau'|$$

Cause of τ' is variability of M.

$$F = \frac{dM}{dx} \implies F \neq 0;\ M = 0$$





Considering the small area dA at the surface A_1 at the upper part cut. (Shaded portion). Let dA be a distance of y from the neutral axis such that the force will be $\frac{M}{I} y \times dA$. To get the the total force on dA integrate to $\int_{A_1} \frac{M}{I} y \, dA$.

Similarly another surface the moment will be M+dM (because, $\frac{dM}{dx} = F$; F & M ≠ 0, M variable with x) So that the total force will be $\int_{A_1} \frac{(M+dM)}{I} y . dA$.

So that the more force due to M+dM will try to shift them to the left. But the section bonded rigidly so that that longitudinal shear (⇆) arises (Due to bonded)

b) Shear stress Equation:-

Equilibrium of Forces:-

$$\int_{A_1} \frac{M}{I} . y . dA + \tau b dx = \int_{A_1} \frac{M+dM}{I} y dA .$$

$$\int_{A_1} \cancel{\frac{M}{I}} \cancel{y} dA + \tau b dx = \int_{A_1} \cancel{\frac{M}{I} y} dA + \int \frac{dM . dA}{I} . y$$

$$\tau . b . dx = \int_{A_1} y . \frac{dM dA}{I}$$

$$\tau = \frac{dM/dx}{I . b} \int_{A_1} y . dA$$

Since $dM/dx = F$

$$\tau = \frac{F}{I . b} \int_{A_1} y . dA$$

$\int_{A_1} y \, dA$ → First moment A_1 area about Neutral Axis

$\sigma = \frac{P}{A}$

$\tau = \frac{T}{J} \times r$

$\sigma = \frac{M}{I} \times y$

$\tau = \left(\frac{F}{I}\right) \times \left(\frac{A\bar{y}}{b}\right)$

$$\begin{pmatrix}\text{First Moment of Area} \\ \text{about Neutral Axis}\end{pmatrix} = A_1 \cdot \bar{y} = \int_{A_1} y\, dA$$

$$\tau = \frac{F}{I \times b} \times (A_1 \cdot \bar{y}) = \underbrace{\left(\frac{F}{I}\right)}_{\text{Constant}} \times \left(\frac{A_1 \bar{y}}{b}\right)$$

F → Shear Force

I → Moment of Inertia of Entire Area about NA.

b → Width of Cut (changing, ▲, ⬮)

A_1 → Area of Cross section on one side of cut.

$$\begin{pmatrix}\text{F.M of } A_1 \\ \text{about N-A}\end{pmatrix} + \begin{pmatrix}\text{F.M. of rem} \\ \text{A about N-A}\end{pmatrix} = 0$$

$$\left| -\begin{pmatrix}\text{F.M. of } A_1 \\ \text{about N-A}\end{pmatrix} \right| = \left| \begin{pmatrix}\text{F.M of Area} \\ \text{remaining about N-A}\end{pmatrix} \right|$$

(F.M of Area) = 0

Page No 48
Q. 2, 3, 9 ?

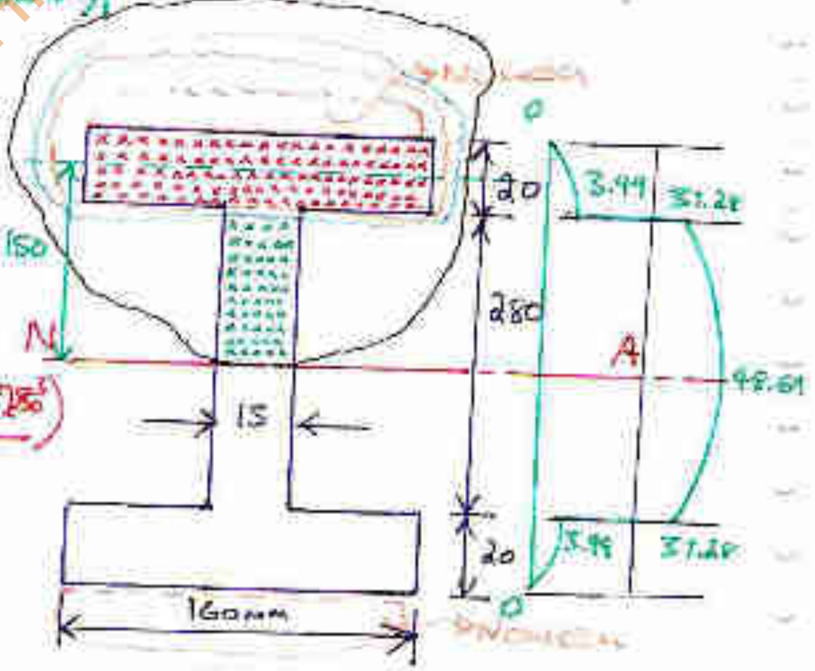
For doubly symmetric section (Symmetric about I_x axis) M.O.I can be easily calculated by,

$$I = \frac{160 \times 340^3}{12} - 2\left(\frac{\frac{(160-15)}{2} \times 280^3}{12}\right)$$

$I = 171653333.3 \text{ mm}^4$

F = 200 kN.

$\frac{F}{I} = \frac{200 \times 1000}{171653333.37} = 1.165 \times 10^{-3}$ N/mm⁴ ($\frac{F}{I}$ → Constant)



Cut on the top and follow one side to determine shear stress. At top & bottom Shear stress is zero. Because total remaining area will have first M.O.I about N-A will be zero.

→ At Top ; $\tau_{Top} = \left(\frac{F}{I}\right)\left(\frac{A\bar{y}}{b}\right) = 0$ → 0 (Zero, no area)

→ At Bottom ; $\tau_{bottom} = \left(\frac{F}{I}\right)\left(\frac{A\bar{y}}{b}\right) = 0$

→ 0 (Zero, also remaining area, FMOA will be;

→ At Junction ; $\tau_{junction} = \left(\frac{F}{I}\right) \times \frac{A_1\bar{y}}{b}$

$\frac{F}{I}$ determined and known as $1.165 \times 10^{-3}\ N/mm^4$

$$[\tau_{junction}]_{Flange} = 1.165 \times 10^{-3} \times \frac{A_1\bar{y}}{b}$$

Cut at the junction ⟶ (Cal by the Flange side)

$\bar{y}$ (from the Neutral axis) $= \frac{280}{2} + \frac{20}{2} = 140+10 = 150mm$

$A_1\bar{y} = (3200) \times 150$; $b = 160$ (at Flange)

$$[\tau_{junction}]_{Flange} = 1.165 \times 10^{-3} \times \frac{(3200 \times 150)}{160}$$

$$[\tau_{junction}]_{Flange} = 3.495\ N/mm^2.$$

$$[\tau_{junction}]_{web} = \left(\frac{F}{I}\right) \times \frac{A_1\bar{y}}{b}.$$

Cut at below junction slightly at web ⟶

$\bar{y} = 140+10 = 150mm$;

$A_1\bar{y} = 160 \times 20 \times 150$; b will become 15

$$[\tau_{junction}]_{web} = 1.165 \times 10^{-3} \times \frac{3200 \times 150}{15} = \underline{\underline{37.28\ N/mm^2}}$$

$$[\tau_{junction}]_{N/A} = \left(\frac{F}{I}\right) \times \frac{A_1\bar{y}}{b}$$

→ area on web

$= $ ~~$1.165 \times 10^{-3} \times (160 \times 20 + 15 \times 190)$~~

$$= \frac{1.165 \times 10^{-3} \times (160 \times 20 \times 150 + 15 \times 140 \times \frac{140}{2})}{15}$$

15 → (width of cut)

$$[\tau_{junction}]_{N/A} = \underline{\underline{48.69\ N/mm^2}}$$

$$[\tau_{junction}]_{N/A} = 48.69\ N/mm^2$$

→ Web Resists 80-90% of Shear Force, Bending Moment are more resisted by Flange (More moment of resistance)

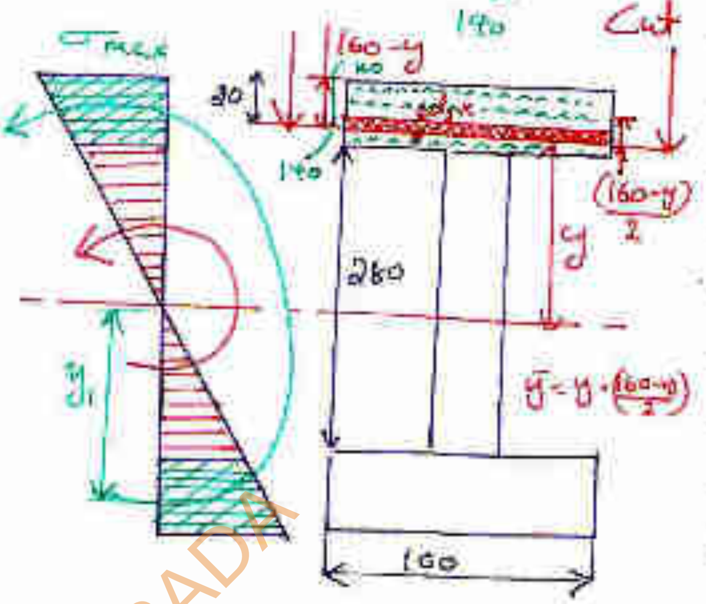
→ Section of Previous problem, what percentage of applied shear Force is being resisted by web and Flanges respectively?

→ Web Resists Max. Shear Force.

$= \int_{140}^{160} ▨ \times \tau \, dy$

Larger Couples/ of resistance So more moment will be resisted by top & bottom flanges.

Shear force resisted by web (80-95%)



To determine, cut at any general distance, At flange a cut is given so that at a distance from Neutral axis

$$(\tau)_{Flange} = \left(\frac{F}{I}\right) \times \frac{A_1 \times \bar{y}}{b}$$

b → Cut at the Flange, so b = 160 mm

$A_1 = (160)(160-y)$

$\bar{y} = y + \left[\frac{(160-y)}{2}\right] = y + 80 - \frac{y}{2}$

$\bar{y} = \frac{y}{2} + 80.$

$A_1\bar{y} = (160)(160-y)\left(\frac{y}{2}+80\right)$

$A\bar{y}\ \left((160)(160-y)\frac{(160+y)}{2}\right)$

$A\bar{y} = 160\,\frac{(160^2-y^2)}{2} = 80(160^2-y^2)$

i.e. $\tau_{Flange} = 1.65 \times 10^{-3} \times \frac{80\,(160^2-y^2)}{160}$

$[\tau]_{\text{Flange}} = 5.825\times10^{-4}(160^2 - y^2]$

$[\tau]$ at y location is known,

Consider the differential area $160\times dy$ → thickness

Force $= [\tau_y] \times$ area.

Force $= \int 5.825\times10^{-4}(160^2 - y^2).\ 160\times dy$

Limit will be 140 → 160

$$\therefore \text{Force} = \int_{140}^{160} (5.825\times10^{-4}\times160^2 - 5.825\times10^{-4}y^2\times160)\,dy.$$

$$\text{Force} = 0.0932\int_{140}^{160} 160^2 - y^2\,dy.$$

$$\text{Force} = 0.0932\left[160^2 y - \frac{y^3}{3}\right]_{140}^{160}$$

$$\text{Force} = 0.0932\left[160^2\times160 - \frac{160^3}{3}\right] - \left[160^2\times140 - \frac{140^3}{3}\right]$$

Force $= \underline{\underline{5.716 \text{ kN}}}$

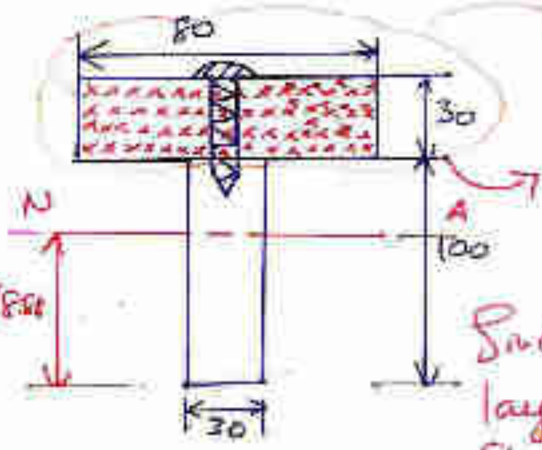
Total Force resisted by 2 flange $= 5.716\times2 = 11.432$ kN

% of Resistant by web $= 100 - $ (% of resistant by 2 flanges)

% of resistant by Flange $= \frac{11.432}{200}\times100 = 5.716\%$

% Resistant by web $= 100 - 5.716 = \underline{\underline{94.284\%}}$

9 The T-section of a timber beam is shown in figure. where the horizontal arm has been fixed to the vertical arm with the help of equally spaced nails. If the section is subjected to shear force of 42 kN and shear resisting capacity of each nail is 2kN. Determine the space between 2 consecutive nails?



Cut the web because maximum critical stress at web so b=30mm

Shearing of nail occurs at the junction of vertical & horizontal arm.

Since we need the critical or large value ~~C web~~ junction below the (b = 30) should be taken.

$(100\times30)\times\frac{100}{2} + (30\times80)\times(50+50+15) = ((100\times30)+(30\times80))\bar{y}$

$\bar{y} = 78.88$ mm from base

$I_T = \frac{80\times30^3}{12} + (80\times30)\times(15+(100-78.8))^2$

$I_T = 3311170.56\text{mm}^4$

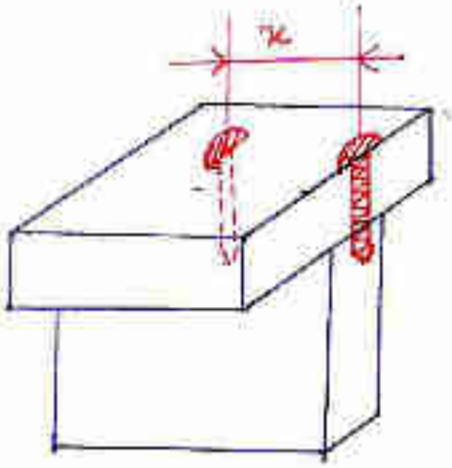
$I_w = \frac{30\times100^3}{12} + (30\times100\times(28.88)^2$

$I_w = 5002163.2\text{mm}^4$

$I = I_T + I_w = 8313333.76\text{mm}^4$

$F = 42\times10^3$

$\frac{F}{I} = \frac{42\times10^3}{8313333.76} = 0.0050521$



$(\tau)_{\substack{web\\Flange}} = \frac{F}{I} \times \frac{A_1\bar{y}}{b}$ [above the flange cal has to be made just below the section]

$A_1 = 30 \times 80 = 2400 mm^{4-2}$

$\bar{y} = (100-78.5)+\frac{30}{2} = 36.2 mm.$

$b = 30$

$(\tau_{web})_{Flange} = 0.0050541 \times \frac{2400 \times 36.2}{30}$

$(\tau_{junction})_{\substack{web\\Flange}} = \underline{\underline{14.63 N/mm^2}}$

$\{(\tau_{junction}) \times (30 \cdot x)\} \le 2000$

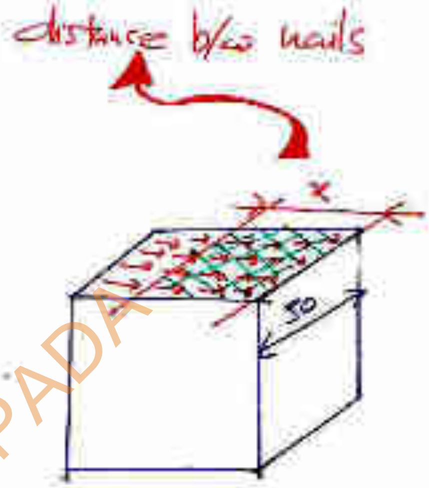
Stress × Area ≤ Shear force

Area → area of cross section

$14.63 \times 30 \times x = 2000$

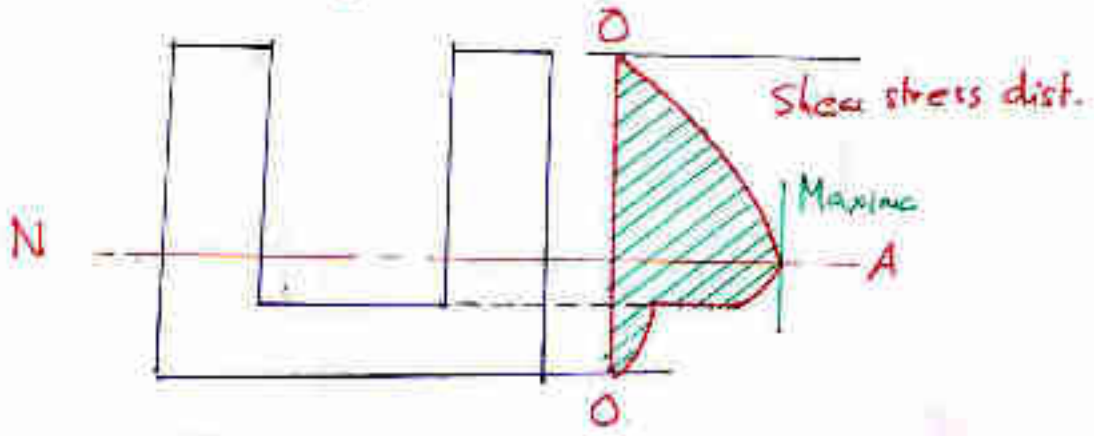
$x = \underline{\underline{4.55 mm}}$

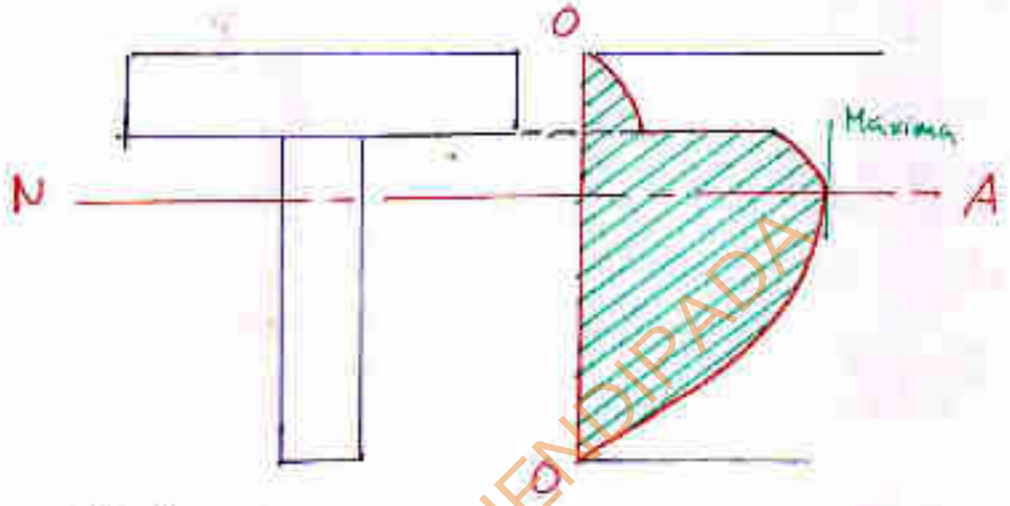
$x \le 4.55mm$

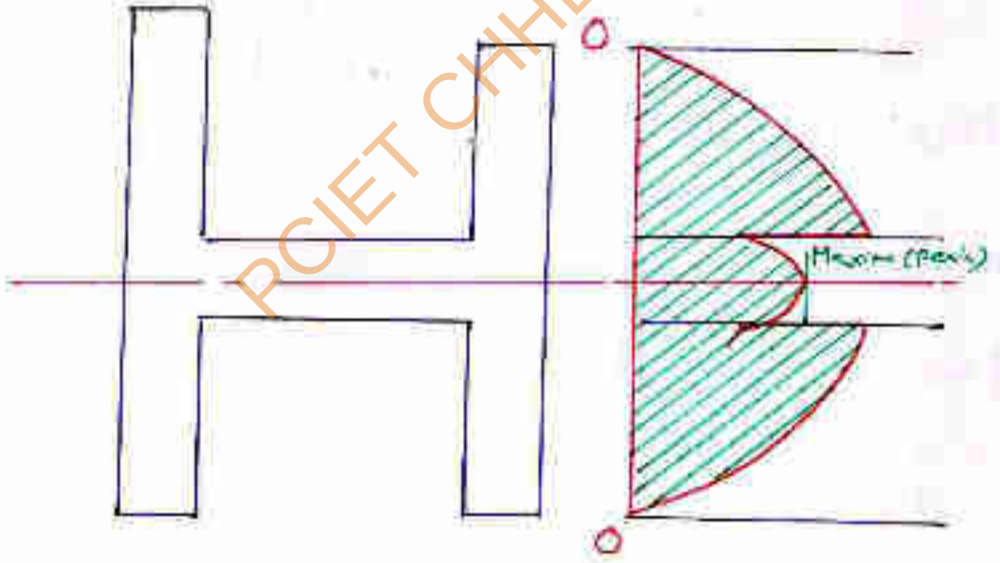


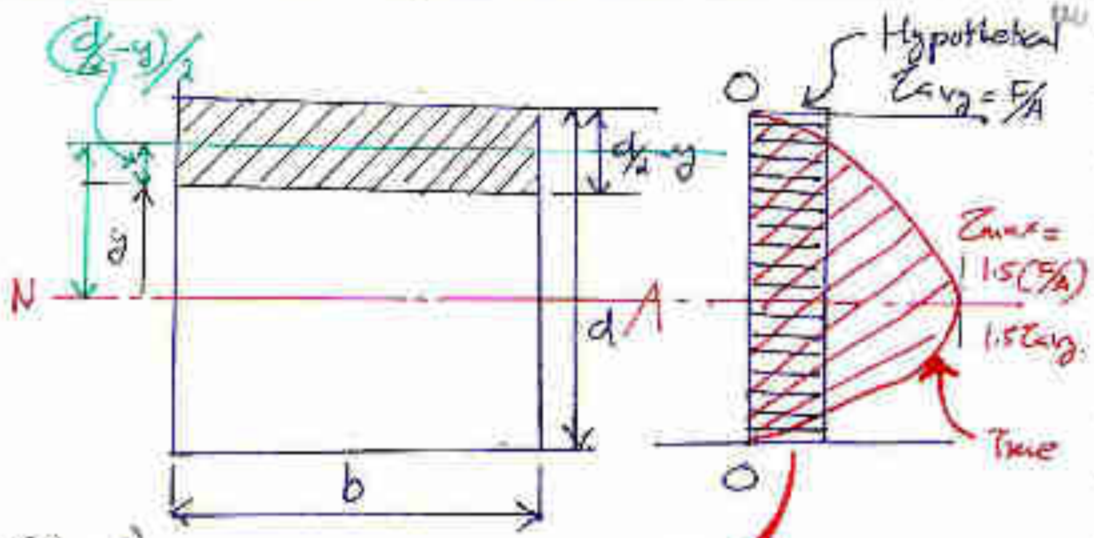
Distance b/w 2nails should be greater than 4.55mm

9) Distribution for Standard Sections:-









$\tau_y = \frac{F}{I}\cdot\left(\frac{A_1\times\bar{y}}{b}\right)$

A_1 - shaded portion $= b\times(\frac{d}{2}-y)$

$\bar{y} = y + \frac{(\frac{d}{2}-y)}{2}$

$\bar{y} = y + \frac{d}{4} - \frac{y}{2} = \frac{\frac{d}{2}+y}{2}$

$\therefore \tau_y = \frac{F}{bd^3/\cancel{12}\,6}\times\left(\frac{\cancel{b}\times(\frac{d}{2}-y)(\frac{y+d/2}{2})}{\cancel{b}}\right)$

$\tau_y = \frac{6F}{bd^3}\left(\frac{d^2}{4}-y^2\right)$ $\tau_y = \frac{6F}{bd^3}\left(\frac{d^2}{4}-y^2\right)$

Without all these theories, the shear stress "τ" will be F/A → i.e. "τ_{avg}" – hypothetical value not true,

Original: $\tau_{max} = 1.5\,\tau_{avg}$ where $\tau_{avg} = F/A$

For Max; $\frac{d\tau_y}{dy}=0$

i.e. $y=0$, τ_y will be maximum @ $y=0$

At $y=0$

$\tau_y = \frac{6F}{bd^3}\left(\frac{d^2}{4}-\cancel{y^2}^{\,0}\right)$

$\tau_y = \frac{6F}{bd^{\cancel{3}}}\times\frac{\cancel{d^2}}{4}$

$\tau_y = \frac{3F}{2\,\boxed{bd}}$ → Area of □

i.e. $\tau_y = 1.5\times\frac{F}{A}$ (Max. Value)

Pg No. 46
Q. 05
?

Max shear force = 140kN

$I_{N/A} = 13\times10^6 \, mm^4$

$\frac{M}{I} = \frac{140\times10^3}{13\times10^6} = 0.010769$

τ_{max} will occur at Neutral Axis

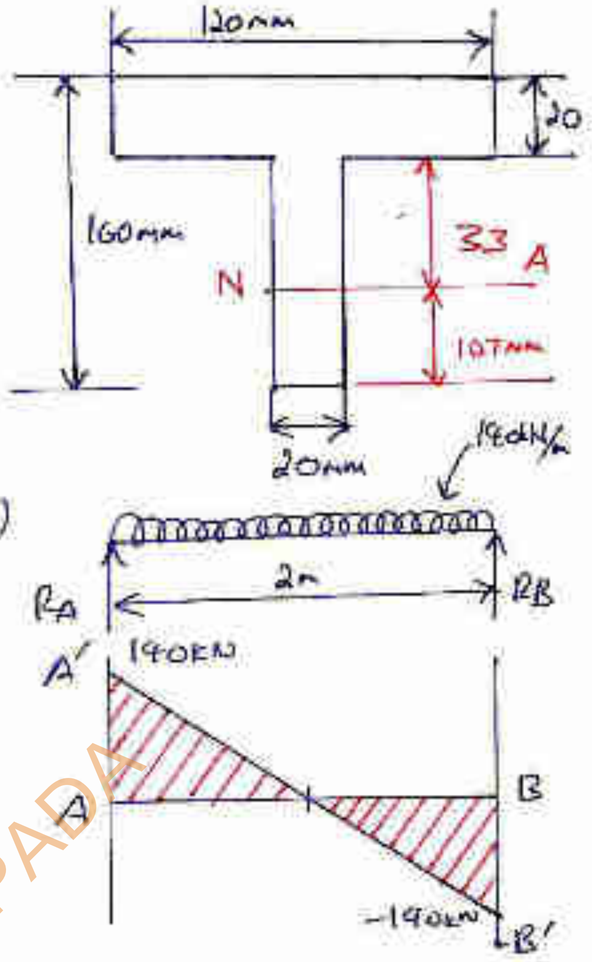
$A\bar{y} = (33\times20)\,33/2 + (20\times120)\,(33+20/2)$

$A\bar{y} = 10890 + 103200 = 114090 \, mm^3$

b will be 20mm due to cutting on the web

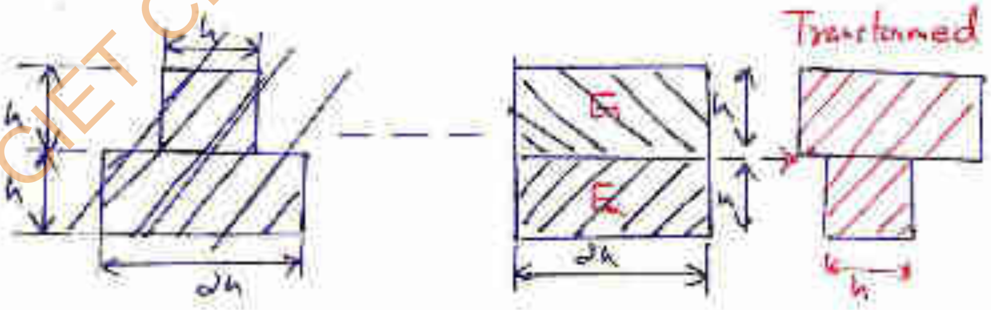
$\tau_{max} = 0.010769 \times \frac{114090}{20}$

$\tau_{max} = \underline{\underline{61.43 MPa}}$



Pg No. 37
Q. 1
?

$\frac{E_1}{E_2} = 2$



~~$I = \frac{bh^3}{12}$~~

$(A_1+A_2)\bar{y} = A_1y_1 + A_2y_2$

$(h^2 + h\times2h)\bar{y} = h^2\times h/2 + 2h\times h \times(h + h/2)$

$(h^2 + 2h^2)\bar{y} = h^3/2 + 2h^2\,(3h/2)$

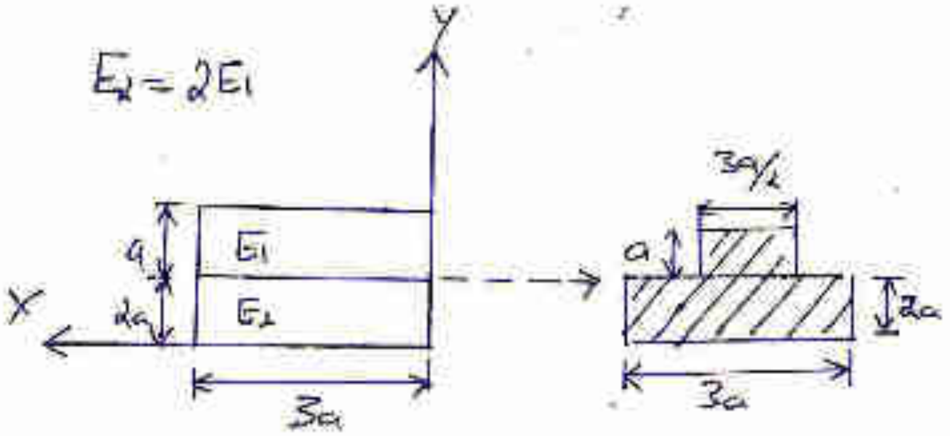
$(h^2 + 2h^2)\bar{y} = h^3/2 + 3h^3$

$(1+2)\bar{y} = \frac{h}{2} + 3h$

$3\bar{y} = 3.5h$

$\bar{y} = \underline{1.167 h \, mm}$

Pg No: 37
Ques. No 2
9



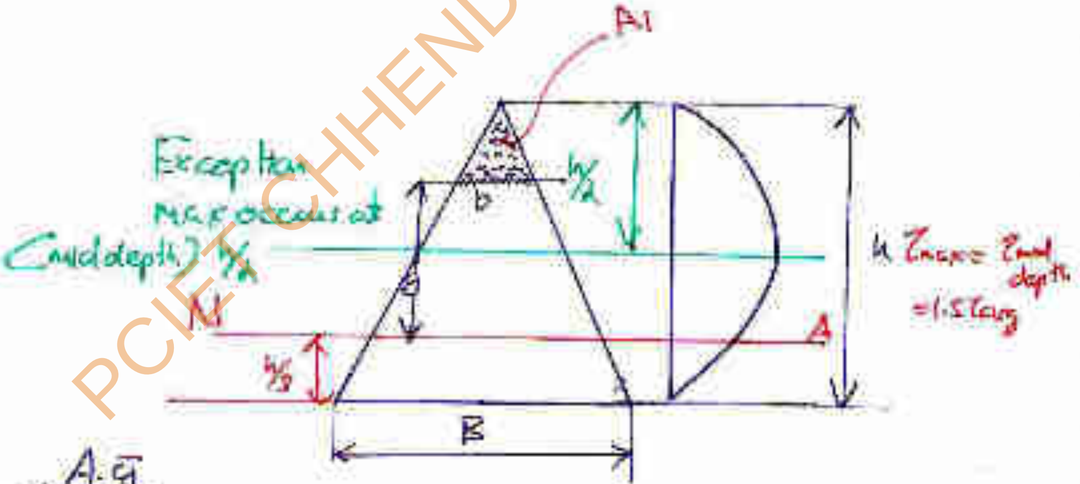
$$(A_1+A_2)\bar{y} = A_1y_1 + A_2y_2.$$

$$\left(2a\times 3a + \frac{3a}{2}\times a\right)\bar{y} = (2a\times 3a)\frac{2a}{2} + \left(\frac{3a}{2}\times a\right)\left(2a+\frac{a}{2}\right)$$

$$\left(6a^2+\frac{3a^2}{2}\right)\bar{y} = (6a^2)\frac{2a}{2} + \left(\frac{3a^2}{2}\right)(2.5a)$$

$$\left(6+\frac{3}{2}\right)\bar{y} = \frac{12a}{2} + \frac{15}{4}a$$

$$\underline{\bar{y} = 1.3a}$$



Exception max occurs at (mid depth) h/2

$\tau_{max} = \tau_{mid\ depth} = 1.5\tau_{avg}$

$$\tau_y = \frac{F}{I}\times\frac{A\cdot\bar{y}}{b}$$

$$I_y = \frac{F}{\frac{Bh^3}{36}}\times\frac{A\times\bar{y}}{b}$$

$$A_1 = \frac{1}{2}\times b\times\left(\frac{2h}{3}-y\right)$$

$$\bar{y} = \left(\frac{1}{3}\left(\frac{2h}{3}-y\right)\right)+y = \left(\frac{1}{3}\left(\frac{2h}{3}-y\right)\right)+y$$

$$\bar{y} = y + \frac{2h}{9} - \frac{y}{3}$$

$$\tau_y = \frac{F}{\frac{B\times h^3}{36}} \times \frac{\frac{1}{2}\times\cancel{b}\times\left(\frac{2h}{3}-y\right)\left(\frac{1}{3}\left(\frac{2h}{3}-y\right)+y\right)}{\cancel{b}}$$

$$\tau_y = \frac{36F}{B\times h^3}\times\frac{1}{2}\times\left(\frac{2h}{3}-y\right)\times\left(\frac{2h}{9}-\frac{y}{3}+y\right)$$

$$\tau_y = \frac{36F}{B\times h^3}\times\frac{1}{2}\left(\frac{2h}{3}-y\right)\times\left(\frac{2h}{9}+\frac{2}{3}y\right)$$

$$\tau_y = \frac{36F}{B\times h^3}\times\frac{1}{2}\times\frac{1}{3}\left(\frac{2h}{3}-y\right)\times\left(\frac{2h}{3}+2y\right)$$

$$\tau_y = \frac{6F}{B\times h^3}\times\left(\frac{2h}{3}-y\right)\left(\frac{2h}{3}+2y\right)$$

$$\tau_y = \frac{6F}{B\times h^3}\times\left(\frac{4h^2}{9}+\frac{4hy}{3}-\frac{2hy}{3}-2y^2\right)$$

$$\tau_y = \frac{6F}{B\times h^3}\times\left(\frac{4h^2}{9}+\frac{2hy}{3}-2y^2\right)$$

$$\tau_y = \frac{6F}{B\times h^3}\times\left(\left(\frac{2h}{3}\right)^2+\frac{2hy}{3}-2y^2\right)$$

$$\cancel{d\tau_y = 0}\quad \cancel{\frac{d\tau_y}{dy}}=0 \qquad \frac{d\tau_y}{dy}=0$$

$$\cancel{0} = \frac{6\cancel{F}}{B\times h^3}\times\left(\cancel{\left(\frac{2h}{3}\right)^2}+\frac{2h}{3}-4y\right)$$

$$\cancel{0} = \cancel{\frac{4h^2}{9}}+\cancel{\frac{2h}{3}}-\cancel{4y} \qquad 0 = \frac{6F}{Bh^3}\left(0+\frac{2h}{3}-4y\right)$$

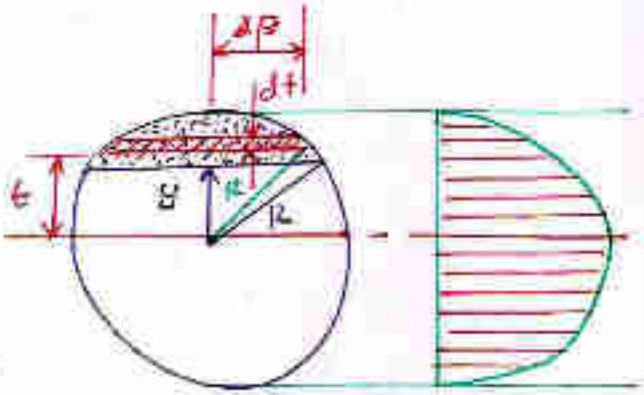
$$\cancel{y} = \cancel{\frac{1}{4}}\left[\cancel{\frac{4h^2}{9}}+\cancel{\frac{2h}{3}}\right] \qquad \frac{2h}{3} = 4y \;;\; y = \frac{h}{3\times 2} = \frac{h}{6}$$

$$\therefore \tau_y \text{ @ } y = \frac{h}{6} = \frac{6F}{Bh^3}\left(\frac{4h^2}{9}+2h\times\left(\frac{h}{3\times 2}\right)+2\times\frac{h^2}{36}\right)$$

$$\tau_y = \frac{6F}{Bh^3}\left(\frac{4h^2}{9}+2h\times\frac{h}{6}+2\times\left(\frac{h}{6}\right)^2\right)$$

$$\tau_y = \frac{\cancel{6}F}{Bh^{\cancel{3}}}\times\frac{1}{\cancel{2}}h^{\cancel{2}} = \frac{3F}{Bh}$$

$$\text{i.e. } \tau_y = \frac{3F}{\frac{2}{2}Bh} = \frac{3}{2}\times\tau_{avg} \underline{\underline{= 1.5\,\tau_{avg}}}$$



Width at cd = $2\times\sqrt{R^2-y^2}$

$\tau_y = \frac{F}{I}\times\frac{A_1\bar{y}}{b}$

Area of strip = $2(\beta\times dt)$

Area of strip, $A_1 = 2\times\beta\times dt$

Limit will be $y \longrightarrow R$.

Consider (green triangle) $y^2+\beta^2 = R^2$.

$\beta^2 = R^2-y^2$; $\beta = \sqrt{R^2-y_t^2}$

$b = \cancel{2\beta} = 2\sqrt{R^2-\cancel{y_t^2}y^2}$

$A_1\bar{y} = \int_{t=y}^{t=R} 2t\,\beta\,dt$

$A_1\times\bar{y} = \int_y^R 2t.\sqrt{R^2-y_t^2}.\,dt$; $b = 2\sqrt{R^2-y^2}$.

$R^2-t^2 = x$

at $t=y$; $x = R^2-y^2$

at $t=R$; $x = R^2-R^2 = 0$

$0-2t.dt = dx$

$\tau_y = \frac{F}{I}\times\frac{\int_y^R 2t.\sqrt{R^2-y^2t^2}\,dt}{2\sqrt{R^2-y^2}}$

$\tau_y = \frac{F}{I}\frac{\int_{R^2-y^2}^{0} -\sqrt{x}\,dx}{2\sqrt{R^2-y^2}}$

$\tau_y = \frac{F}{I\times2} - \frac{2}{3}\left[x^{3/2}\right]_{R^2-y^2}^{0}$

$\tau_y = \frac{F}{3\times I}\times{}^{-}(0-(R^2-y^2))$

$\tau_y = \frac{F}{3\times I}\times(R^2-y^2)$

$\frac{d\tau_y}{dy} = 0$; i.e τ_{max} at $y=0$

$\tau_y = \frac{F}{3I}\times R^2$

$\tau_y = \frac{F}{3\times\frac{\pi D^4}{\cancel{64}16}}\times\frac{D^2}{\cancel{4}}$

$\tau_y = \frac{F}{3\times\frac{\pi D^{\cancel{4}2}}{4\times4}}\times\cancel{D^2}$

$\tau_y = \frac{4}{3}\times\frac{F}{\pi D^2/4}$

$\tau_y = 1.33\ F/A$

$\tau_{max} = 1.33\times F/A$

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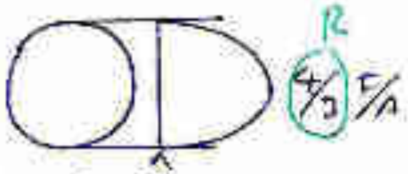
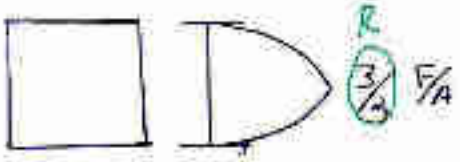
Q) Since rectangle is always better than Circle.





$\tau_{max} \le \hat{\tau}$

$1.33\ F/a \le \hat{\tau}$

$F_c \le \frac{\hat{\tau}.a}{1.33}$

(Circle)

$\tau_{max} \le \hat{\tau}$

$1.5 \times F/a \le \hat{\tau}$

$F_R \le \frac{\hat{\tau} \times a}{1.5}$

(Rectangle)

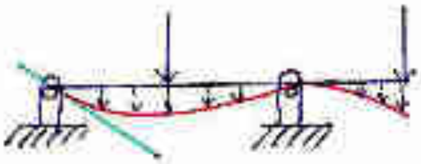
Re $\frac{F_c}{F_R} = \frac{\frac{\hat{\tau}_c a}{1.33}}{\frac{\hat{\tau}_R a}{1.5}}$ (Since cross sectional area)

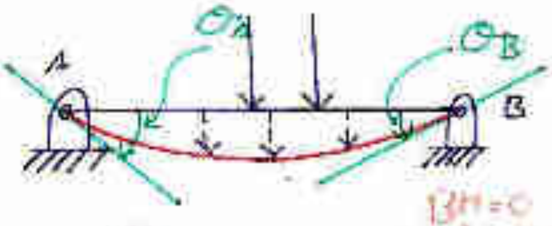
$\frac{F_c}{F_R} = \frac{1.5}{1.33} \times \frac{\hat{\tau}_c}{\hat{\tau}_R}$ (Since material)

$F_c = 1.127\ F_R$

Circular section can resist More shear than rectangle. But rectangle normally chosen because the Bending is more critical than shear.

3.7 SLOPE & DEFLECTION :-

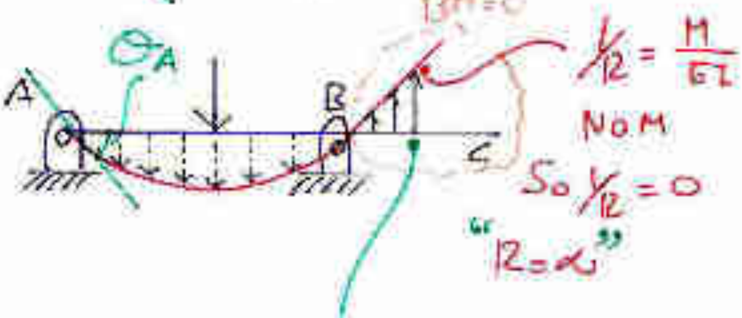




Overhanging portion having no load.

✳

Overhanging beam, at end No load, So ~~xx~~ 'R' will be ∞, No Bending moment i.e. M will be "Zero".



$\frac{1}{R} = \frac{M}{EI}$

NO M

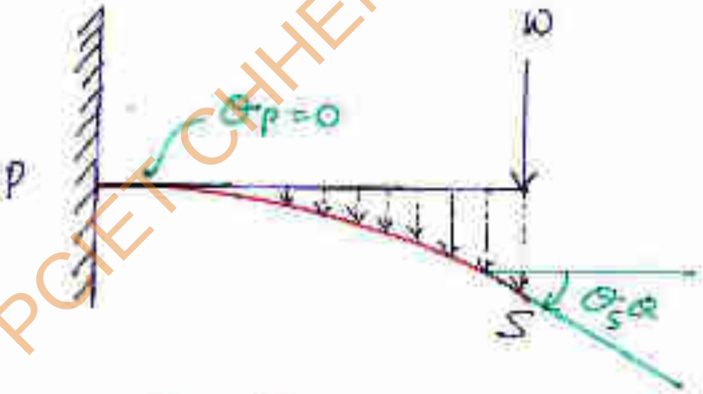
So $\frac{1}{R} = 0$

"$R = \infty$"

BM at BC will be zero

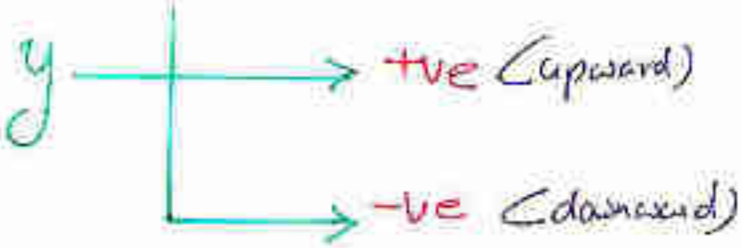
"   "

B.M is Zero, Zero Curvature



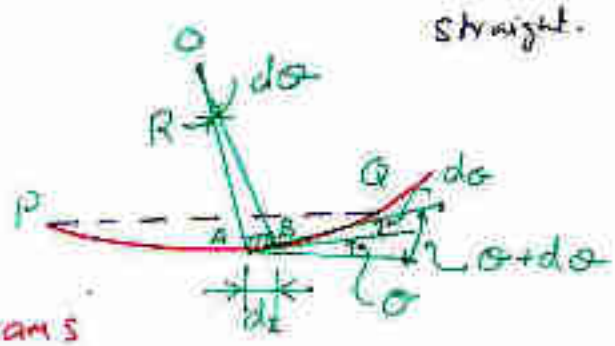
a) DEFLECTION (A fn. of x) :-

y = (lateral displacement) = (Deflection) = f(x)



→ $y' = \frac{dy}{dx} = \tan\theta = \text{Slope} = \theta(x)$; $\frac{dy}{d\theta_x} \approx \theta = \tan\theta$

Beams are almost straight.



→ dθ = differential change in Slope,

$y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ → For beams

$y'' = \frac{d\theta}{dx}$

θ is measured from +ve x-Axis

$d\theta = dx/R$

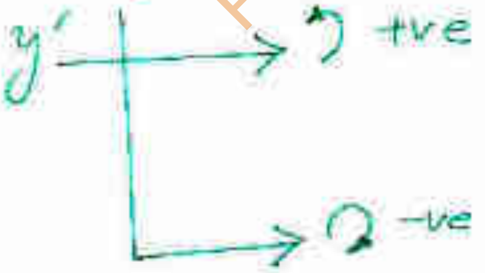
$y'' = \frac{dx/R}{dx}$

$y'' = 1/R = \frac{M}{EI}$

$y'' = \frac{M}{E.I}$

b) Differential Equation of Beam/Elastic Curvature:

$E.I.y'' = M$ $y' = \tan\theta \approx \theta = \text{Slope} = \theta(x)$



Method Of Integration:-

$$E I . y'' = M$$

On Integration

$$E I y' = \int M dx. + A$$

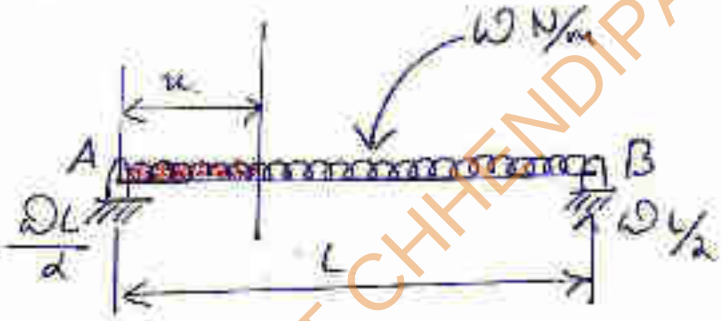
$$E I y' = \int M dx + A$$

$$E I y = \int\int (M dx + A) dx + B$$

$$E I y = \int\int M dx dx + Ax + B$$

$$E . I y = \int\int M dx dx + Ax + B$$

?



$$BM_x = + \frac{\omega L}{2} \times x + - \omega x \times x/2.$$

$$BM_x = \frac{\omega L}{2} x + - \omega x^2/2$$

$$M_x = \frac{\omega L x}{2} - \frac{\omega x^2}{2}$$

$$E I y'' = M = \frac{\omega L}{2} x - \frac{\omega x^2}{2}.$$

$$\int y''. E I = \int \frac{\omega L}{2} x - \frac{\omega x^2}{2} dx.$$

$$E I . y' = \frac{\omega L x^2}{2 \times 2} - \frac{\omega x^3}{6} + A.$$

$$E I . y = \frac{\omega L x^3}{6 \times 2} - \frac{\omega x^4}{24} + Ax + B$$

$$E I . y = \frac{\omega L x^3}{12} - \frac{\omega x^4}{24} + Ax + B$$

To know value of A and B.

At Support $y=0$; θ is allowed (at hinge)

$y=0$; $\theta=0$ (at fixed end).

$$E\cdot Iy = \frac{wL}{12}x^3 - \frac{wx^4}{24} + Ax + B.$$

At $y=0$, $x=0$

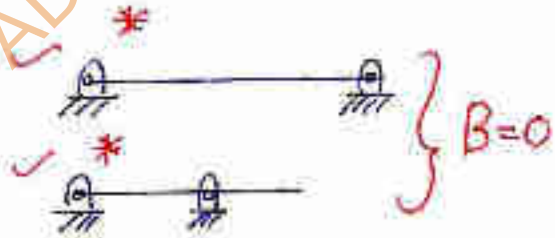
$$E\times 0\times I = 0 - 0 + A\times 0 + B$$

$B=0$

At $y=0$; $x=l$.

$$EI\times 0 = \frac{wL}{12}\times L^{\cancel{3}2} - \frac{w\times L^{\cancel{4}3}}{24} + A\times\cancel{L} + \cancel{B}^{\,0}$$

$$-\frac{wL^3}{12} + \frac{wL^3}{24} = A$$



$$A = \frac{wL^3}{24} - \frac{wL^3}{12}$$

$$A = \frac{-wL^3}{24}$$

$$EIy = \frac{wL}{12}x^3 - \frac{wx^4}{24} - \frac{wL^3}{24}\times x + \cancel{B}^{\,0}$$

$$EIy' = \frac{wLx^2}{4} - \frac{wx^3}{6} + A$$

In Simply supported beam θ will be max either A or B

θ_{max} = at $y'=\theta$ $x=0$.

$EIy' = A$.

$$y' = \frac{-wL^3}{24EI}$$

$\boxed{\theta_{max} = \frac{-wL^3}{24EI}.}$ (at $x=0$)

y_{max} at $L/2$.

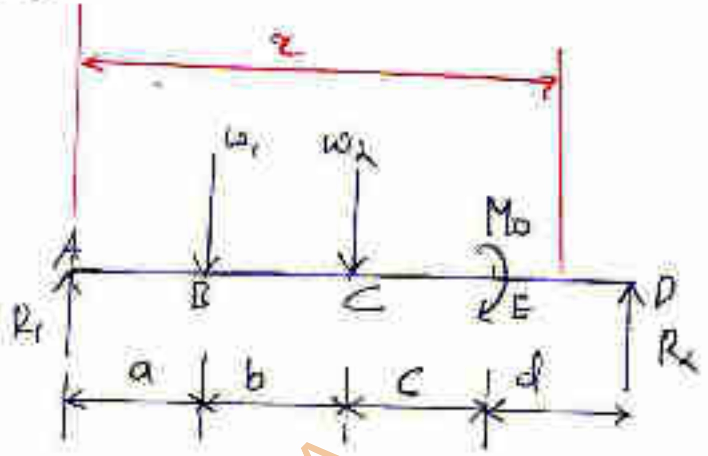
$$EIy = \frac{wL}{12}\left(\frac{L}{2}\right)^3 - \frac{w}{24}\times\left(\frac{L}{2}\right)^4 - \frac{wL^3}{24}\times\frac{L}{2}$$

$$EIy = \frac{wL}{12} \times \frac{L^3}{8} - \frac{w}{24} \cdot \frac{L^4}{16} - \frac{wL^3}{24} \times \frac{L}{2}.$$

$$EIy = \frac{wL^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48}$$

$$\boxed{y = -\frac{5wL^4}{384EI}}$$ (downward)

Standard Value



$\cancel{AB}$

$\cancel{\theta A}$ 8 $\cancel{M_1} = \cancel{R_1} x$

BC

$\cancel{\theta A}$: $\cancel{M_1 x}$

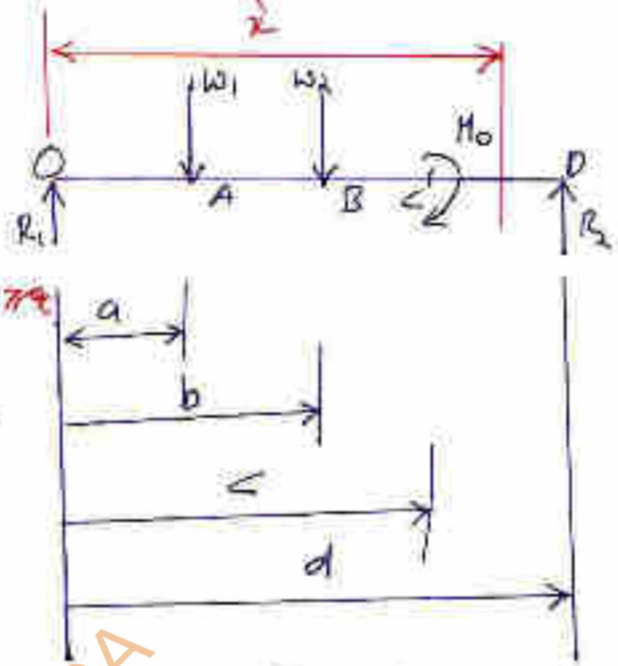
d) MACAULAY'S METHOD (MOD I - METHOD) :- (Always use this method)

$OA: M_1 = R_1 x \qquad 0 \leq x \leq a$

$AB: M_2 = R_1 x - \omega_1 \langle x-a \rangle \qquad a \leq x \leq b$

$BC: M_3 = R_1 x - \omega_1 \langle x-a \rangle - \omega_2 \langle x-b \rangle \qquad b \leq x \leq c$

$CD: M_4 = R_1 x - \omega_1 \langle x-a \rangle - \omega_2 \langle x-b \rangle + M_0 \langle x-c \rangle^0 \qquad x \geq d$



RULE 1: Bending Moment Equation is to be written for the last segment keeping quantities less than x within special brackets.

RULE 2: While applying the boundary conditions or determining slope/deflection at any point, the quantity within special brackets is to be dropped or omitted, if it turns negative.

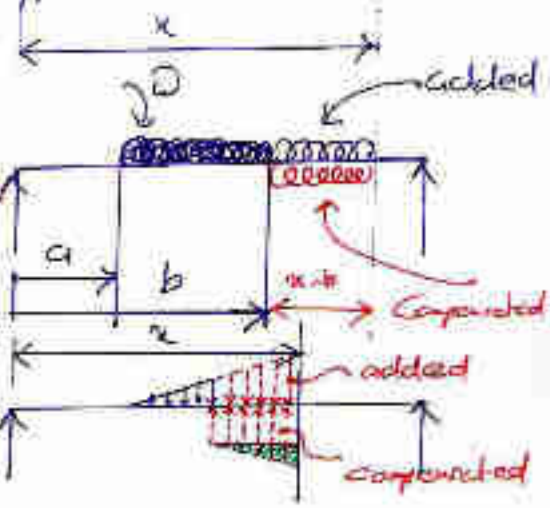
RULE 3: Couple, if any is to be multiplied by a distance raised to zero.

RULE 4: Quantities within special brackets are to be integrated as a whole.

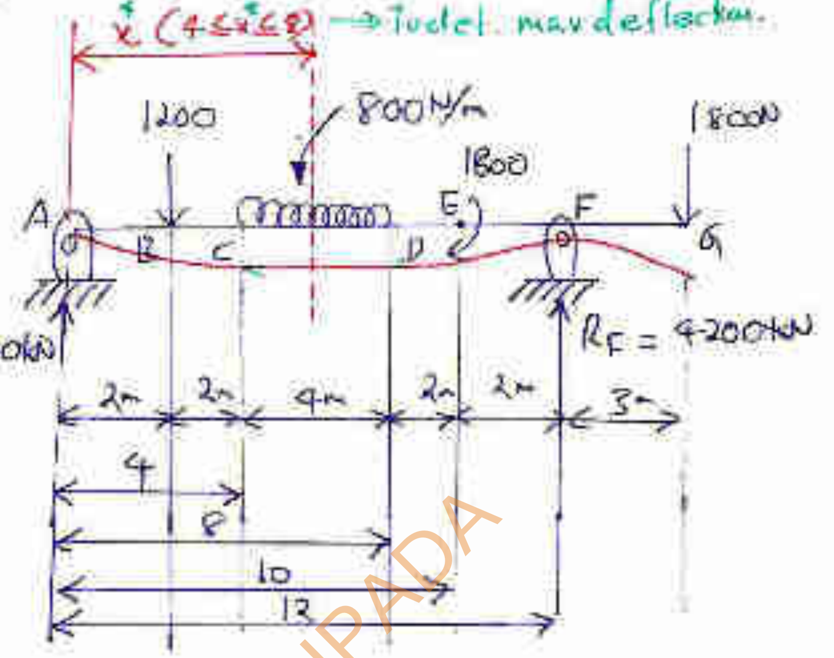
RULE 5: Distributed load, if any, is to be extended till the last segment and to be compensated by equal and opposite load.

Without Extending : $-\omega \langle b-a \rangle$

After Extending : $-\omega \langle x-a \rangle + \omega \langle x-b \rangle$



9 An overhanging beam is loaded as shown in figure. If Flexural rigidity of beam 'EI' is 24,000 kNm²/mm² then determine i) slope at A. ii) Slope at C. iii) Deflection at C, D and G. iv) Locate the point of max. deflection and also find the maximum deflection.



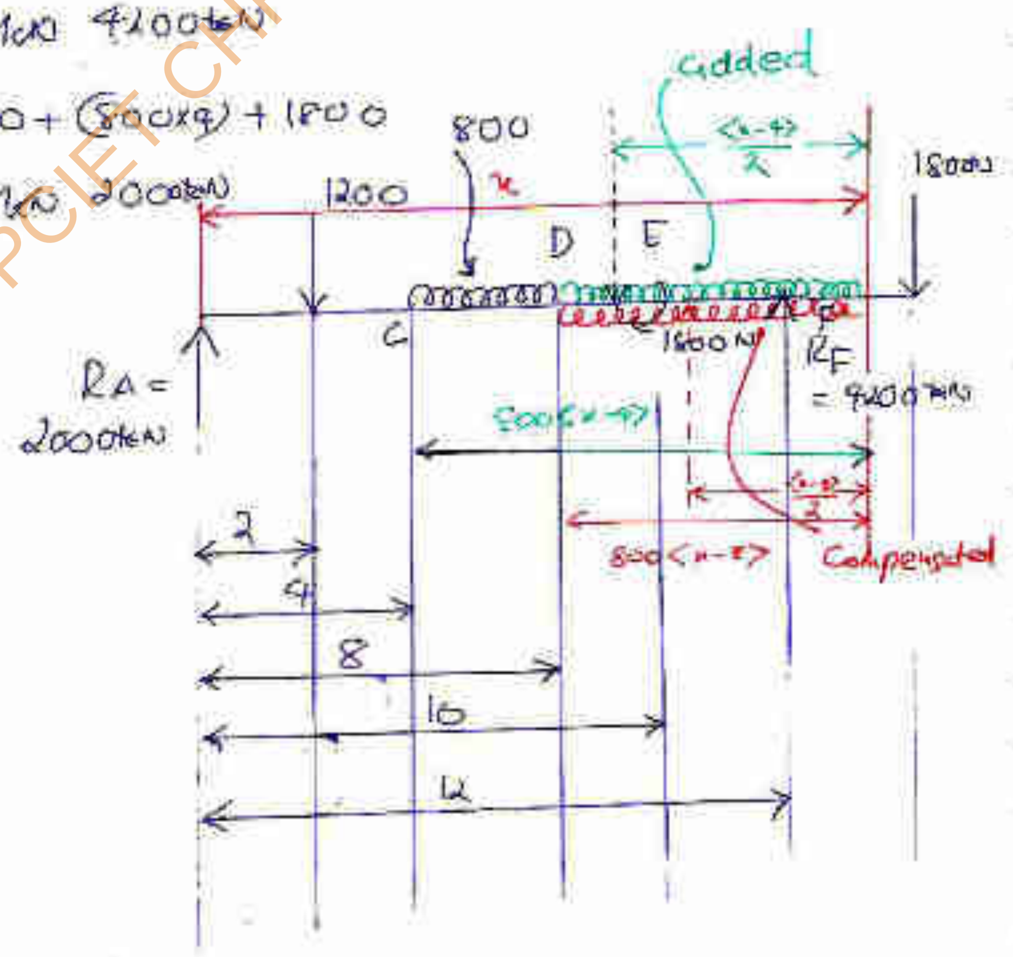
$\Sigma M_A = 0$

$1200\times2 + (800\times4)(6) + 1800 + 1800\times15 = R_F\times12$

$R_F =$ ~~4183.33 kN~~ 4200 kN

$R_A + R_F = 1200 + (800\times4) + 1800$

$R_A =$ ~~2016.67 kN~~ 2000 kN



About left Section.

$$EIy'' = M = 2000x - 1200\langle x-2\rangle - 800\langle x-4\rangle \frac{\langle x-4\rangle}{2} +$$
$$4200\langle x-12\rangle + 1800\langle x-10\rangle^0 + 800\langle x-8\rangle\frac{\langle x-8\rangle}{2}$$

$$EIy'' = M = 2000x - 1200\langle x-2\rangle - 400\langle x-4\rangle\langle x-4\rangle + 400\langle x-8\rangle^2 +$$
$$1800\langle x-10\rangle^0 + 4200\langle x-12\rangle$$

On Integrating.

$$EIy' = 1000x^2 - 600\langle x-2\rangle^2 - \frac{400}{3}\langle x-4\rangle^3 + \frac{400}{3}\langle x-8\rangle^3 + 1800\langle x-10\rangle^1$$
$$+ 2100\langle x-12\rangle^2 + A,$$

On further integration

$$EIy = \frac{1000x^3}{3} - 200\langle x-2\rangle^3 - \frac{100}{3}\langle x-4\rangle^4 + \frac{100}{3}\langle x-8\rangle^4 + 900\langle x-10\rangle^2$$
$$+ 700\langle x-12\rangle^3 + Ax + B$$

B.C. → At $x=0$; $y=0$

-ve dropped -ve dropped -ve dropped -ve dropped

$$0 = 0 - 200\langle 0-2\rangle^3 - \frac{100}{3}\langle 0-4\rangle^4 + \frac{100}{3}\langle 0-8\rangle^4 + 900\langle 0-10\rangle^2$$

-ve dropped

$$+ 700\langle 0-12\rangle^3 + A\times 0 + B$$

$\therefore B = 0$

B.C. → At $x=12$; $y=0$

$$0 = 576000 - 200000 - 136533.33 + 8533.33 + \cancel{76}3600 + 700\times 0$$
$$+ A\times 12$$

$$\underline{A = -20966.66} \; ; B = 0$$

~~EIy''~~

$$EIy' = 1000x^2 - 600\langle x-2\rangle^2 - \frac{400}{3}\langle x-4\rangle^3 + \frac{400}{3}\langle x-8\rangle^3 + 1800\langle x-10\rangle^1 +$$
$$2100\langle x-12\rangle^2 - 20966.66.$$

$$EIy = \frac{1000}{3}x^3 - 200\langle x-2\rangle^3 - \frac{100}{3}\langle x-4\rangle^4 + \frac{100}{3}\langle x-8\rangle^4 + 900\langle x-10\rangle^2 +$$
$$700\langle x-12\rangle^3 - 20966.66x + \cancel{B}^{\nearrow 0}$$

→ Slope At A (i.e. $x=0$).

$EIy' = 0 - 0 - 0 + 0 + 0 + 0 - 20966.66.$ (-ve terms [illegible] are dropped)

$$y' = \frac{-20966.66}{EI}$$
EI → Flexural rigidity.

$$y' = \frac{-20966.66}{2400\cancel{0}\times10^3} = \underline{\underline{-8.736\times10^{-\cancel{4}3}}} C$$

→ Deflection At C (i.e. $x=4$)

$$EIy = \frac{1000\times4^3}{3} - 200\langle 4-2\rangle^3 - 0 + 0 + 0 + 0 - 20966.66\times4.$$

$$y = \frac{-64133.3067}{EI}$$

$$y = \frac{-64133.3067}{2400\times10^3} = \underline{\underline{-0.0267\,m}} = \underline{\underline{-26.72\,mm}}$$ (downward deflection)

check by slope

⊕ --slope--> ⊖
Max. deflection

→ To get Max deflection Slope changes sign.

Take a section x^* from 0 to section C-D

$$\theta @ x^* = (y')_{x^*} = 1000x^{*2} - 600\langle x^*-2\rangle^2 - \frac{400}{3}\langle x^*-4\rangle^3$$
$$-20966.66$$

$(y')_{x^*} \rightarrow 0$

Slope → 0
↳ Max. deflection.

$1.e = 0 = 1000x^2 - 600(x^2 - \overset{4x}{\cancel{2x}} + 4) - \frac{400}{3}(x^3 - \overset{12}{\cancel{3x}}x^2 + \cancel{12x}\ 48x - 64)$

-20966.666

$20966.66 = 1000x^2 - 600x^2 \cancel{+4}200x + -2400 - \frac{400}{3}x^3 + 1600x^2 +$

$-6400x + \frac{25600}{3}$

$20966.66 = 2000x^2 - \frac{400}{3}x^3 - \cancel{8}200x + \frac{18400}{3}$

$\frac{400}{3}x^3 - 2000x^2 + 8200x + 14833.34 = 0$

$x^3 - 15x^2 + \overset{15}{\cancel{34}}x + 111.25 = 0$

$x_1 = 11.5795$ ✗

$x_2 = -1.849$ ✗

$x_3 = 5.25$ ✓ (B/w C+D)

~~$EIy = 1000 \times \frac{5.25^3}{3} - 200 < 5.25 - 2 >^2 - \frac{400}{3} < 5.25 - 4 >^3 - 20966.66$~~

~~$EI.y = 2489479$~~

~~$y = \frac{24894.79}{2400 \times 1000}$~~

Max. deflection at x = 5.25m.

$EI.y = \frac{1000}{3} \times 5.25^3 - 200 < 5.25 - 2 >^3 - \frac{100}{3} < 5.25 - 4 >^4 + 0 + 0 + 0 \mp 20966.66 \times 5.25$

$EI.y = 48234.375 - 6865.625 - 81.38 \mp 110074.965$

$EIy = \cancel{151361.335} - 68786.97$

$y = \frac{\cancel{151361.335}\ -68786.97}{2400 \times 1000} = \underline{\underline{\cancel{63.06}}}\ mm\ -28.66$ (downward)

Deflection at x = 15

$EIy = 1000 \times \frac{15^3}{3} - 200 \times 13^3 - \frac{100}{3} \times 11^4 + \frac{100}{3} \times 7^4 + 400 \times 5^2 + 700 \times 3^3 - 20966.66 \times 15$

$EIy = 4000.1$

$y = \frac{4000.1}{2400 \times 10^3} = \underline{\underline{1.66\ mm}}$

→ Deflection at C :- (x=4)

$$EI\,y = \frac{1000}{3} \times 4^3 - 200\langle 4-2\rangle^3 + 0 + 0 + 0 + 0 - 20966.66 \times 4$$

$$EI.y = 21333.33 - 1600 - 83866.64$$

$$y = \frac{-64133.31}{2400 \times 1000} = \underline{\underline{-26.72\,mm.}}$$

→ Deflection at D : (x=8)

$$EI\,y = \frac{1000}{3} \times 8^3 - 200\langle 8-2\rangle^3 + \frac{-\overset{1}{8}00}{3}\langle 8-4\rangle^3 + 0 + 0 + 0 - 20966.66 \times 8$$

$$EI\,y = 170666.667 - 43200 - 2133.33 - 167733.28$$

$$y = \frac{-42399.943}{2400 \times 1000} = \underline{\underline{-17.66\,mm.}}$$

→ Slope at C (i.e. x=4)

$$EI\,y' = 1000 \times 4^2 - 600\langle 4-2\rangle^2 - 0 + 0 + 0 + 0 - 20966.66$$

$$y' = \frac{-7366.66}{2400 \times 1000} = \underline{\underline{-3.06 \times 10^{-3}}}$$

#1) $M_A = 0$

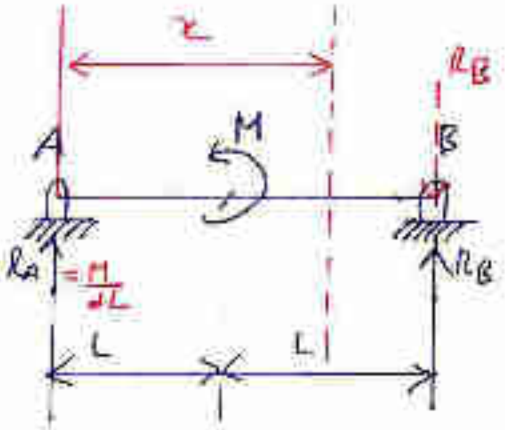
$M + R_B \times 2L = 0.$

$R_B \times 2L = -M$

$R_B = \frac{-M}{2 \times L}$

$R_A + R_B = 0$

$R_A = -R_B = \frac{M}{2L}$



$$M_x = +R_A \times x + M\langle x-L\rangle^0$$

$$M_x = \frac{M}{2L} \times x - M\langle x-L\rangle^0$$

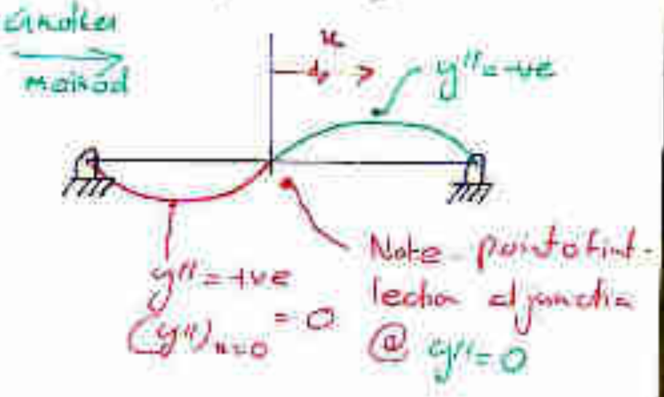
$$EIy'' = \frac{M}{2L} \times x - M\langle x-L\rangle^0$$

$$EIy' = \frac{M}{2L} \times \frac{x^2}{2} - M\langle x-L\rangle$$

$$EIy' = \frac{M}{4L} \times x^2 - M\langle x-L\rangle + A$$

$$EIy = \frac{M}{4L} \times \frac{x^3}{3} - M\frac{\langle x-L\rangle^2}{2} + Ax + B.$$

Same question:



at $x=0;\ y=0.$

$$0 = 0 - \cancel{M\frac{\langle 0-L\rangle^2}{2}} + 0 + B. \quad \text{dropped}$$

$$\cancel{\frac{ML^2}{2}} = \cancel{B}\ ;\ \cancel{B = \frac{ML^2}{2}} \qquad B = 0.$$

at $x = 2L;\ y = 0.$

$$0 = \frac{M}{2L \times 2} \times \frac{(2L)^3}{3} - \frac{M\langle 2L-L\rangle^2}{2} + A \times 2L + \cancel{B}^{\to 0}$$

$$0 = \frac{M}{4 \times \cancel{L}} \times \frac{8L^{\cancel{3}2}}{3} - \frac{ML^2}{2} + A \times 2L + 0$$

$$0 = \frac{2ML^2}{3} - \frac{ML^2}{2} + A \times 2L$$

$$A \times 2\cancel{L} = \frac{-1}{6}ML^{\cancel{2}}$$

$$\underline{\underline{A = -\frac{ML}{12}}}$$

i.e

$$EIy' = \frac{M}{4L} \times x^2 - M\langle x-L\rangle - \frac{ML}{12}$$

$$EI.y = \frac{M}{12L} \times x^3 - M\frac{\langle x-L\rangle^2}{2} - \frac{ML}{12} \times x$$

Slope at mid point $\cancel{y' @ x = L/2}$ $y' @ x = L$

$$E\cancel{I}y' = \frac{\cancel{M}}{4L} \times \left(\frac{L}{2}\right)^2 - \cancel{M}\langle \cancel{\tfrac{L}{2}} - L/2\rangle^{\to -ve} - \frac{\cancel{M}L}{12}.$$

$$\cancel{EI}y = \frac{ML}{8 \times 2} - \frac{ML}{12}$$

$$E\cancel{I}y' = -\frac{ML}{48}$$

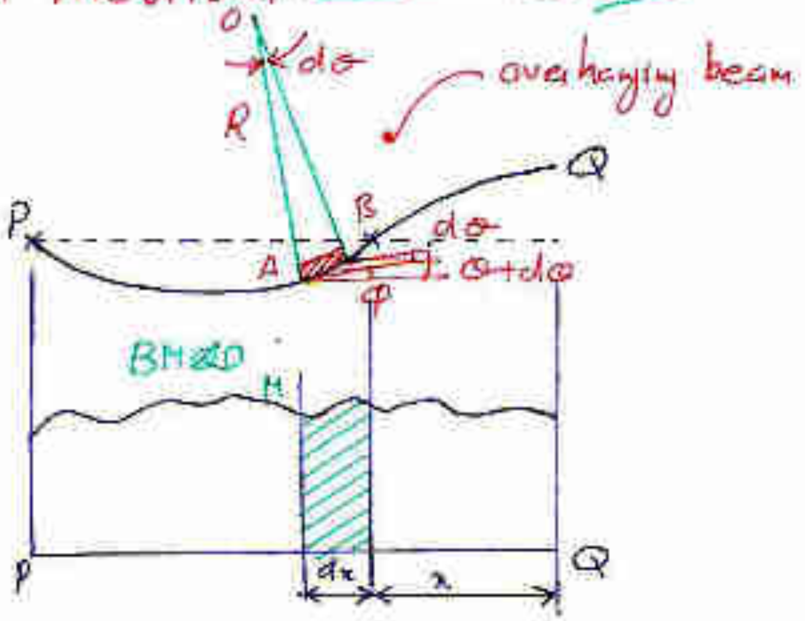
$$EIy' = \frac{M}{4L} \times L^{\cancel{2}} - 0 - \frac{ML}{12}.$$

$$EIy' = \frac{ML}{4\cancel{L}} - \frac{M}{12}$$

$$\underline{\underline{y = \frac{ML}{6EI}}}$$

21/07/2017

Moment Area method:- <Cantilever>



$d\theta = \left(\text{differential change in slope}\right) = \frac{AB}{R}$

AB is dx i.e. $d\theta = \frac{dx}{R}$

To obtain total integrate from P→Q

$$\int_P^Q d\theta = \int_P^Q \frac{dx}{R}$$

$$\theta_Q - \theta_P = \int_P^Q \frac{dx.M}{EI} \qquad \left[\frac{1}{R} = \frac{M}{EI}\right]$$

$$\theta_Q - \theta_P = \frac{1}{EI}\int_P^Q M.dx$$

Mdx = Area of shaded area.

$$\theta_Q - \theta_P = \frac{1}{EI}\int_P^Q dA_{BMD}$$

$$\theta_Q - \theta_P = \frac{A[BMD]_P^Q}{EI}$$

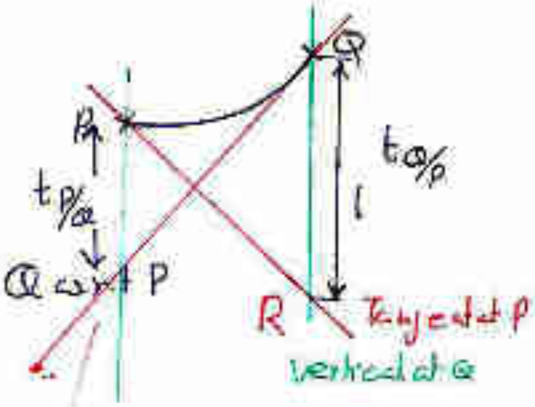
→ $$\theta_Q - \theta_P = \frac{A[BMD]_P^Q}{EI}$$

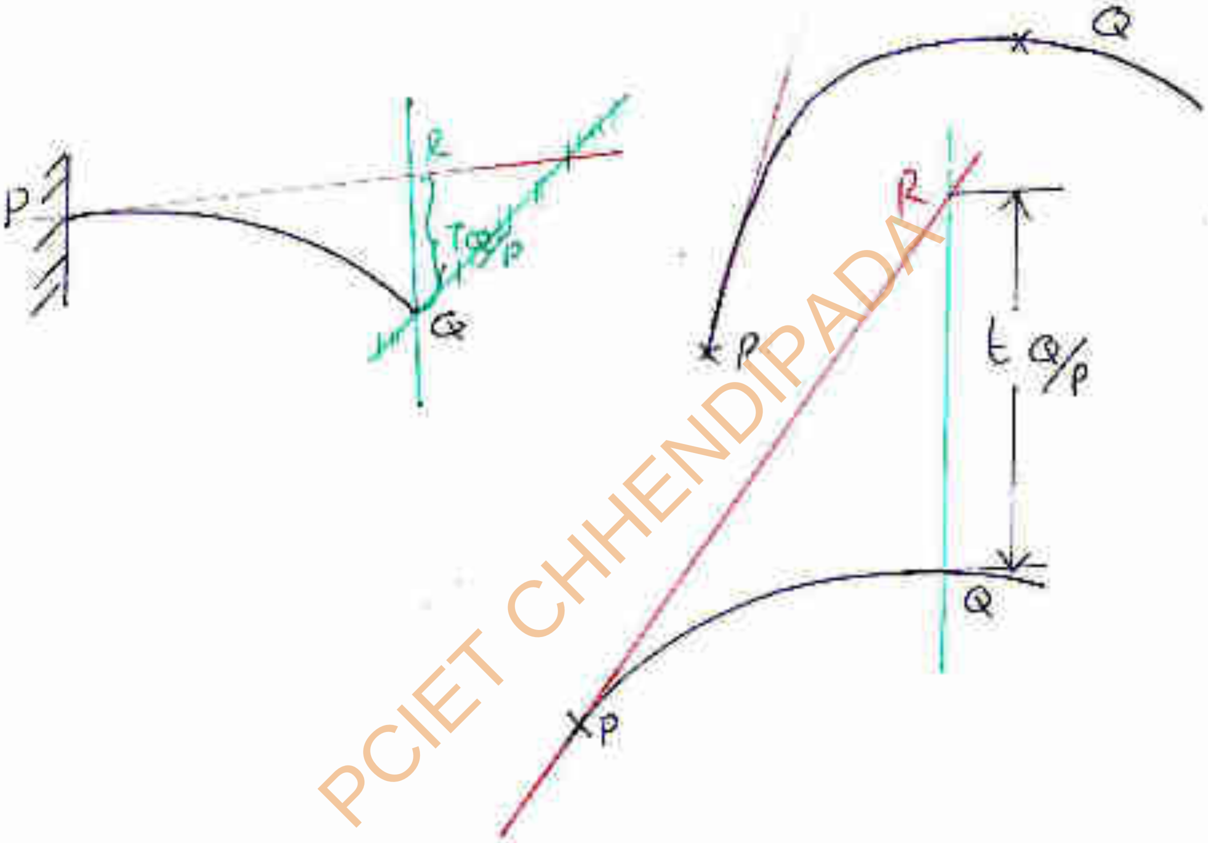
Tangential Deviation:-

Draw a tangent through P

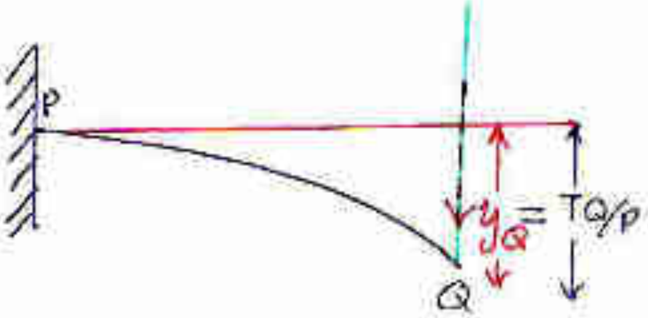
Draw a vertical at Q.

R ⟶ tangential deviation Q w.r.t P

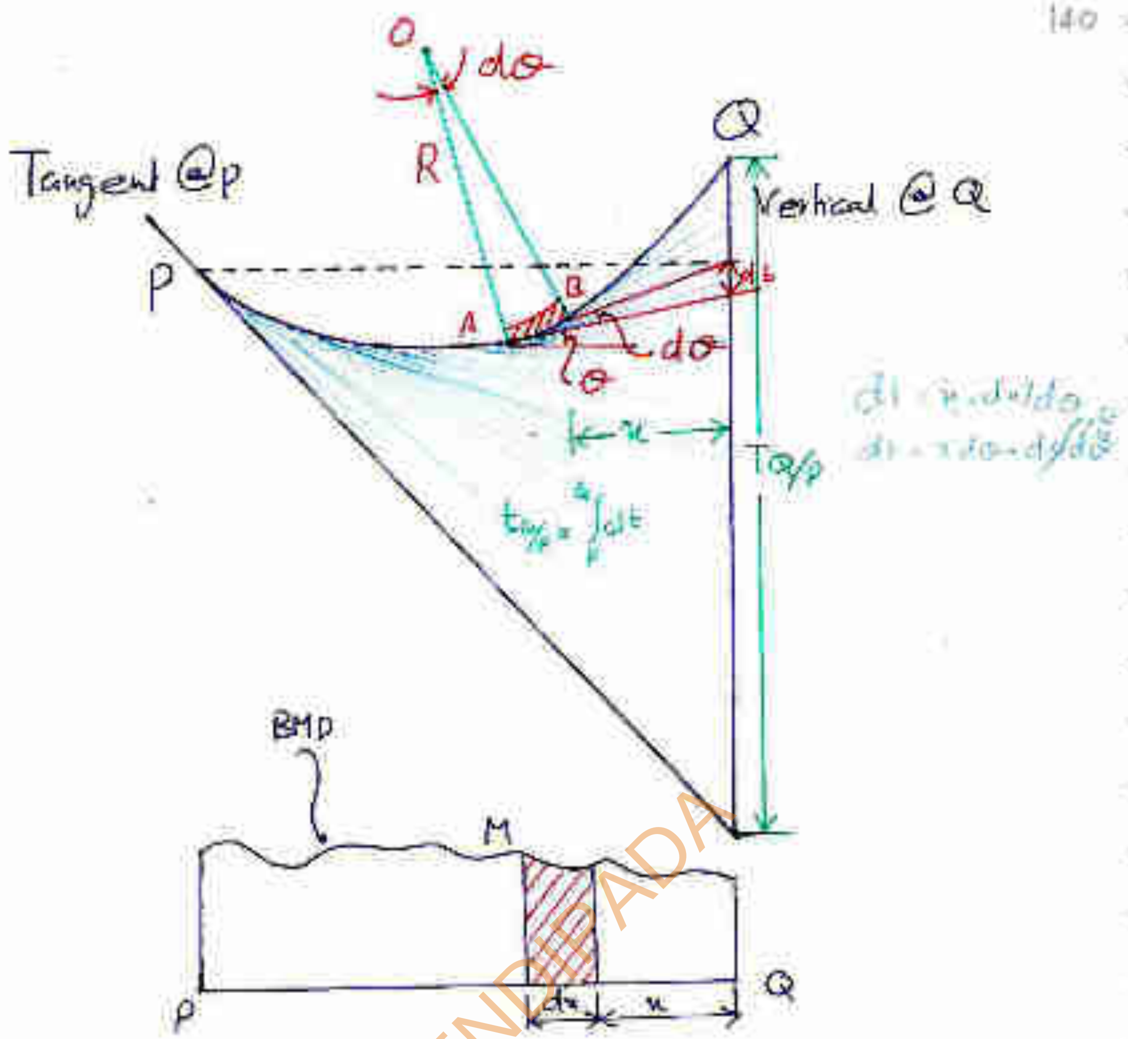




→ For a Cantilever beam $T_{Q/P} = y_Q$.



Only for Cantilever beams



$t_{Q/P}$ = Tangential Deviation = y_Q → In case of Cantilever beam with p marked as fixed point.

$$t_{Q/P} = \int_P^Q dt = \int_P^Q x\,d\theta = \int_P^Q x \times \frac{dx}{R}.$$

$$t_{Q/P} = \int_P^Q x \times \frac{M}{EI}\,dx$$

$$t_{Q/P} = \frac{1}{EI}\int_P^Q x.M\,dx \qquad [M\,dx = dA]$$

i.e. $$t_{Q/P} = \frac{1}{EI}\int_P^Q x.\,dA$$

$\int_P^Q x\,dA$ → First moment of area about Q

→ $$t_{Q/P} = \frac{1}{EI} \times A\bar{x}_Q$$

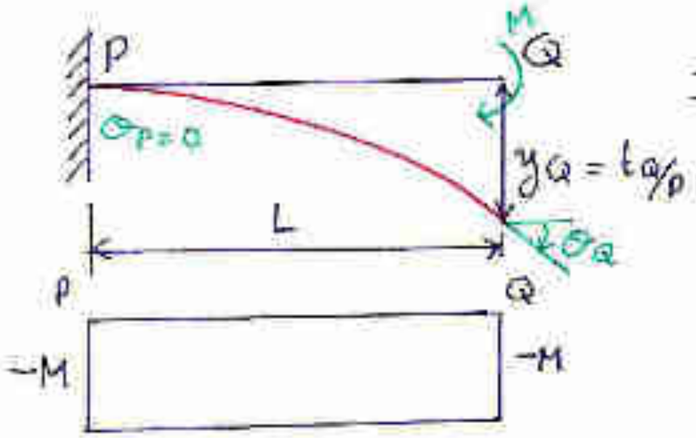
P → at fix support only.

$\bar{x}$ → from Q'

I) Theorem I: $\theta_Q - \theta_P = [A]_{BMD}^{Q} / EI$

II) Theorem II: $t_{Q/P} = \dfrac{\left\{ [A_{BMD}]_P^Q \times \bar{x}_Q \right\}}{EI}$

→ Case (i) :-



I) $-\theta_P + \theta_Q = \dfrac{A_{[BMD]_P^Q}}{EI} = \dfrac{-ML}{EI}$

$\theta_Q - \theta_P = \dfrac{-ML}{EI}$

Here $\theta_P = 0$ because straight at P

$\theta_Q - \overset{\to 0}{\cancel{\theta_P}} = -\dfrac{M\cancel{E}L}{EI}$

$\theta_Q = -\dfrac{ML}{EI}$ (-ve should be considered in area)

$\theta_{max} = \dfrac{-ML}{EI}$

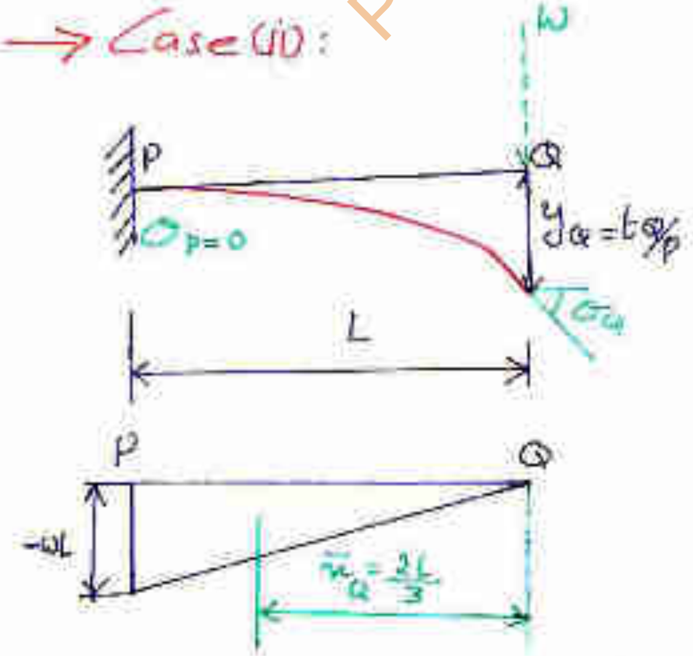
II) $t_{Q/P} = \dfrac{A_{[BMD]_P^Q} \times \bar{x}_Q}{EI}$

For a cantilever beam $t_{Q/P} = y_Q$.

i.e. $y_Q = \dfrac{(-ML)\cdot L/\cancel{2}2}{EI}\ \overbrace{[\text{from } Q]}^{\bar{x}_Q}$

i.e. $y_{max} = \dfrac{-ML^2}{2EI}$ $\therefore y_{max} = \dfrac{-ML^2}{2EI}$

→ Case (ii) :



I) $\theta_Q - \theta_P = \dfrac{A_{[BMD]_P^Q}}{EI} = \dfrac{-wL^2/2}{EI}$

$\theta_P = 0$:-

$\theta_Q = \dfrac{-wL^2}{2EI}$ i.e. $\theta_{max} = \dfrac{-WL^2}{EI}$

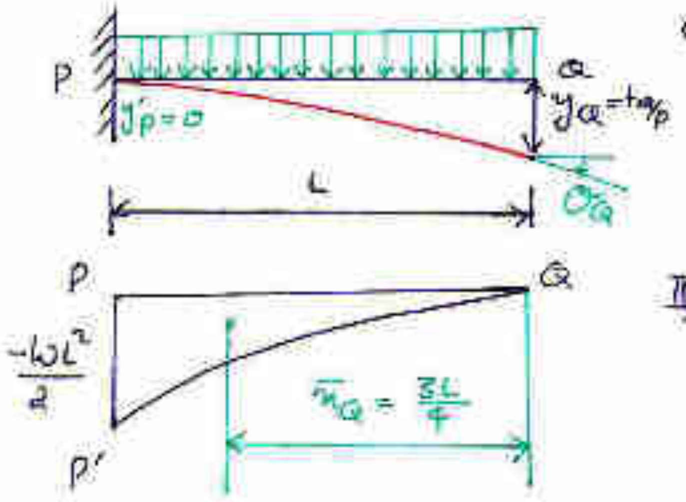
II) $t_{Q/P} = \dfrac{A_{[BMD]_P^Q} \times \bar{x}_Q}{EI}$

Cantilever $t_{Q/P} = y_Q$.

$y_Q = \dfrac{\dfrac{-wL^2}{\cancel{2}} \times \dfrac{\cancel{2}L}{3}}{EI} = \dfrac{-wL^3}{3EI}$

$y_{max} = \dfrac{-WL^3}{3EI}$

→ Case (iii):



I) $\theta_Q - \theta_P = \frac{A_{[BMD]_P^Q}}{EI}$

$\theta_P = y_P = 0$, Cantilever

$\theta_Q = \frac{\frac{1}{3} \times L \times \frac{-wL^2}{2}}{EI}$

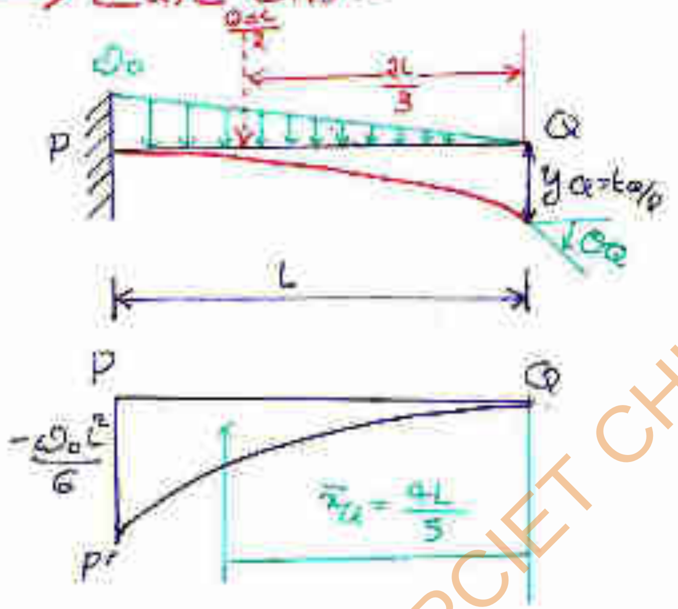
$\theta_Q = \frac{-wL^3}{6EI}$ i.e. $\theta_{max} = \frac{-wL^3}{6EI}$

II) $t_{Q/P} = \frac{A_{[BMD]_P^Q} \times \bar{x}_Q}{EI}$

$y_Q = \frac{\frac{1}{3} \times L \times \frac{-wL^2}{2} \times \frac{3L}{4}}{EI}$

$y_Q = y_{max} = \frac{-wL^3}{8EI}$

→ Case (iv):-



I] $\theta_Q - \theta_P = \frac{A_{[BMD]_P^Q}}{EI}$

$\theta_P = y_P = 0$, Cantilever.

$\theta_Q - 0 = \frac{\frac{1}{4} \times \frac{-\omega_0 L^2}{6} \times L}{EI}$

$\theta_Q = \frac{-\omega_0 L^3}{24EI}$

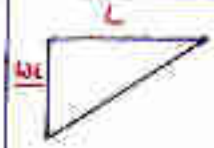
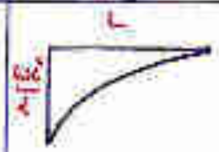
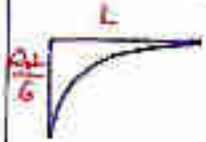
$\theta_{max} = -\frac{\omega_0 L^3}{24EI}$

II] $t_{Q/P} = \frac{A_{[BMD]_P^Q} \times \bar{x}_Q}{EI}$

$t_{Q/P} = \frac{\frac{1}{4} \times -\frac{\omega_0 L^2}{6} \times L \times \frac{4L}{5}}{EI}$

$t_{Q/P} = \frac{-\omega_0 L^4}{30EI}$

$y_{max} = \frac{-\omega_0 L^4}{30EI}$

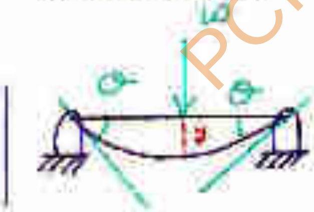
Beam	B.M.D	$\lvert\theta_{max}\rvert$	$\lvert y_{max}\rvert$
		$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$
		$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$
		$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$
		$\frac{w_0L^3}{24EI}$	$\frac{w_0L^4}{30EI}$

Eg: For UVL, y

$y = \frac{w_0L^4}{30EI}$ $\frac{\quad}{Nm^2}$

↓ fixed number

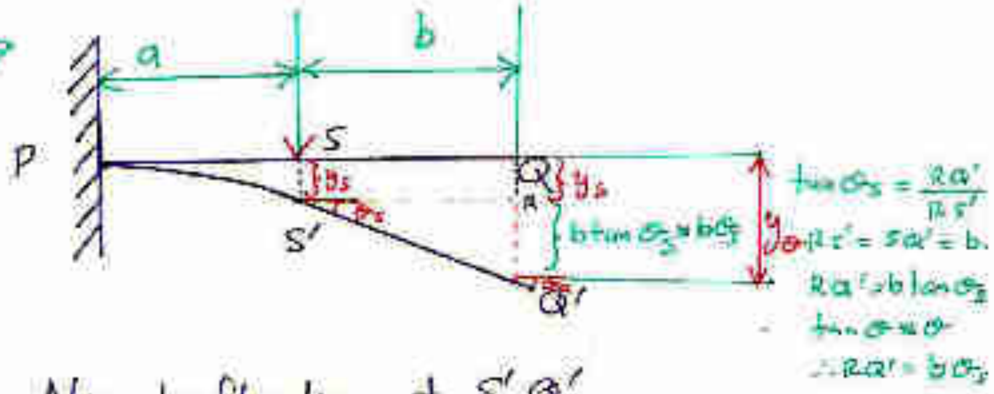
$\overbrace{\underset{M}{1}\ \underset{W}{2}\ \underset{UDL}{6}\ \underset{UVL}{24}}^{\theta}$ $\overbrace{\underset{M}{2}\ \underset{W}{3}\ \underset{UDL}{8}\ \underset{UVL}{30}}^{y}$



$\frac{WL^2}{16EI}$ $\frac{WL^3}{48EI}$

$\frac{wL^3}{24EI}$ $\frac{5wL^4}{384EI}$

→



No deflection at S' Q'

So $\theta_S = \theta_Q$

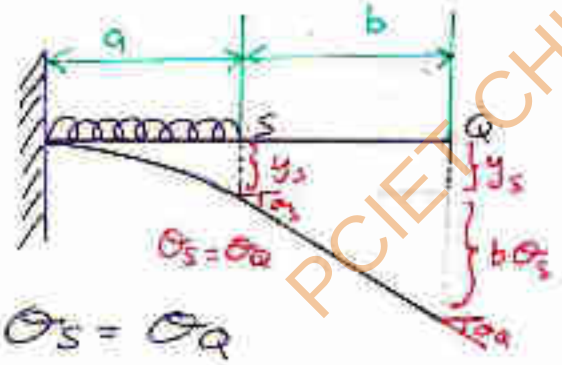
$y_Q = y_S + b\tan\theta_S$

$\tan\theta_S \approx \theta_Q$

i.e $y_Q = y_{\cancel{Q}S} + b.\theta_S$

i.e. $y_{\cancel{S}Q} = -\left(\frac{Wa^3}{3EI} + b\frac{Wa^2}{2EI}\right)$ (-ve due to clockwise rotation, & deflection downward.)

→



$\theta_S = \theta_Q$

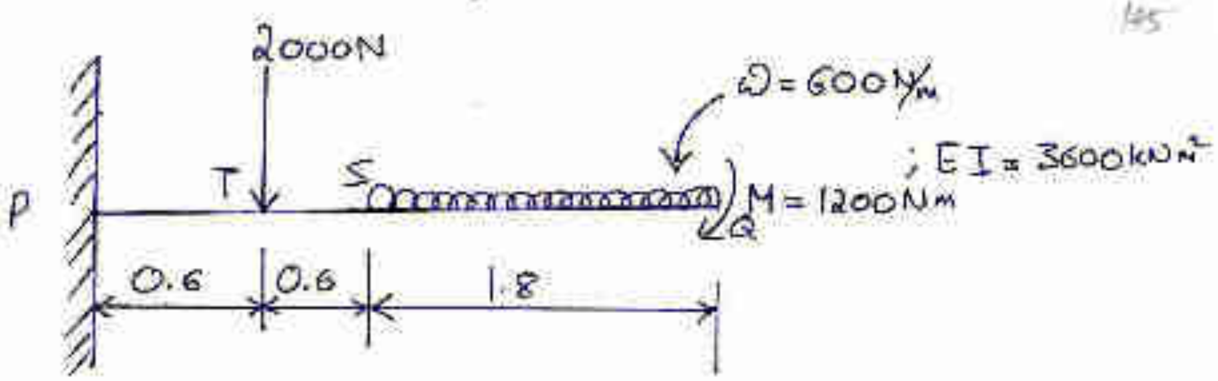
θ_S for UDL $= \frac{-\omega a^3}{6EI}$

$y_Q = y_S + b\tan\theta_S.$

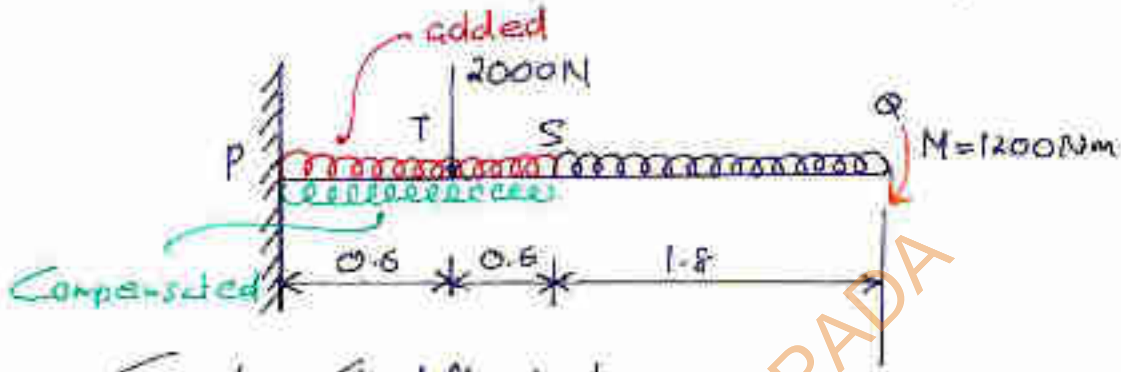
$y_Q = y_S + b.\theta_S$

$y_Q = -\left[\frac{\omega a^4}{8EI} + \frac{b.\omega a^3}{6EI}\right]$

Q

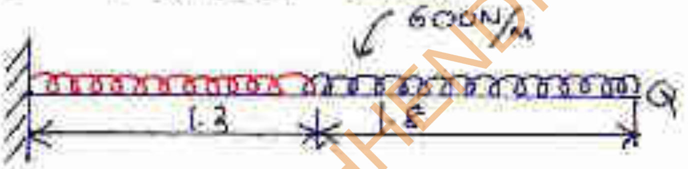


First of all extend UDL to Fix support & give compensation.



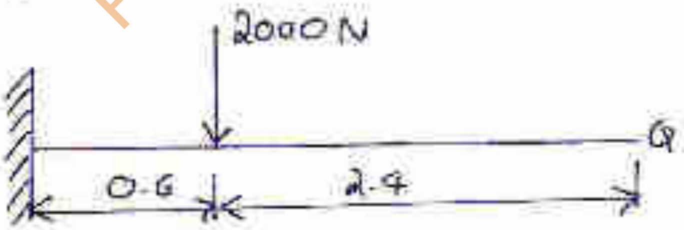
Consider 4 different beams

i)

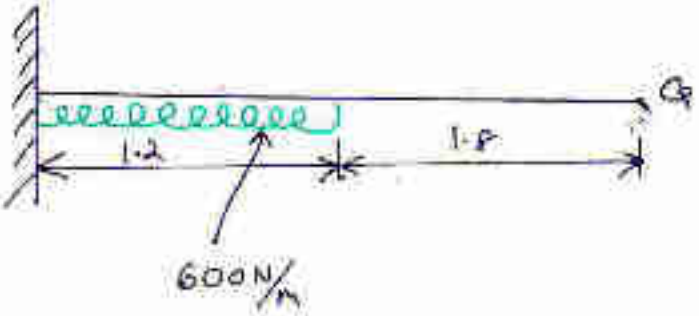


ii)

iii)



iv)



$$\theta_Q = \left[(\theta_Q)_{i} + (\theta_Q)_{ii} + (\theta_Q)_{iii} + (\theta_Q)_{iv}\right]$$

$$\theta_Q = \frac{1}{EI}\left[-\frac{600\times 3^3}{6} - (1200\times 3) - \left(\frac{2000\times 0.6^2}{2}\right) + \left(\frac{600\times 1.2^3}{6}\right)\right]$$

-ve due clockwise rotation of load

+ve due to anticlockwise rot. of load.

$$\theta_Q = \frac{1}{EI}\left[-2700 - 3600 - 360 + 172.8\right]$$

$$\theta_Q = \frac{-6487.2}{3600\times 1000}$$

$$\theta_Q = -1.802\times 10^{-3}$$

$$y_Q = \left[(y_Q)_{i} + (y_Q)_{ii} + (y_Q)_{iii} + (y_Q)_{iv}\right]$$

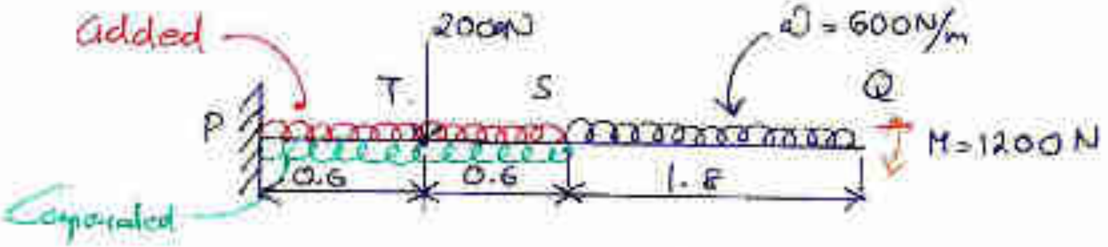
$$y_Q = \frac{1}{EI}\left[-\frac{600\times 3^4}{8} + -\left(\frac{1200\times 3^{\overset{2}{\cancel{3}}}}{2}\right) - \left(\frac{2000\times 0.6^3}{3} + \frac{2.4\times 2000\times 0.6^2}{\cancel{3}2}\right) + \left(\frac{600\times 1.2^4}{8} + 1.8\times\left(\frac{600\times 1.2^3}{6}\right)\right)\right]$$

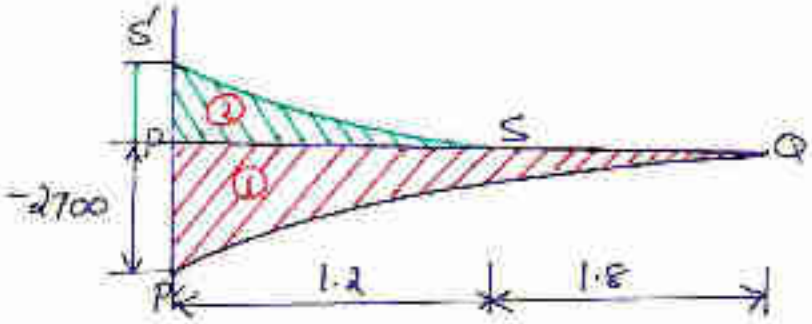
$$y_Q = \frac{1}{EI}\left[-6075 - \underset{5400}{\overset{\cancel{16200}}{\cancel{8100}}} - \overset{1008}{\cancel{720}} + 466.56\right]$$

$$y_Q = \frac{\cancel{-6328.44}\ -12016.44}{3600\times 1000} = -3.33\,mm$$

Moment Area Method:-

Note- Area can be $\boxed{-ve}$ or $\boxed{+ve}$ Never change





$A_1 = \frac{1}{3} \times 3 \times -2700 = -2700 m^2$

$\bar{x}_1 = \frac{3}{4} \times 3 = 2.25 m$

$A_2 = \frac{1}{3} \times 1.2 \times 4\theta\theta 32 = 4172.8 m^2$

$\bar{x}_2 = \frac{4}{3} \times \frac{3}{4} \times 1.2 = 0.9 m$

$A_3 = 3 \times -1200 = -3600 m^2$

$\bar{x}_3 = \frac{3}{2} = 1.5 m$

$A_4 = \frac{1}{2} \times 0.6 \times -1200 = -360 m^2$

$\bar{x}_4 = (0.6+1.8)+(\frac{2}{3} \times 0.6)$

$\bar{x}_4 = 2.8 m$

According to Moment Area theorems

$$\theta_Q - \overset{0}{\cancel{\theta_P}} = \frac{\Sigma A}{EI} = \frac{-2700-3600-360+172.6}{3600 \times 1000}$$

$$\theta_Q = -1.802 \times 10^{-3} c$$

$$y_Q = t_{Q/P} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 + A_4\bar{x}_4}{EI}$$

$$y_Q = \frac{(-2700 \times 2.25) + (172.8 \times 0.9) + (-3600 \times 1.5) + (-360 \times 2.8)}{3600 \times 1000}$$

$$y_Q = \underline{\underline{-3.92 mm}}$$

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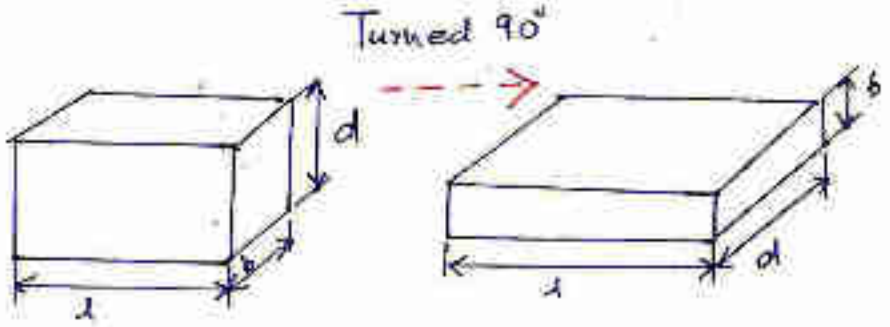
Q.1 ?



$y_{central} = y$ – deflection

Rotated by 90°, only I changes.

$$\frac{(y_{central})_{II}}{(y_{central})_{I}} = \frac{I_1}{I_{II}} = \frac{\cancel{b}d^{\cancel{3}}/\cancel{12}}{\cancel{d}b^{\cancel{3}}/\cancel{12}} = d^2/b^2$$

$$(y_{central})_{II} = (y_{central})_{I} \times d^2/b^2$$

$$y_{II} = y \times d^2/b^2 = (d/b)^2 \times y$$

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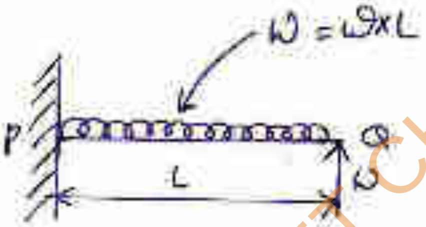
Q.2 ?



$$y_Q = (y_Q)_{UDL} + (y_Q)_W$$

$$y_Q = \frac{-wL^4}{8EI} + \frac{W \times L^3}{3EI}$$

Here $wL = W$

$$y_Q = \frac{-WL^3}{8EI} + \frac{WL^3}{3EI}$$

$$y_Q = \frac{+5WL^3}{24EI} \text{ (upward)}$$

Page No. 67
Q.N.3
Q.

$y = 18$mm. ; $\frac{WL^3}{6EI} = 0.02$.

$$\frac{WL^4}{8EI} = 18$$

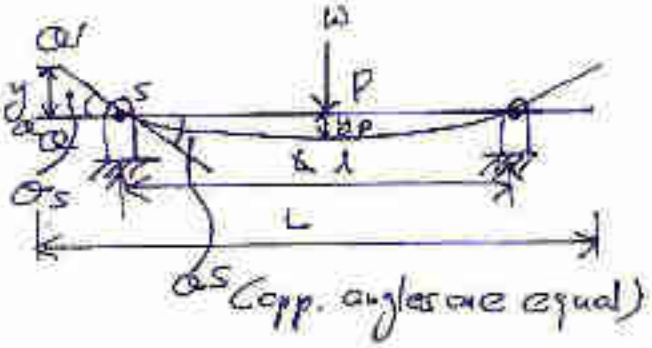
$$\frac{WL^3}{6EI} \times \frac{6L}{8EI} = 18$$

$0.02 \times \frac{6L}{8} = 18 = 1200$mm $=$ 1.2m

31/7/2017
Day No 67

Q.9 Simply and Symmetrically supported.

here $|y_P| = |y_Q|$



Beam between the support

Simply supported beam.

$y_P = \frac{WL^3}{48EI}$

Q's have No bendig moment So Q's will remains straight.

$\tan\theta_S = \frac{y_Q}{b}$

$y_Q = b \tan\theta_S$

i.e. $y_Q = b.\theta_S$

$b = L - l/2$

$\therefore \theta_S = \theta_S$ for simply supported beam.

$\theta_S = \frac{Wl^2}{16EI}$

$y_Q = \frac{L-l}{2} \times \frac{Wl^2}{16EI}$

$y_P = y_Q$.

$\frac{\cancel{W}l^2}{\cancel{16EI}} \times \frac{L-l}{2} = \frac{\cancel{W}l^3}{\cancel{48EI}_{3}}$

$\frac{L+l}{\cancel{2}} = \cancel{\frac{-L}{3}}$

$\cancel{\frac{L}{2}} + l/3 = \cancel{1/2}$

$\frac{(3+2)L}{\cancel{6}} = \cancel{1/2}$

$\cancel{L/l} = \cancel{3/5}$

$\left(\frac{L-l}{2}\right) \cdot \frac{Wl^2}{16EI} = \frac{Wl^3}{48EI}$

$\frac{\cancel{W}L l^2}{32EI} - \frac{\cancel{W}l^3}{32EI} = \frac{\cancel{W}l^3}{48EI}$

$\frac{Ll^2}{32\ 16} - \frac{l^3}{32\ 16} = \frac{l^3}{48\ 24}$

$\frac{L}{16} = \frac{l}{24} + \frac{l}{16}$

$\frac{L}{\cancel{16}} = \frac{40l}{24 \times 16}$

$$L/l = 5/3$$

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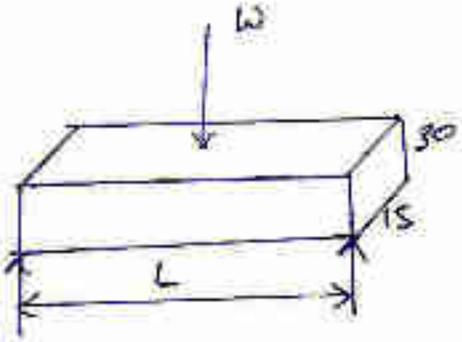
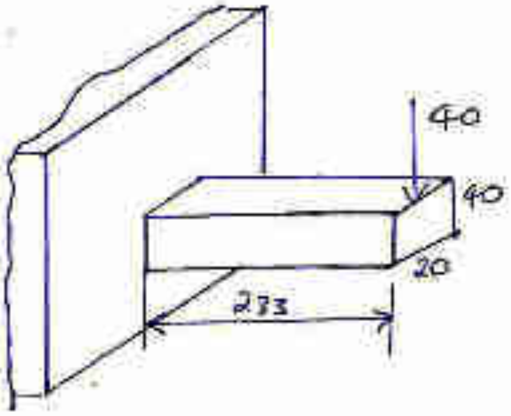
Q.No 6
2





$$\frac{\cancel{40}\times 233^3}{\cancel{3}E\times \frac{30^3\times 15}{\cancel{12}}} = \frac{\cancel{40}\times L^3}{\cancel{48}_{16}E\times \frac{40^3\times 20}{\cancel{12}}}$$

$$L = \underline{\underline{900 mm}}$$

Methods To be Used: (To get y/θ)

→ Simply Supported Beam → Macaulay's, M.O.S

→ Beam with at least 1 fixed support → MAM, MOS

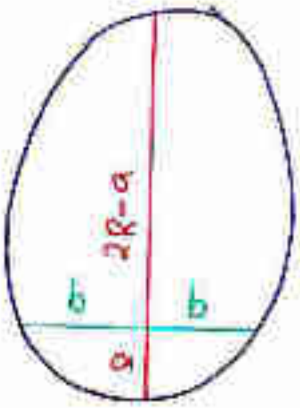
→ Frame → Castigliano's Theorem.

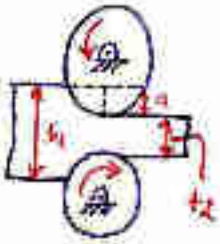
→ Length & 1/R (Curvature) is given → $y_{max} = \frac{L^2}{8R} = \frac{ML^2}{8EI}$

→

Design of Leaf Springs $\rightarrow (2R-a)a = b^2$

and also rolling process to determine draft.





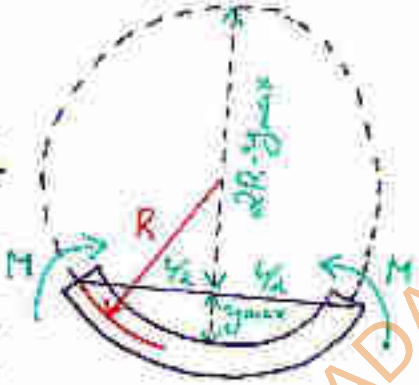
$$a(2R-a) = b^2$$

$$y_{max}(2R - y_{max}) = \frac{L^2}{4}$$

At $R \to \infty$ $y_{max} \approx 0$

$$\therefore y_{max} \times 2R = L^2/4$$

$$y_{max} = \frac{L^2}{8R} = \frac{ML^2}{8EI}$$ → Can be used one and only L and R given. Any beam can be used.



Beams carved, Almost horizonta beams curvature will be zero R will be infinity.

7 A simply supported reinforced concrete beam of L=10m Sags. Assume curvature to be 0.004 m⁻¹ along the span, max. deflection (in m) of the beam at mid span is.

— $y_{max} = \frac{L^2}{8R}$; L=10

$1/R = 0.004$

$$y_{max} = \frac{10^2 \times 0.004}{8}$$

$$y_{max} = \underline{\underline{0.05m}}$$

3.8 STATICALLY INDETERMINATE BEAMS:-

→ Propped Cantilever:-

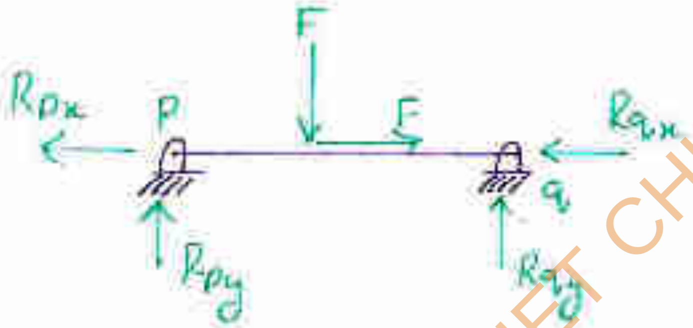
Degree of indeterminacy=1

Here there are 4 unknown and 3 equilibrium equation So that the problem be come Statically indeterminate. So Compatibility will be came.

Rpx
Rpy
RQ
Mp

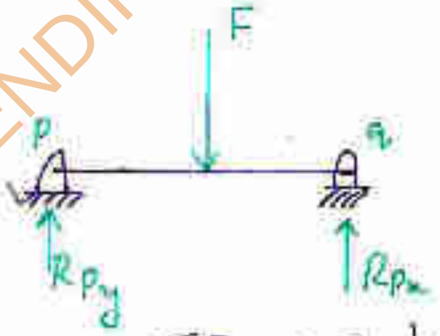
$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M = 0$

i.e. $y_Q = 0$ --> Compatibility Conditions.



$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M = 0$
} 4 unknowns, 3 equations

Statically indeterminate.

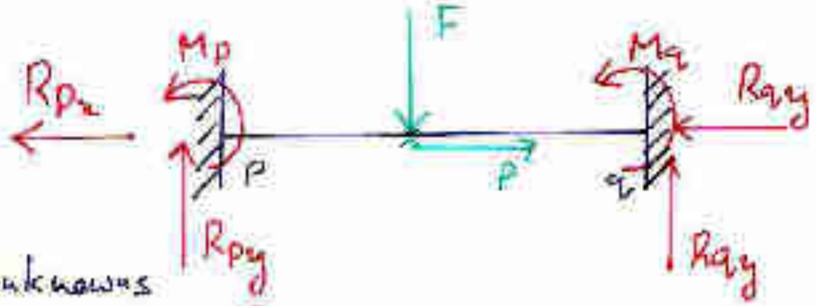


$\Sigma F_y = 0$
$\Sigma M = 0$
} 2 unknowns, 2 equation

Determinate

NOTE: Because of loading and support reactions depend upon the static indeterminacy.

→ FIXED BEAM:



Unknowns:

R_{Px} R_{Qx}
R_{Py} R_{Qy} } 6 Unknowns
M_P M_Q

(for both axial + transverse) → Degree of Indeterminacy = 3

$\Sigma F_x = 0$
$\Sigma F_y = 0$ } 3 static equations
$\Sigma M = 0$

3 More Equations needed :-

i.e $y_Q = 0$ $y_Q = 0$

$y_L = 0$ $\delta_L = 0$

$\theta_Q = 0$ $\theta_Q = 0$

→ If there are axial loading the problem will become:

Unknowns :-

R_{Py} M_P
R_{Qy} M_Q } 4 unknowns

(only transverse) → Degree of Indeterminacy = 2

$\Sigma F_y = 0$
$\Sigma M = 0$ } 2 static Equ.

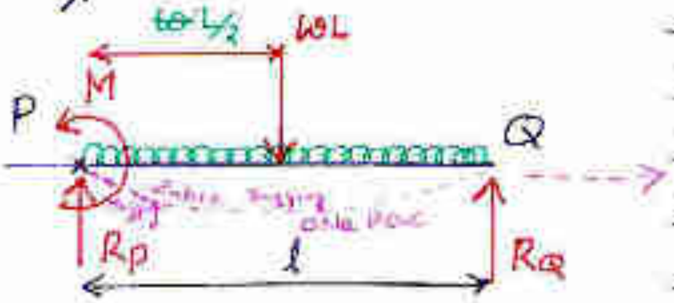
$y_Q = 0$: $\Sigma F_y = 0$

$\theta_Q = 0$: $\Sigma M = 0$

GATE:

? Determine the reactions?

NOTE - Should determine reactions for SF and BM diagram, Same way.



Indeterminate

Equilibrium:

$\Sigma F=0: \; R_p + R_q = \omega L$

$\Sigma M=0: \; M + R_Q \times l = \omega L \times L/2$

$M + R_Q \times l = \omega L^2/2$

Compatibility:

$y_Q=0: \; \frac{+\omega L^4}{8EI} = \frac{R_Q \times L^3}{3EI}$

From this

$\frac{R_Q \times L^{\cancel{3}}}{3 \times \cancel{EI}} = \frac{+\omega L^{\cancel{4}}}{8 \times \cancel{EI}}$

$R_Q = \frac{3\omega L}{8}$

$M + \frac{3\omega L}{8} \times L = \frac{\omega L^2}{2}$

$M = \frac{\omega L^2}{2} - \frac{3\omega L^2}{8}$

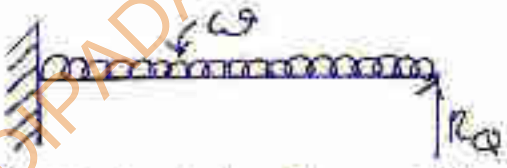
$M = \frac{\omega L^2}{8}$

$R_p + R_q = \omega L$

$R_p = \omega L - \frac{3\omega L}{8}$

$R_p = \frac{5}{8}\omega L$

Think the beam as a cantilever So that.



By Method of Superposition.

$y_Q = 0.$

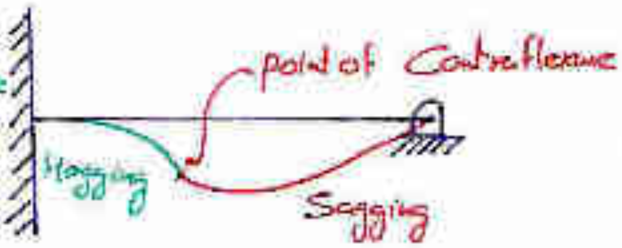
$\frac{-\omega L^4}{8EI} + \frac{R_Q \times L^3}{3EI} = 0$

$R_p = 5/8 \times \omega L$

$R_q = \frac{3\omega L}{8}$

$M = \frac{\omega L^2}{8}$

NOTE- For a proped Cantilever, there will be always a 1 point of contraflexure



At r, SF = 0

SF at R: $\frac{5}{8}\omega l - \omega x = 0.$

$\frac{5}{8}\omega l = \omega x$

$x = \frac{5}{8} l$

BM at P$_+$: $BM_p = M = \frac{-\omega L^2}{8}$

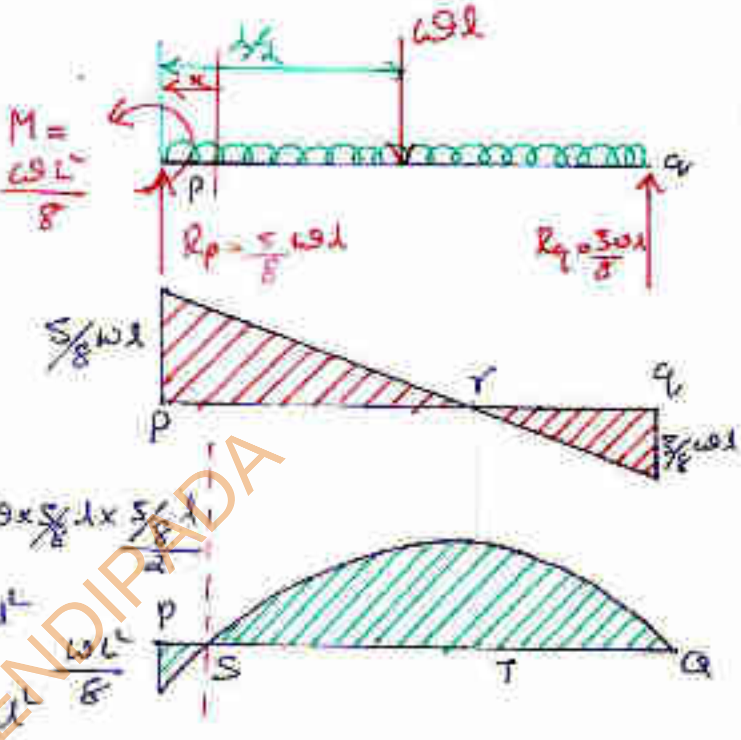
BM at T: $BM_T =$ i.e at $\frac{5}{8}L$

$$BM_T = \frac{-\omega L^2}{8} + \frac{5}{8}\omega l \times \frac{5}{8}l - \omega \times \frac{5}{8}l \times \frac{\frac{5}{8}l}{2}$$

$$BM_T = \frac{-\omega L^2}{8} + \frac{25}{64}\omega l^2 - \frac{25}{64 \times 2}\omega l^2$$

$$BM_T = \frac{25}{64}\omega l^2 - \frac{\omega l^2}{8} - \frac{25}{128}\omega l^2$$

$BM_T = \frac{9}{128}\omega L^2$ (max. BM)



Assume at distance S be point of inflection.

$$BM_x = \frac{5}{8}\omega L x - \frac{\omega L^2}{8} - \omega x.\frac{x}{2} = 0$$

$$\frac{5}{8}\omega L x - \frac{\omega L^2}{8} - \frac{\omega x^2}{2} = 0$$

$$\frac{5}{8}Lx - \frac{L^2}{8} - \frac{x^2}{2} = 0.$$

$$\frac{-x^2}{2} + \frac{5}{8}Lx - \frac{L^2}{8} = 0.$$

$$x^2 - \frac{5}{4}Lx + \frac{L^2}{4} = 0.$$

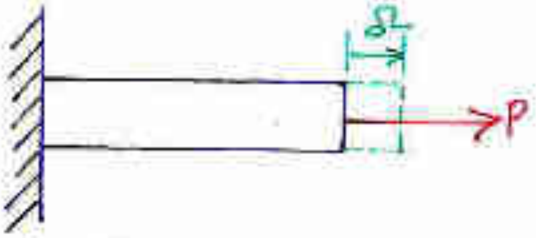
$$x = \frac{\left(\frac{5}{4} \mp \sqrt{\left(\frac{5}{4}\right)^2 - 4 \times 1 \times \frac{1}{4}}\right)L}{2 \times 1}$$

$$x = \frac{\left(\frac{5}{4} \pm 0.75\right)L}{2}$$

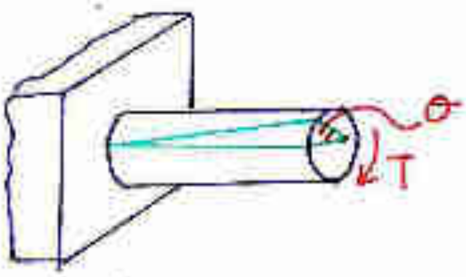
$x = \underline{\underline{0.25L}}$

4.6 ENERGY METHODS:

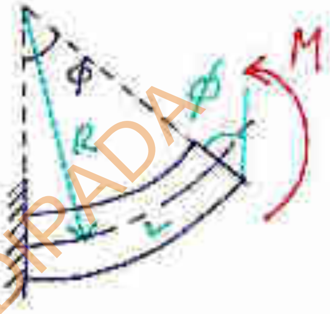
AXIAL: $\delta L = \frac{PL}{AE}$



TORSION: $\theta = \frac{TL}{GJ}$



BENDING: $\phi = \frac{ML}{EI}$



4.1 STRAIN ENERGY:-

AXIAL : $U = W = \frac{1}{2} \cdot P \times \delta L = \frac{P^2 L}{2EA}$ $\left[\delta L = \frac{PL}{AE}\right] = \int \frac{P^2 \cdot dx}{2EA}$

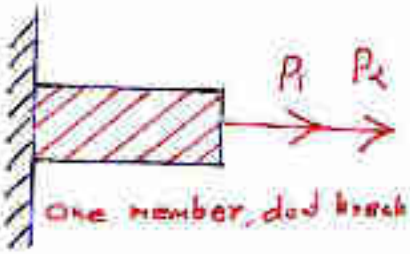
TORSION : $U = W = \frac{1}{2} \times T \times \theta = \frac{T^2 L}{2GJ}$ $\left[\theta = \frac{TL}{GJ}\right] = \int \frac{T^2 dx}{2GJ}$

BENDING : $U = W = \frac{1}{2} \times M \times \phi = \frac{M^2 L}{2EI}$ $\left[\phi = \frac{ML}{EI}\right] = \int \frac{M^2 dx}{2EI}$

SHEAR : $U = W = \frac{F^2 L}{2GA} = \int \frac{F^2 dx}{2EI}$

* $\longrightarrow$ $U_{Total} \neq \frac{P_1^2 L}{2EA} + \frac{P_2^2 L}{2EA}$

$U_{TOTAL} = \frac{(P_1 + P_2)^2 L}{2AE}$



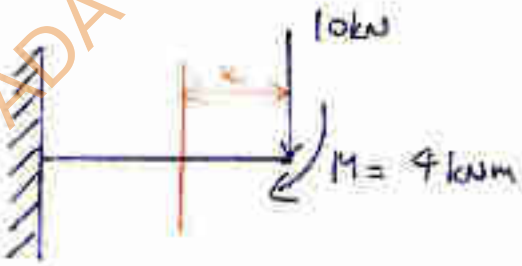
One member, don't break load

* $U_{Total} = (U)_{\substack{NET \\ AXIAL}} + (U)_{\substack{NET \\ TORSION}} + (U)_{\substack{NET \\ BENDING}} + (U)_{\substack{NET \\ SHEAR}}$ (can be neglected as compared to bending)

*

→ When Shear and Bending both are acting on beam Then energy due to shear by default be neglected in Comparison of energy due to bending. $U_{SF} <<<< U_{B.F}$ $\searrow 0$

? $EI = 3600\ kNm^2$



$M_x = (-4 - 10x)$

$U_{Total} = U_{Shear} + U_{Bending}$

$U_{total} = U_{Bending}$

$U_{total} = \int \frac{M^2 \cdot dx}{2 \times EI}$

$U_{total} = \int_0^2 \frac{(-4-10x)^2 dx}{2 \times 3600}$

$U_{total} = \frac{1}{7200} \int_0^2 +16 + 80x + 100x^2\, dx$

$U_{total} = \frac{1}{7200} \left[16x + \frac{80x^2}{2} + \frac{100x^3}{3}\right]_0^2$

$U_{total} = \underline{\underline{0.0637}}\ kNm = 63.7\ Nm$

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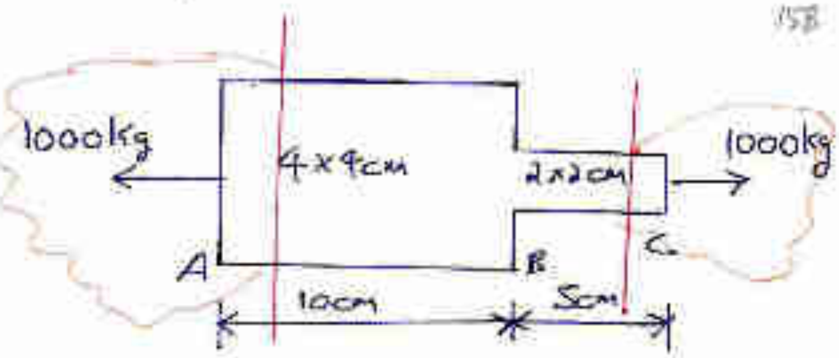
Q. No 1

? $E = 4.1 \times 10^6\ kg/cm^2$

$\nu = 0.3$; $P = 1000\,kg$



$U_{Total} = U_{AB} + U_{BC}$

$$U_{Total} = \frac{P^2 \times 10}{2 \times 4.1 \times 10^6 \times 4 \times 4} + \frac{P^2 \times 5}{2 \times 4.1 \times 10^6 \times 4 \times 4}$$

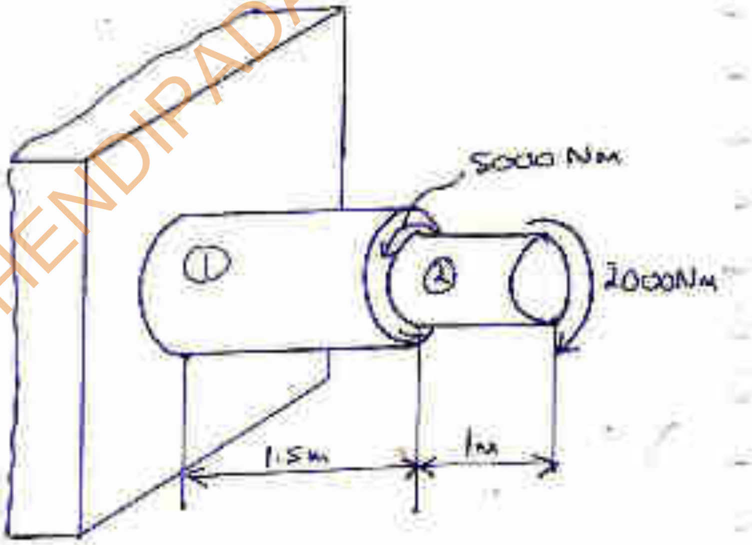
$$U_{Total} = P^2 \times \left(\frac{10}{2 \times 4.1 \times 10^6 \times 16} + \frac{5}{2 \times 4.1 \times 10^6 \times 4} \right)$$

$U_{Total} = \underline{\underline{0.228}}\ kg\,cm$

? $d_1 = 50mm$

$d_2 = 30mm$

$G = 77\,GPa$



$U = U_1 + U_2$

$$U_1 = \frac{T_1^2 L_1}{2G \cdot J_1} \qquad U_2 = \frac{T_2^2 \times L_2}{2G J_1}$$

~~$U_1 = \frac{5000^2 \times 1.5 \times 10^3 \times 10^{3\times2}}{2 \times 77 \times \frac{\pi}{32} \times 50^4}$~~

$T_1 = 3000$

~~$U_2 = (2000 \times 16)$~~

$$U_1 = \frac{(\overset{3}{\cancel{8}}000)^2 \times 1.5}{2 \times 77 \times 10^9 \times \frac{\pi}{\cancel{4}32} \times (50 \times 10^{-3})^4} = \overset{142}{\cancel{396}}\ Nm.$$

$$U_2 = \frac{(2000)^2 \times 1}{2 \times 77 \times 10^9 \times \frac{\pi}{32} \times (30 \times 10^{-3})^4} = 326\ Nm$$

$U = U_1 + U_2 = \underline{\underline{469.49\ Nm}}$

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Q.No 5

? $U = U_1 + U_2$.

$$U = \frac{T_1^2 L_1}{2 G J_1} + \frac{T_2^2 L_2}{2 G J}$$

$$U = \frac{(10\times10^3)^2\times100}{2\times80\times10^3\times\frac{\pi\times50^4}{32}} + \frac{(10\times10^3)^2\times100}{2\times80\times10^3\times\frac{\pi\times26^4}{32}}$$

$U = 0.101 + 1.39$

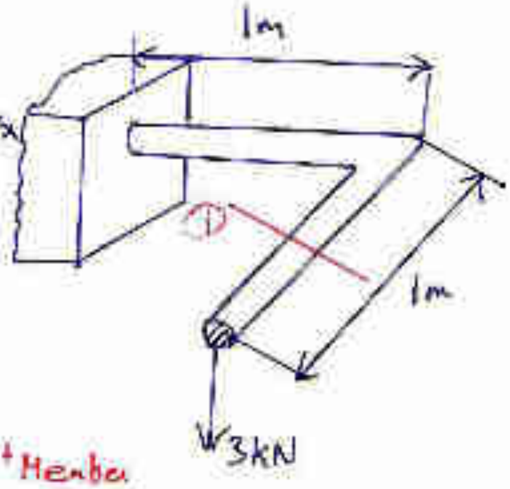
$U = \underline{\underline{1.49\ Nmm}}$

? IF the modulus of elaticity and poissons ratio of member loaded are 200 GPa and 0.3 respectievely, determine total strainenergy in Nm?

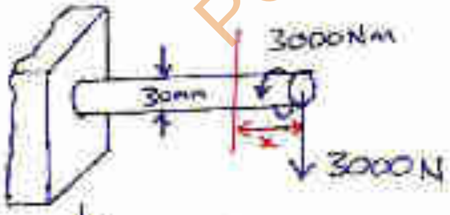
$U = U_1 + U_2$.

$$\cancel{U = \frac{(3\times10^3)^2 x\, dx}{2\times200\times10^9\times\frac{\pi\times20^4}{64}} + 3\times10^3 x}$$

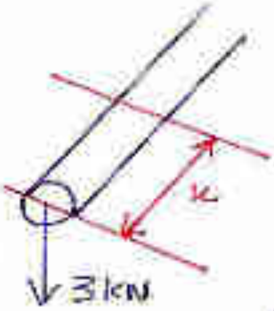
$E = 2G(1+\upsilon)$: $E = 200\times10^9\ Pa$.



2nd Member



1st Member



$$U_1 = \int_0^1 \frac{(3000x)^2 dx}{2EI}$$

$$U_2 = \int_0^1 \frac{(3000x)^2 dx}{2EI} + \frac{(3000)^2\times1}{2\times GJ}$$

$E = 2G(1+\upsilon)$

$200\times10^9 = 2\times G(1+0.3)$

$G = 7.69\times10^{10}\ Pa$

$\text{Total} = U_1 + U_2$

$$U_{Total} = 2\int_0^1 \frac{(3000x)^2 dx}{2EI} + \frac{(3000)^2\times1}{2GJ}$$

$$U_{total} = \frac{3000^2}{EI}\int_0^1 x^2 dx + \frac{3000^2}{2GJ}$$

$$U_{total} = \frac{9\times10^6}{EI}\times\frac{1}{3} + \frac{3000^2}{2GJ}$$

$$I = \frac{\pi d^4}{64}$$

$$I = \frac{\pi\times(30\times10^{-3})^4}{64} = 3.97\times10^{-8}\,m^4$$

$$EI = 200\times10^9\times3.97\times10^{-8} = 7952.156\,Nm^2$$

$$J = \frac{\pi d^4}{32}$$

$$J = \frac{\pi\times(30\times10^{-3})^4}{32} = 7.95\times10^{-8}\,m^4$$

6194.64

$GJ = 7.69\times10^9\times7.95\times10^{-8} =$ ~~611.5208×10~~ $=$ ~~$6115208\,Nm^2$~~.

$$U_{total} = \frac{9\times10^6}{3\times7952.156} + \frac{9\times10^6}{\cancel{6115.208}\ 6194.64\times2}$$

$U_{total} =$ ~~524.51 Nm~~ 377 + 726.43 = 1103.68 Nm

4.2 CASTIGLIANO'S THEOREM:

\# $\delta l = \frac{PL}{EA}$

$\delta A_{\rightarrow} = \frac{PL}{EA}$; displacement of A in horizontal direction.

$U = \frac{P^2 L}{2EA}$

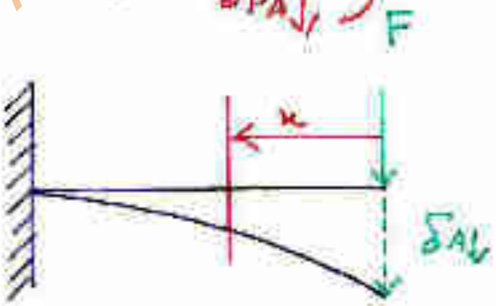
$\frac{\partial U}{\partial P_{A\rightarrow}} = \frac{PL}{EA}$

i.e. $\frac{\partial U}{\partial P_{A\rightarrow}} = \delta L_{A\rightarrow}$

$\frac{\partial U}{\partial P_{A\rightarrow}} = \delta_{A\rightarrow}$

$\theta_A = \frac{\partial U}{\partial M}$ --> Slope

$\left.\begin{array}{l}\delta_{A\rightarrow} = \frac{\partial U}{\partial P_{A\rightarrow}} \\ \delta_{A\downarrow} = \frac{\partial U}{\partial P_{A\downarrow}}\end{array}\right\}$ deflection



\# $M = -F \times x$

$U = \int_0^L \frac{(-Fx)^2 dx}{2EI}$

$U = \frac{F^2}{2EI}\left[\frac{x^3}{3}\right]_0^L$

$U = \frac{F^2}{2EI} \times \frac{L^3}{3}$

$U = \frac{F^2 L^3}{6EI}$

$\frac{\partial U}{\partial P_{A\downarrow}} = \frac{\partial U}{\partial F} = \frac{FL^3}{3EI}$

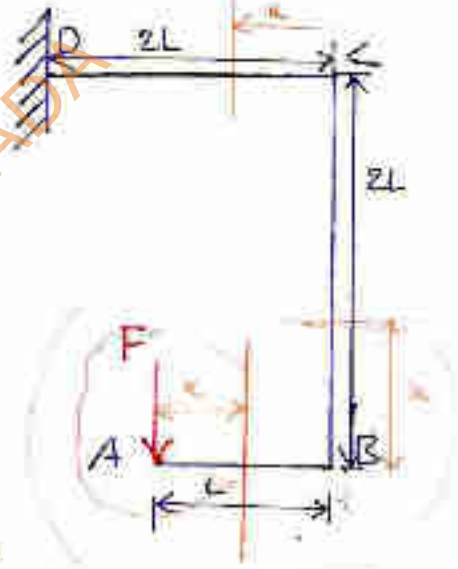
$\frac{\partial U}{\partial P_{A\downarrow}} = \frac{FL^3}{3EI}$

$\frac{\partial U}{\partial P_{A\downarrow}} = \delta_{A\downarrow}$

→ Statement:-

"According to Castigliano's theorem total strain energy of structure when partially differentiated with respect to load gives the displacement of the point where the load is acting in the direction of the load."

Q Determine the vertical displacement of point A of the frame loaded as shown in figure Q



Step 0: Introduce a dummy force if required.

Step 1: Determine BM for each segment

Consider AB: $M = Fx$ (Valid L) (No sign needed)

Consider BC: $M = F.L$ (Valid for 2L)

Consider CD: $M = \cancel{F.x}$ (Valid for 2L)

$M = F(L-x)$

Step 2: Express U_{total}

$$U_{total} = U_{AB} + U_{BC} + U_{CD}$$

$$U_{total} = \int_0^L \frac{(Fx)^2 dx}{2EI} + \frac{(F.L)^2 . 2L}{2EI} + \int_0^{2L} \frac{(F(L-x))^2 dx}{2EI}$$

$$U_{total} = \int_0^L \frac{(F^2 x) dx}{2EI} + \frac{(FL)^2 . 2L}{2EI} + \int_0^{2L} \frac{(F(L-x))^2 dx}{2EI}$$

} * Don't Integrate

Step 3: Apply Castigliano's theorem

$$\delta A{\downarrow} = \frac{\delta U_{total}}{\delta P_{A\downarrow}} \xrightarrow{} F \quad = \frac{\delta U_{total}}{\delta F}$$

$$\text{i.e. } \delta_{A\downarrow} = \int_0^L \frac{\cancel{2}F . x^2 dx}{\cancel{2}EI} + \frac{2F L^2 . 2L}{\cancel{2} EI} + \int_0^{2L} \frac{\cancel{2}F (L-x)^2 dx}{\cancel{2}EI}$$

$$\delta_{A\downarrow} = \int_0^L \frac{F . x^2 dx}{EI} + \frac{F L^2 . 2}{EI} + \int_0^{2L} \frac{F(L-x)^2 dx}{EI}$$

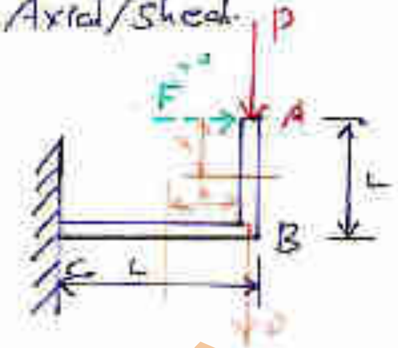
$$\delta_{A\downarrow} = \frac{F}{EI} \times \frac{L^3}{3} + \frac{2FL^3}{EI} + \int_0^{2L} \frac{F(L^2 - 2Lx + x^2)}{EI} dx$$

$$\delta_{A\downarrow} = \frac{FL^3}{3EI} + \frac{2FL^3}{EI} + \frac{F}{EI}\left[L^2x - Lx^2 + \frac{x^3}{3}\right]_0^{2L}$$

$$\delta_{A\downarrow} = \frac{FL^3}{3EI} + \frac{2FL^3}{EI} + \frac{F}{EI}\left[2L^3 - 4L^3 + \frac{8L^3}{3}\right]$$

$$\delta_{A\downarrow} = \frac{3FL^3}{EI}$$

? Determine horizontal displacement at point A for the beam loaded as shown in figure? Neglect Axial/shear.



Introduce the dummy force :-

AB: $M = Fx$ C valid at L).

BC: $M = (F \times L + Px)$ C valid at L

$$U_{total} = \int_0^L \frac{(Fx)^2}{2EI}dx + \int_0^L \frac{(FL+Px)^2}{2EI}dx.$$

~~$$U_{total} = \int_0^L \frac{F^2x^2dx}{2EI} + \int_0^L \frac{F^2L^2 + Px(xFL) + P^2x^2}{2EI}dx$$~~

~~$$\frac{\partial U}{\partial F_{A\rightarrow}} = \int_0^L \frac{2F\overset{0}{x^2}dx}{2EI} + \int_0^L \frac{2F\overset{0}{L^2} + 2PxFL + P^2x^2}{2EI}dx$$~~

~~Apply $F = 0$~~

~~$$\frac{\partial u}{\partial F_{A\rightarrow}} = 0 + \int_0^L \frac{P^2x^2}{2EI}dx.$$~~

~~$$\frac{\partial u}{\partial F_{A\rightarrow x}} = 0 + \frac{P^2}{2EI}\frac{L^3}{3} = \frac{P^2L^3}{6EI}$$~~

$$U_{total} = \int_0^L \frac{(Fx)^2}{2EI}dx + \int_0^L \frac{(FL+Px)^2dx}{2EI}$$

Apply Castigliano's theorem

$$\frac{\partial u}{\partial A_{\rightarrow}} = \int_0^L \frac{\cancel{2}\overset{0}{F}x^2}{\cancel{2}EI}dx + \int_0^L \frac{\cancel{2}(\overset{0}{FL}+Px)^2Ldx}{\cancel{2}EI}$$

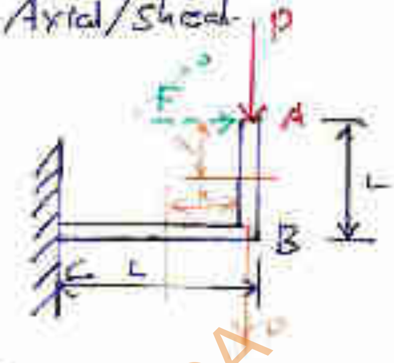
$$\frac{\partial u}{\partial A_{\rightarrow}} = \int_0^L \frac{PxLdx}{EI} = \frac{PL^3}{2EI}$$

$$\delta_{A\downarrow} = \frac{FL^3}{3EI} + \frac{2FL^3}{EI} + \frac{F}{EI}\left[L^2x - Lx^2 + \frac{x^3}{3}\right]_0^{2L}$$

$$\delta_{A\downarrow} = \frac{FL^3}{3EI} + \frac{2FL^3}{EI} + \frac{F}{EI}\left[2L^3 - 4L^3 + \frac{8L^3}{3}\right]$$

$$\delta_{A\downarrow} = \frac{3FL^3}{EI}$$

? Determine horizontal displacement at point A for the beam loaded as shown in figure? Neglect Axial/shear.



Introduce the dummy force :-

AB: $M = Fx$ (valid at L).

BC: $M = (F \times L + Px)$ (Valid at L)

$$U_{total} = \int_0^L \frac{(Fx)^2}{2EI}dx + \int_0^L \frac{(FL+Px)^2}{2EI}dx.$$

~~$U_{total} = \int_0^L \frac{F^2x^2dx}{2EI} + \int_0^L \frac{F^2L^2 + Px \times FL \times 2 + P^2x^2}{2EI}dx$~~

~~$\frac{\partial u}{\partial F_{A\rightarrow}} = \int_0^L \frac{2F \cdot x^2 dx}{2EI} + \int_0^L \frac{2FL^2 + 2Px \cdot FL + P^2x^2}{2EI}dx$~~

~~Apply $F = 0$~~

~~$\frac{\partial u}{\partial F_{A\rightarrow}} = 0 + \int_0^L \frac{P^2x^2}{2EI}dx.$~~

~~$\frac{\partial u}{\partial F_{A\rightarrow}} = 0 + \frac{P^2}{2EI}\frac{xL^3}{3} = \frac{P^2L^3}{6EI}$~~

$$U_{total} = \int_0^L \frac{(Fx)^2 dx}{2EI} + \int_0^L \frac{(FL+Px)^2 dx}{2EI}$$

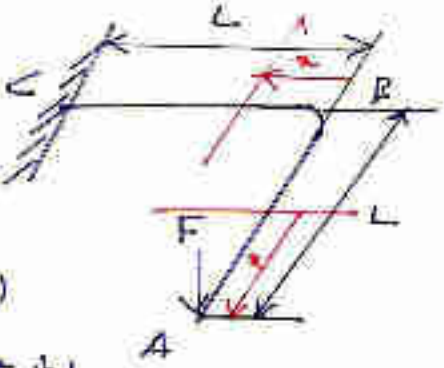
Apply Castigliano's theorem

$$\frac{\partial u}{\partial A\rightarrow} = \int_0^L \frac{2\overset{\rightarrow 0}{F}x^2}{2EI}dx + \int_0^L \frac{2(\overset{0}{F}L+Px)L\,dx}{2EI}$$

$$\frac{\partial u}{\partial A\rightarrow} = \int_0^L \frac{PxL\,dx}{EI} = \frac{PL^3}{2EI}$$

11/8/15

* ? Same problem; by Castigliano Method:



$AB;\ M = Fx$ (valid at L)

$BC;\ \overset{F}{M} = (FL + Fx)$ (valid at L)

$\lfloor T = FL;\ M = Fx$ (valid for L)

$U_{Total} = U_{AB} + U_{BC}$

$$U_{Total} = \int_0^L \frac{(Fx)^2 dx}{2EI} + \frac{(FL)^2 \cdot L}{2GJ} + \int_0^L \frac{(Fx)^2 dx}{2EI}$$

$$U_{Total} = 2\int_0^L \frac{(Fx)^2 dx}{2EI} + \frac{(FL)^2 L}{2GJ}$$

$$U_{Total} = \int_0^L \frac{(Fx)^2 dx}{EI} + \frac{F^2 L^3}{2GJ}.$$

$$U_{Total} = \int_0^L \frac{(Fx)^2 dx}{EI} + \frac{F^2 L^3}{2G \times 2I}.$$

GATE+ES →
Frame - deflection
Use castigliano's theorem

Apply Castigliano's theorem.

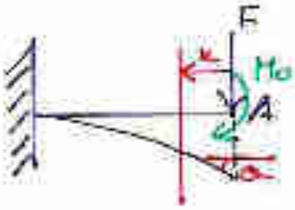
$$\frac{\partial U}{\partial F\downarrow} = \delta_{A\downarrow r} = \int_0^L \frac{2Fx^2 dx}{EI} + \frac{\cancel{2}FL^3}{2G \times 2I}$$

$$\delta_{A\downarrow r} = \int_0^L \frac{2F.x^2 dx}{EI} + \frac{FL^3}{2GI}$$

$$\delta_{A\downarrow r} = \frac{F}{2I}\left[\frac{2L^3}{E3} + \frac{L^3}{2G}\right]$$

$$\delta_{A\downarrow} = \frac{F}{I}\left[\frac{2L^3}{3.E} + \frac{2L^3}{4G}\right]$$

$$\delta_{A\downarrow} = \frac{2FL^3}{I}\left[\frac{1}{3E} + \frac{1}{4G}\right]$$



→ ? Determine θ at A.

Step 1: Introduce a dummy moment of value Mo = 0

There is shear Force and BM at F. Neglect shear Force.

Step 2: $M_{net} = -M_0 - Fx$

Step 3: $U = \int_0^L \frac{M^2 dx}{2EI}$

$$= U = \int_0^L \frac{(-M_0+Fx)^2}{2EI}\,dx$$

$$\Longrightarrow U = \int_0^L \frac{(M_0+Fx)^2}{2EI}\,dx.$$

Step 4: Apply Castiglianos theorem : $\theta_A = \frac{\partial U}{\partial M_A}$

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial}{\partial M_0}\int_0^L \frac{(M_0+Fx)^2\,dx}{2EI}.$$

$$\theta_A = \int_0^L \frac{\cancel{2}(M_0+Fx)\,1}{\cancel{2}EI}.dx.$$

Apply $M_0 = 0$

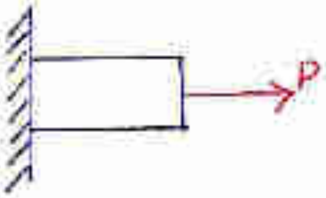
$$\theta_A = \int_0^L \frac{(\cancel{M_0}^{\,0}+Fx)}{EI}\,dx$$

$$\theta_A = \frac{F}{EI}\int_0^L x\,dx.$$

$$\theta_A = \frac{F}{E\cdot I}\left[\frac{x^2}{2}\right]_0^L$$

$$\underline{\underline{\theta_A = \frac{FL^2}{2EI}}}$$

4.4 RESILIENCE & TOUGHNESS:



$$U = \frac{P^2 \cdot LA}{2E\, A \times A}$$

$$U = \frac{P^2 \cdot LA^2}{2E A^2}$$

LA – Volume.

$$U = \frac{P^2 \cdot \forall}{2E A^2} \qquad \forall \rightarrow \text{Volume}$$

$$\frac{U}{\forall} = \frac{P^2}{2E A^2}$$

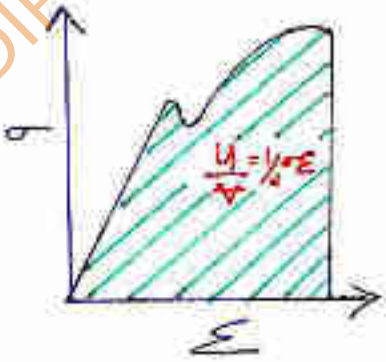
$$\frac{U}{\forall} = \frac{P}{2} \times \frac{P}{E A^2} \left\{ \frac{P}{A} = \sigma \right\}.$$

$$\frac{U}{\forall} = \frac{\sigma}{2} \times \frac{\sigma}{E \cdot A} \left\{ \frac{\sigma}{\varepsilon} = E \right\}$$

$$\text{i.e. } \frac{U}{\forall} = \frac{1}{2} \cdot \sigma \cdot \varepsilon$$

Strain Energy/Volume = $\frac{1}{2} \times \sigma \times \varepsilon$

$$\frac{U}{\forall} = \frac{1}{2} \times \sigma \times \varepsilon = \text{A}_{\sigma-\varepsilon} \text{ curve}$$



Fixed for one material upto specified limit of stress

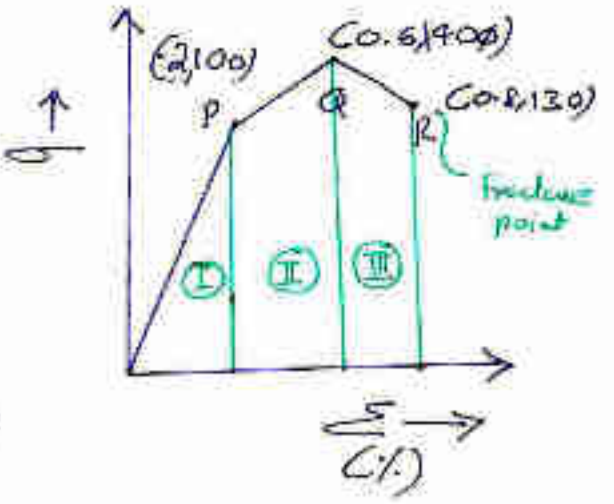
→ We found that energy absorbed by unit volume of material equals area under stress-strain curve. Since Area under stress-strain curve when determined upto some specified limit of σ, is a fixed value for the given material, we conclude that it must represent a material property.

This material is. Energy/volume (U/∀) is known as -

Resilience when determined upto proportionality limit and Toughness when determined upto "Fracture point"

GATE-16.

? P – proportionality limit.
Q – Ultimate tensile strength
R – Fracture point.
Determine toughness?



Area = $A_1 + A_2 + A_3$ = Toughness

Toughness = $A_1 + A_2 + A_3$

Toughess $= \left(\frac{1}{2} \times 0.2 \times 100\right) + \left(\frac{100+140}{2} \times (0.6-0.2)\right) + \left(\frac{140+130}{2}\ (0.8-0.6)\right)$

Toughness $= \left[\left(\frac{1}{2} \times 0.2 \times 100\right) + (120 \times 0.4) + (135 \times 0.2)\right] / 100.$

Toughness $= \underline{\underline{0.85 \text{ MJ}}}$

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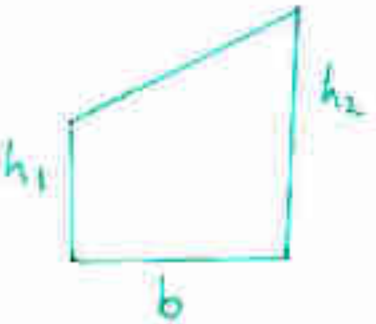
? Resilience $= \left(\frac{U}{V}\right)_{\text{proportionality}} = \frac{1}{2} \times 0.004 \times 70 = 014\ \text{N/mm}.$

$\frac{U}{V} = 14 \times 10^4\ \text{Nm/m}^3$

$\left(\frac{U}{V}\right)_{\text{Fracture}}$ = Toughess = $14\times10^4 + \left(\frac{120+110}{2} \times (0.012-0.004)\right)\times10^4$

$= 14\times10^4 + 96\times10^4 = 90\times10^4\ \text{Nm/m}.$



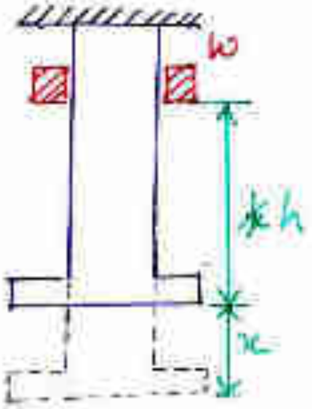
Area $= \left(\frac{h_1+h_2}{2}\right) \times b$

4.4. IMPACT LOADING:

x → Elongation

W → weight

h → Initial height



During impact loading it extends the collar by a distance.

Change in potential Energy $\Delta PE = W(h+x)$

Change in Strain Energy $\Delta SE = \frac{\sigma^2}{2E} \times AL$

$$\Delta PE = \Delta SE$$

$$W(h+x) = \frac{\sigma^2}{2E} \times AL$$

$$\frac{2E}{AL} W(h+x) = \frac{\cancel{2E}}{\cancel{AL}} \times \frac{\sigma^2}{\cancel{2E}} \times \cancel{AL}$$

$$\frac{2E}{AL} Wh + \frac{2E}{AL} \cdot Wx = \sigma^2$$

$$\frac{2E}{AL} \times Wh + \frac{2\cancel{E}}{A\cancel{L}} \cdot W \times \frac{\sigma \cancel{L}}{\cancel{E}} = \sigma^2$$

$$\frac{2WhE}{2AL} + \frac{2W}{A} \cdot \sigma = \sigma^2$$

$$\sigma^2 - \left(\frac{2W}{A}\right)\sigma - \frac{2WhE}{AL} = 0$$

$$\sigma = \frac{\frac{2W}{A} + \sqrt{\frac{\cancel{4}W^2}{A^2} + \cancel{4} \times 1 \times \frac{2WhE}{AL}}}{\cancel{2} \times 1}$$

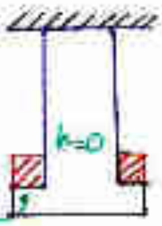
$$\sigma = \frac{W}{A} + \sqrt{\frac{W^2}{A^2} + \frac{2WhE}{AL}}$$

→ Impact with Zero momentum is known sudden loading

$$\boxed{\begin{array}{l} \text{Impact load} : \frac{W}{A} + \sqrt{\frac{W^2}{A^2} + \frac{2WhE}{AL}} \\ \quad \vee \\ \text{Sudden load} : 2W/A \\ \quad \vee \\ \text{Gradual load} - W/A \end{array}}$$

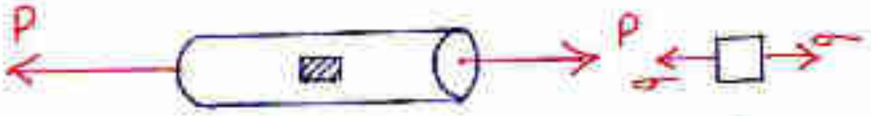
Impact : i.e $\sigma = \frac{W}{A} + \sqrt{\frac{W^2}{A^2} + \frac{2WhE}{AL}}$; $\geq W/A$

For Sudden load ; $\sigma = \frac{2W}{A}$ [i.e. $h=0$]



5. COMBINED LOADING:

Axial:



$\sigma = \frac{P}{A}$

Torsional:

$\tau = \frac{T}{J} \times r$

Bending:

$\sigma = \frac{M}{I} \times y$

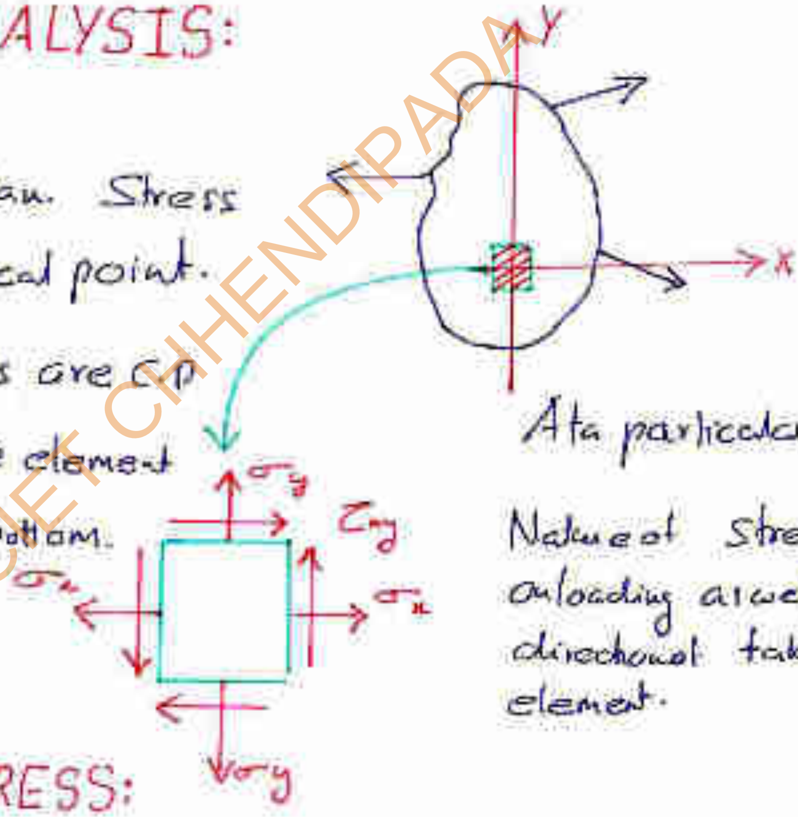
5.1 STRESS ANALYSIS:

The point of Max. Stress is called critical point.

Axial - All points are CP

Torsional - Surface element

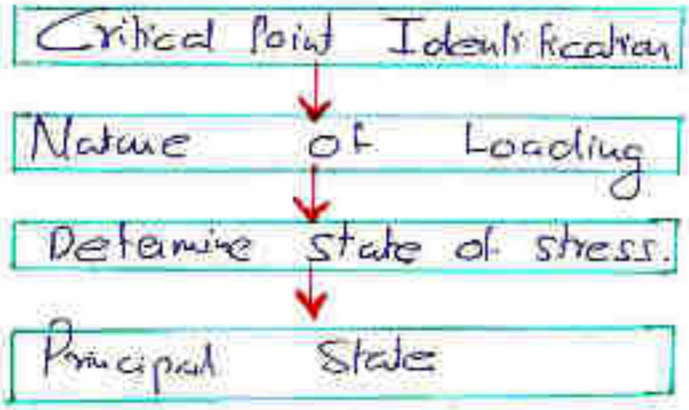
Bending - Top or bottom.



At a particular point

Nature of stress depends on loading as well as the direction of taking the element.

a) STATE OF STRESS:



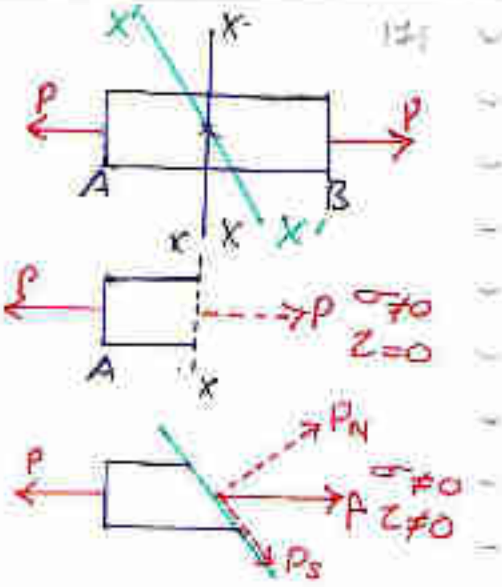
Critical Point Identification

Nature of Loading

Determine state of stress.

Principal State

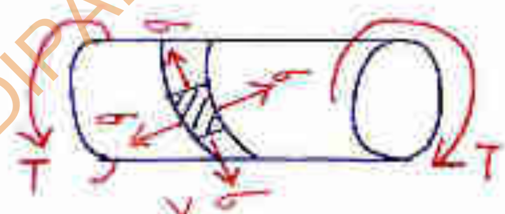
→ Consider a force P applied a material. Sections are being selected X-X and X-X'. At the section X-X only Normal stress occur. and shear will be zero. At X-X' because the section is known inclined-plane. They have 2 components of force and will have a shear stress.



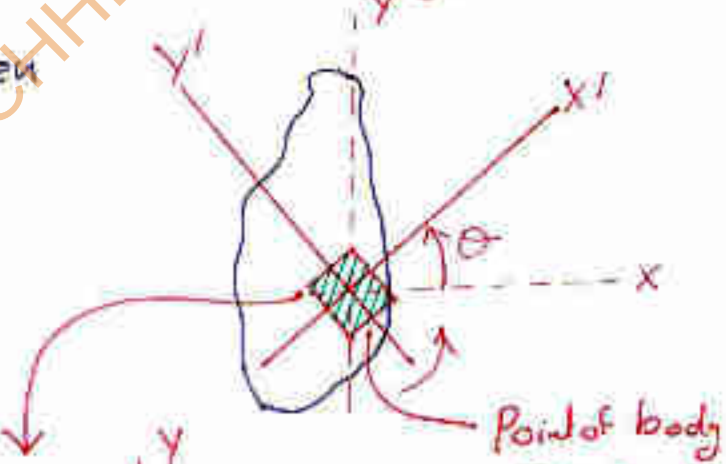
→ Similarly, at an angle of shear there will stress ⊥. to each other. There will be σ also present at other angle.

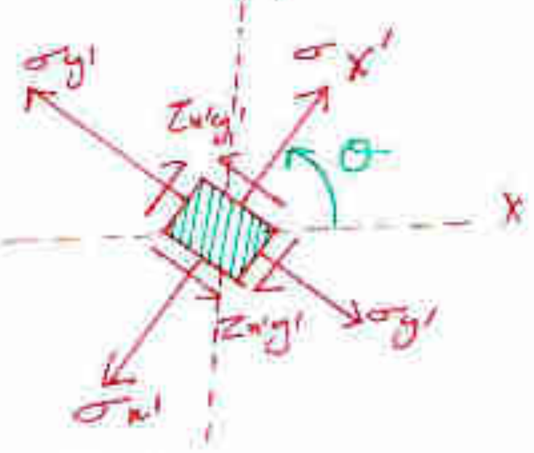
$\sigma \neq 0 ; \tau \neq 0$

Not a pure shear.



Another axis is taken





Transformed Axis

b) TRANSFORMATION EQUATION:

$\sigma_{x'}$ and $\sigma_{y'}$ are the new stresses in X'-Y' direction. They are being twisted by an angle θ.

$$\sigma_{x'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \text{ -----(i)}$$

$$\sigma_{y'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) +^{-} \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \text{ -----(ii)}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right).\sin 2\theta + \tau_{xy}\cos 2\theta. \text{ -----(iii)}$$

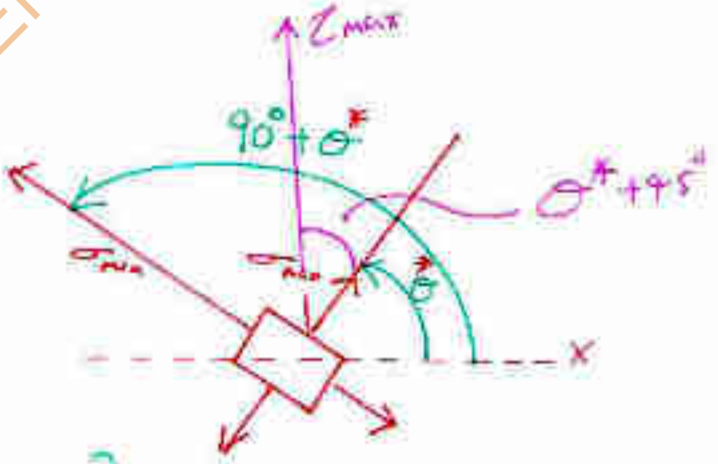
→ At some orientation σ will become maximum. That orientation will be principal plane and maximum value will become principal stress.

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \Leftarrow \text{(i)} + \text{(ii)}$$ (Analogy $I_{x'} + I_{y'} = I_x + I_y$)

→ "1st Stress invariance."
(Never change)

$$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2 = \sigma_x\sigma_y - \tau_{xy}^2$$

→ "2nd Stress Invariance"



Principal plane orientation.

c) PRINCIPAL STRESSES:

$$\frac{d\sigma_{x'}}{d\theta}=0$$

$$\frac{d}{d\theta}\left(\frac{\sigma_x+\sigma_y}{2}+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta+\tau_{xy}\sin 2\theta\right)=0.$$

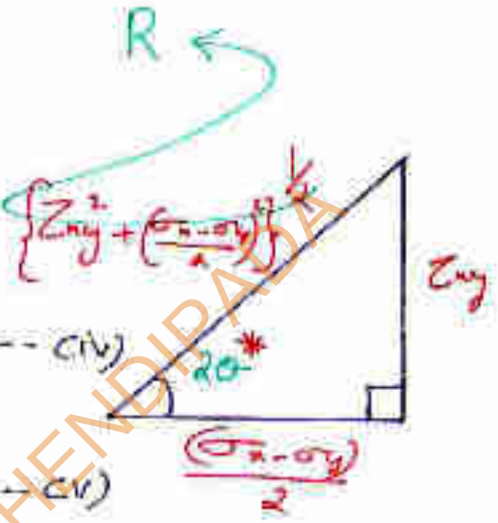
$$\frac{\sigma_x-\sigma_y}{2}-\sin 2\theta.\cancel{2}+\cancel{2}\tau_{xy}\cos 2\theta=0$$

$\theta_1=\theta^*$

$\theta_2=\theta^*+90$

$$\frac{\sin 2\theta}{\cos 2\theta}=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

$$\tan 2\theta^*=\frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$$



$$\therefore \sin 2\theta=\frac{\tau_{xy}}{R}\text{ ----(iv)}$$

$$\cos 2\theta=\frac{\frac{\sigma_x-\sigma_y}{2}}{R}\text{ ----(v)}$$

Sub. (iv) & (v) in $\sigma_{x'}$ →

$$\sigma_{x'}=\frac{\sigma_x+\sigma_y}{2}+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta+\tau_{xy}\sin 2\theta$$

$$\sigma_{x'}=\frac{\sigma_x+\sigma_y}{2}+\left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{\sigma_x-\sigma_y}{R}+\tau_{xy}\cdot\frac{\tau_{xy}}{R}.$$

$$\sigma_{x'}=\frac{\sigma_x+\sigma_y}{2}+\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}{R}$$ $\left\{\text{Since } R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\right\}$

i.e. $$\sigma_1=\frac{\sigma_x+\sigma_y}{2}+\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\sigma_1=\sigma_{avg}+R$$

$$\sigma_1+\sigma_2=\frac{\sigma_x+\sigma_y}{2}\cdot 2$$

$$\sigma_1+\sigma_2=2\sigma_{avg}$$

$$\sigma_{avg}+R+\sigma_2=2\sigma_{avg}$$

$$\sigma_2 = \sigma_{avg} - R$$

Sub (iv) & (v) in (iii) ⟶

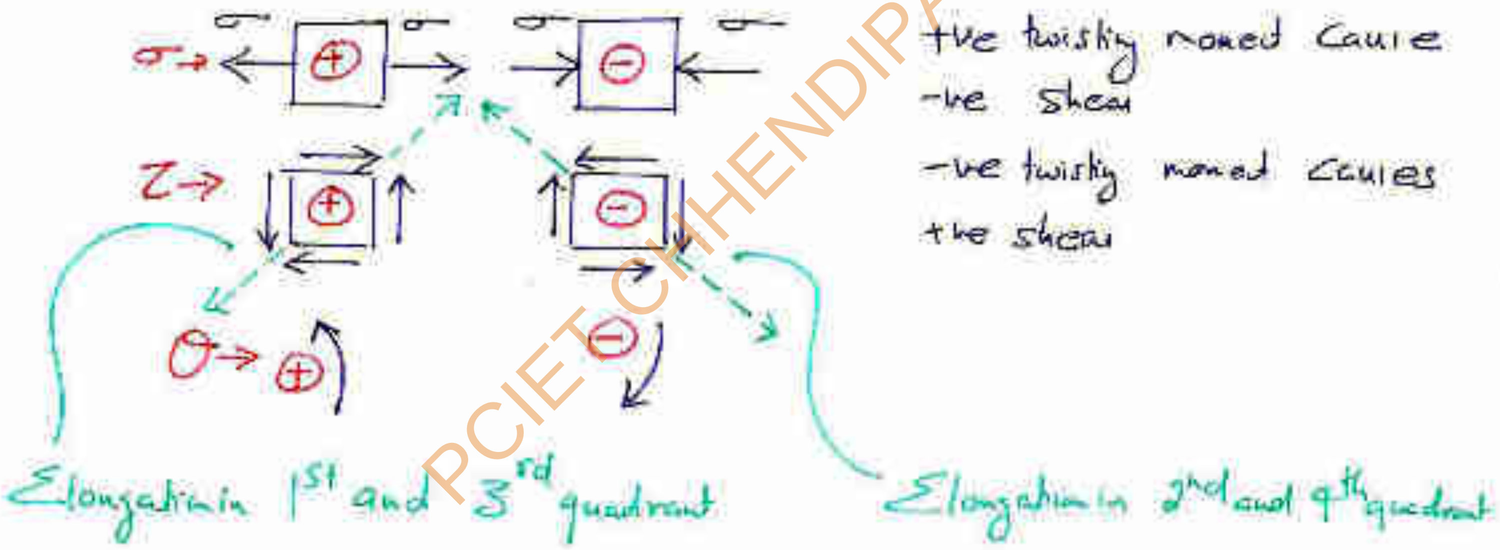
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{d}\right)\frac{\tau_{xy}}{R} + \tau_{xy}\cdot\left(\frac{\sigma_x - \sigma_y}{d}\right)$$

$$\tau_{x'y'} = 0$$

NOTE: Principal plane have no shear.

NOTE: To determine plane at which σ_{max} occurs put θ value on the $\sigma_{x'}$ and $\sigma_{y'}$

CONVENTION:



+ve twisting moment cause -ve shear

-ve twisting moment causes +ve shear

To determine the angle direction.

→ Draw the x-axis

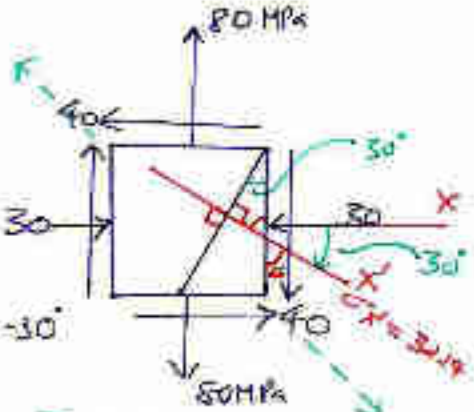
→ Draw ⊥ to the plane.

→ Angle orientation with x-axis.

→ 9 The State of a stress at critical point of loaded body is shown in figure. Determine the normal and shear stress on the plane represented at 30°. Also determine principal stresses and represent them on corresponding principal plane.

$\sigma_x = -30 MPa ; \sigma_y = 80 MPa ; \tau_{xy} = -40 MPa$

$\theta = -30°$.



To determine direction of angle draw +ve x-axis and draw ⊥ to the plane

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} = \frac{-30+80}{2} + \left(\frac{-30-80}{2}\right)\cos \times 2 \times -30° + \tau_{xy}\sin 2 \times -30°$$

$$\sigma_{x'} = 25 + \overset{5}{25} \times \cos -60° + 40 \sin -60°$$

$$\sigma_{x'} = \underline{\underline{32.14 MPa}}.$$

$$\tau_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_y = -\left(\frac{-30-80}{2}\right)\sin(2\times-30) + -40\cos(2\times-30)$$

$$\tau_{xy} = -\left(\frac{-110}{2}\right)\sin(-60) - 40\cos(-60)$$

$$\tau_{xy} = -\underline{\underline{67.63 MPa}}$$

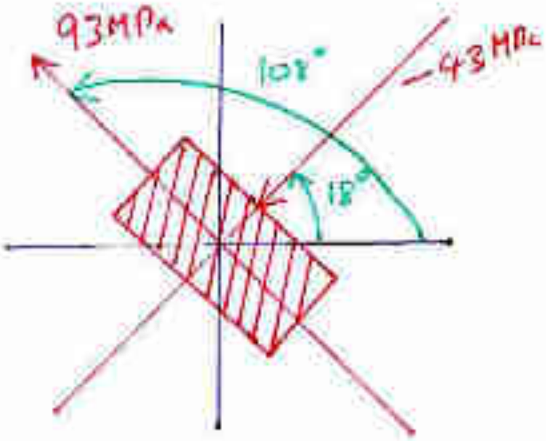
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma' V = \text{Tan}\,2\theta^* = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\text{Tan}\,2\theta^* = \frac{-40}{(-30+-80)/2}$$

$$\theta_1^* = 18.01°$$

$$\theta_2^* = \theta_1^* + 90 = 108.01°$$



$$\sigma_1 = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{-30-80}{2}\right)^2 + 40^2}$$

$$R = \underline{\underline{68.007}}\ \text{MPa.}$$

$$\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2} = \frac{-30+80}{2} = 25\cancel{8}\ \text{MPa.}$$

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_1 = 25\cancel{8} + 68.007 = 93\ \text{MPa}$$

$$\sigma_2 = \sigma_{avg} - R$$

$$\sigma_2 = 25\cancel{8} - 68.007 = -43\ \text{MPa}$$

Important: To determine where σ_1 and σ_2 occur put value of $\underline{\theta}$ in $\underline{\sigma_{x'}\ \text{eq}}$

$$\sigma_{x'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

Check $\theta_p^* = 18^\circ$

$$\sigma_{x'} = \left(\frac{-30+80}{2}\right) + \left(\frac{-30-80}{2}\right)\cos 2\times 18^\circ + -40\sin 2\times 18^\circ$$

$$\sigma_{x'} = \underline{\underline{-43.007}}\ \text{MPa.}$$

So at 18° $\sigma_{x'}$ will -43.007 MPa

and at 108° σ will become 93 MPa

d) Max. Shear And Its Plane:-

$$\frac{d\tau_{x'y'}}{d\theta} = 0$$

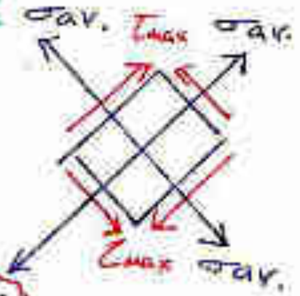
$$-\frac{(\sigma_x-\sigma_y)}{2}\,\overset{\cos}{\sin 2\theta}.4 + \tau_{xy} - \sin 2\theta . 2 = 0$$

$$-\left(\frac{\sigma_x-\sigma_y}{2}\right).\overset{\cos}{\sin 2\theta}. + \tau_{xy} - \sin 2\theta = 0.$$

$$-\tau_{xy}.\sin 2\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right).\cos 2\theta$$

$$\tan 2\theta = -\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\tau_{xy}}$$

$$\tan 2\theta^{\circledast} = -\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\tau_{xy}} \dashrightarrow \text{Max. Shear}$$



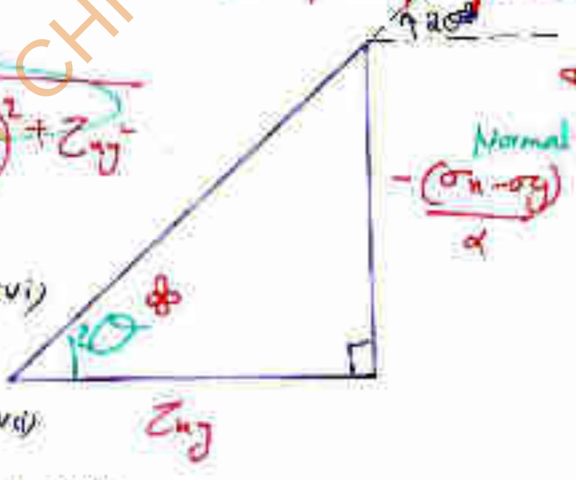
$$\tan 2\theta^{\circledast}.\tan 2\theta^{*} = -1$$

$$\theta^{\circledast} = \theta^{*} + 45^\circ$$

σ_{av}

NOTE:- Max shear plane occurs 45° to principal plane at θ^*

$$\xrightarrow{} R \quad \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$\sigma_{av} = \frac{\sigma_x+\sigma_y}{2}$$

Normal stress on the shear plane (Max)

$$-\frac{(\sigma_x-\sigma_y)}{2}$$

$$\sin 2\theta^{\circledast} = \frac{-\frac{(\sigma_x-\sigma_y)}{2}}{R} \quad \text{---(vi)}$$

$$\cos 2\theta^{\circledast} = \frac{\tau_{xy}}{R} \quad \text{---(vii)}$$

Sub (vi) and (vii) and (iii) ---→

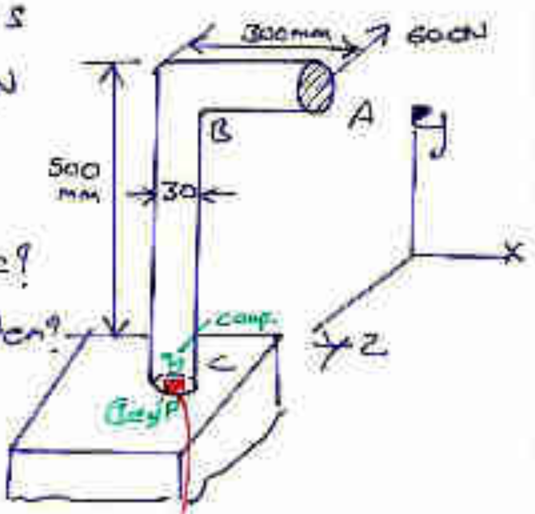
$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right) - \frac{(\sigma_x-\sigma_y)}{2.R} + \tau_{xy}.\frac{\tau_{xy}}{R}$$

$$\tau_{x'y'} = \frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2 \rightarrow R^2}{R}$$

$$\tau_{x'y'} = \sqrt{R}\ R$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = R = \frac{\sigma_1-\sigma_2}{R}$$

9. An L-shaped frame of dia. 30mm is fixed at the bottom then a force of 600N is applied in Z direction.



i) Identify the critical point on the frame?

ii) Determine the state of stress in XY system?

iii) Calculate major/minor principal stresses along with their orientations?

i) Critical point at C (BC member) (600×500 → BM at P tension q - comp)

600N transverse loading. $\xrightarrow{\text{1st step}}$ Select Critical Element first

Important

ii) $\sigma_y = \frac{M\,\Sigma\,y}{I}$

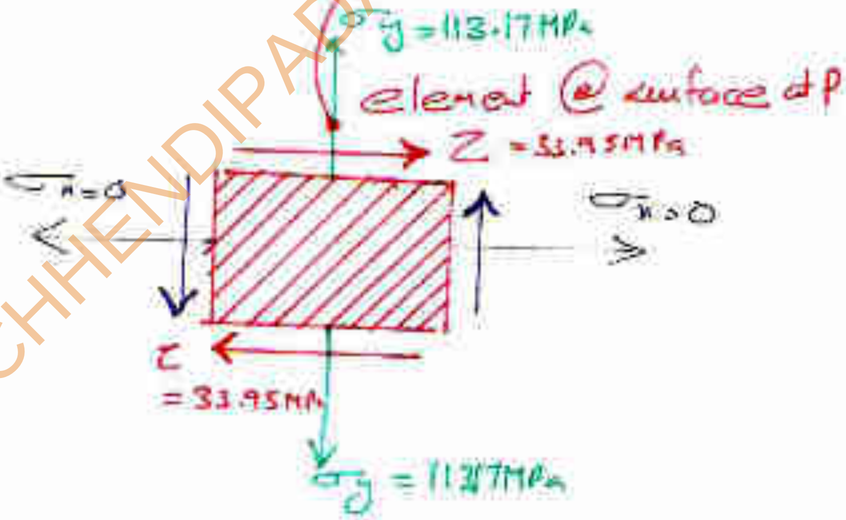
$\sigma_y = \frac{M}{I} \cdot y_{max}$

$\sigma_y = \frac{600\times500}{\frac{\pi\times30^4}{64}} \times \frac{30}{2}$

$\sigma_y = \underline{113.17\ MPa}$

$\tau = \frac{16\times T}{\pi d^3}$

$\tau = \frac{16\times600\times300}{\pi\times30^3} = 33.95\ MPa$



iii) $\tan 2\theta^* = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$

$\sigma_{avg} = \frac{113.17+0}{2} = 56.58\ MPa.$

$R = \sqrt{\left(\frac{0-113.17}{2}\right)^2 + 32.95^2} = 65.98\ MPa.$

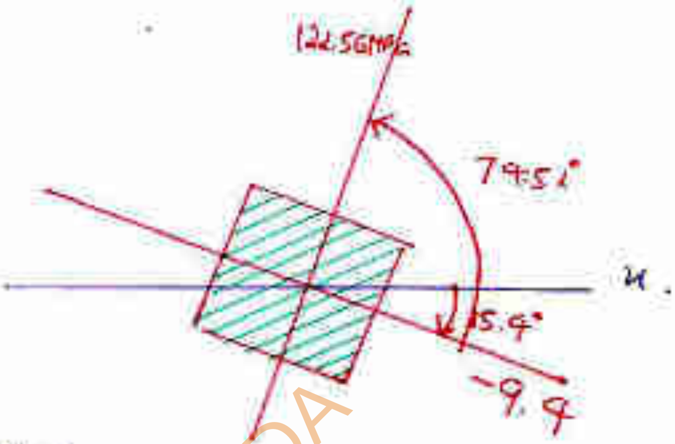
$\sigma_1 = \sigma_{avg} + R = 122.56\ MPa.$

$\sigma_2 = \sigma_{avg} - R = -9.4\ MPa$

$$\tan 2\theta^* = \frac{\tau_{xy}}{\left(\frac{\sigma_x-\sigma_y}{2}\right)}$$

$$\tan 2\theta^* = \frac{33.99}{\left(\frac{0-113.96}{2}\right)}$$

$$\underline{\underline{\theta^* = -15.9^\circ}}$$



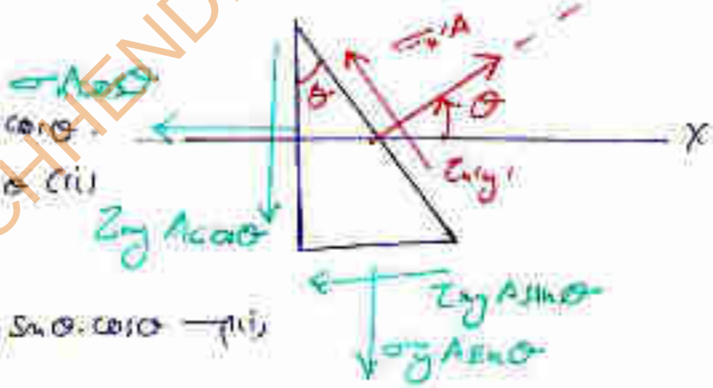
Equilibrium.

$$\Sigma F_{x'} = 0$$

$$\sigma_{x'} A \cos\theta = \sigma_x A\cos\theta + \tau_{xy} A\sin\theta$$

$$-\tau_{x'y'} A\sin\theta + \sigma_{x'} \cos\theta = \sigma_x \cos\theta + \tau_{xy} \sin\theta \quad \text{—(i)}$$

Force Diagram:



$$\Sigma F_y = 0$$

$$\sigma_{x'} A\sin\theta + \tau_{x'y'} A\cos\theta = \sigma_y A\sin\theta + \tau_{xy} A\cos\theta.$$

$$\sigma_{x'} \sin\theta + \tau_{x'y'} \cos\theta = \sigma_y \sin\theta + \tau_{xy} \cos\theta \quad \text{(ii)}$$

(i) × $\cos\theta$ ⇒.

$$\sigma_{x'} \cos^2\theta - \tau_{x'y'} \sin\theta.\cos\theta = \sigma_x \cos^2\theta + \tau_{xy} \sin\theta.\cos\theta \quad \text{—(iii)}$$

(ii) × $\sin\theta$ ⇒

$$\sigma_{x'} \sin^2\theta + \tau_{x'y'} \cos\theta.\sin\theta = \sigma_y \sin^2\theta + \tau_{xy} \cos\theta.\sin\theta \quad \text{(iv)}$$

(iii) + (iv) →.

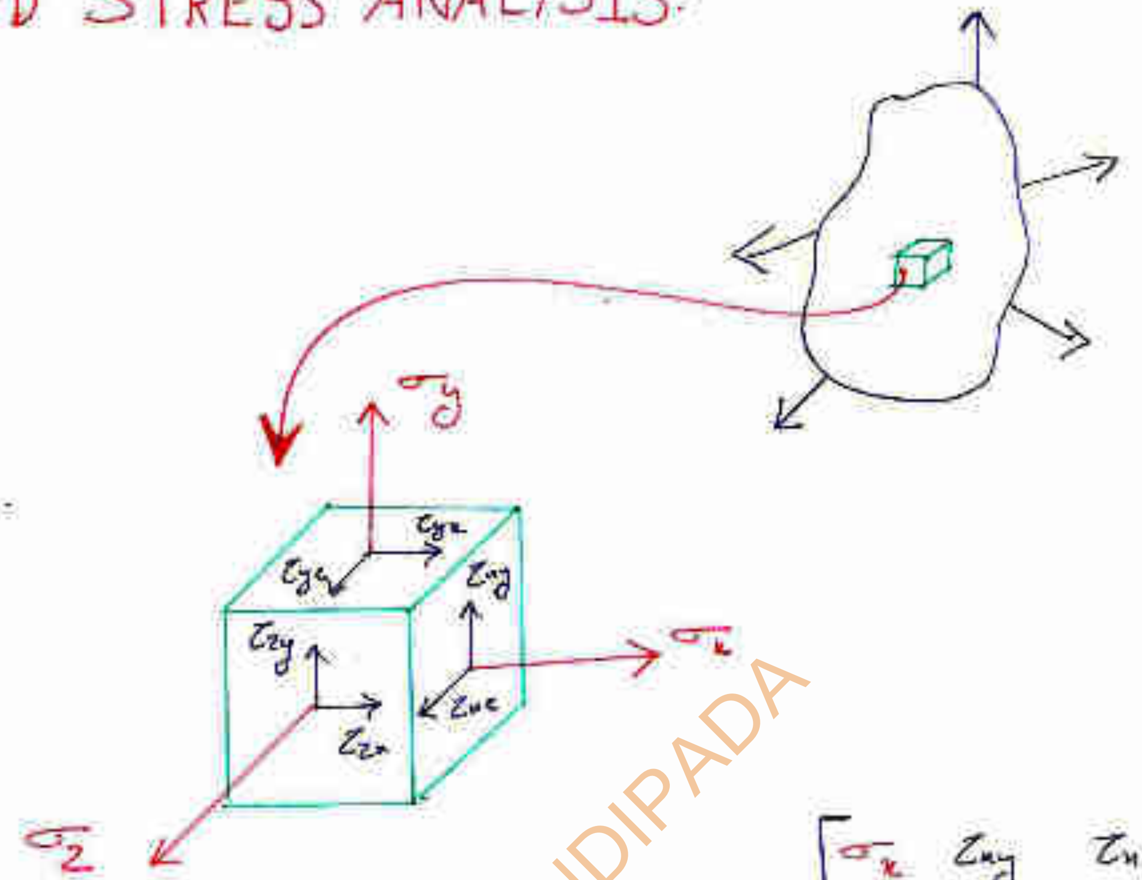
$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$\sigma_{x'} = \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy} \sin 2\theta$$

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

26/8/2017

5.2. 3-D STRESS ANALYSIS:



$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Stress is a 2nd order Tensor quantity. we need 9 components to describe 2nd order tensor quantity. But the stress is an example of the quantity but 6 quantities are needed.

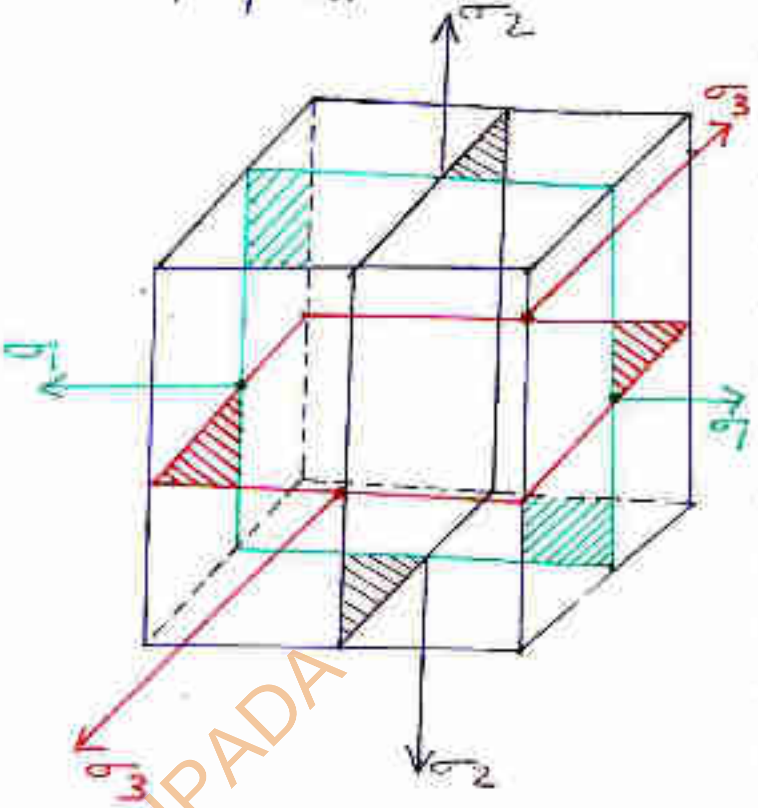
$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

First stress invariance $I_1 = \sigma_x + \sigma_y + \sigma_z$

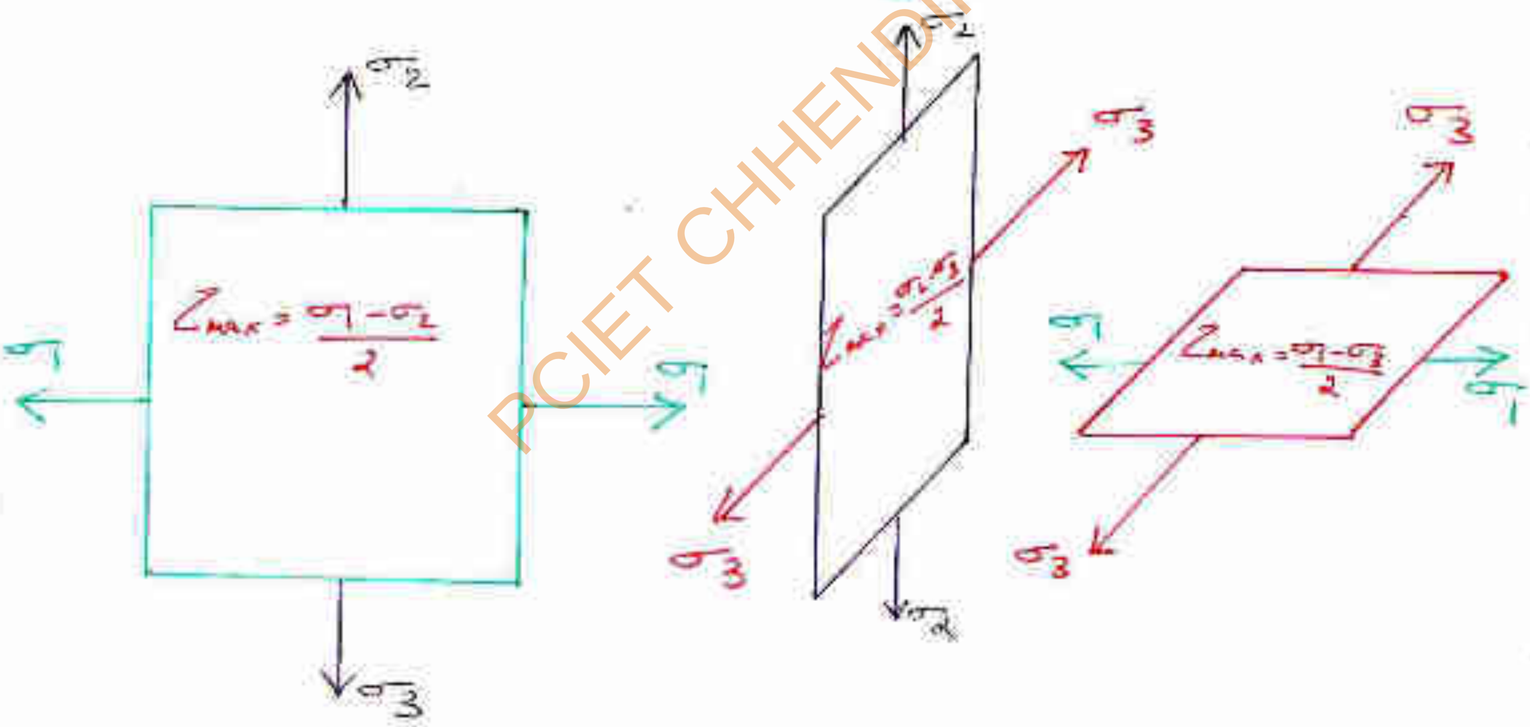
Second stress invariance $I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2$

Third stress invariance
$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix}$$

A 3-D problem canbe divided into 3 - 2-D problem i.e XYZ as XY plane, YZ plane and XZ plane.



- → If $\sigma_1, \sigma_2, \sigma_3$ are given use τ_{max} eqn. derived.
- → If σ_1, σ_2 are given put $\sigma_3 = 0$ in τ_{max} equation.
- → If describing inplane max. shear $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$



$$\tau_{max} = \text{Max}\left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|\right]$$

In plane Max shear, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

9. A Shaft is Subjected to an axial tensile load of 10kN and a twisting moment of 2400~~0~~Nm. The shaft diameter is 40mm. Determine

i) The state of the stress at a point on surface of shaft.

ii) Mayor and minor principal stresses.

iii) Maximum shear stress.

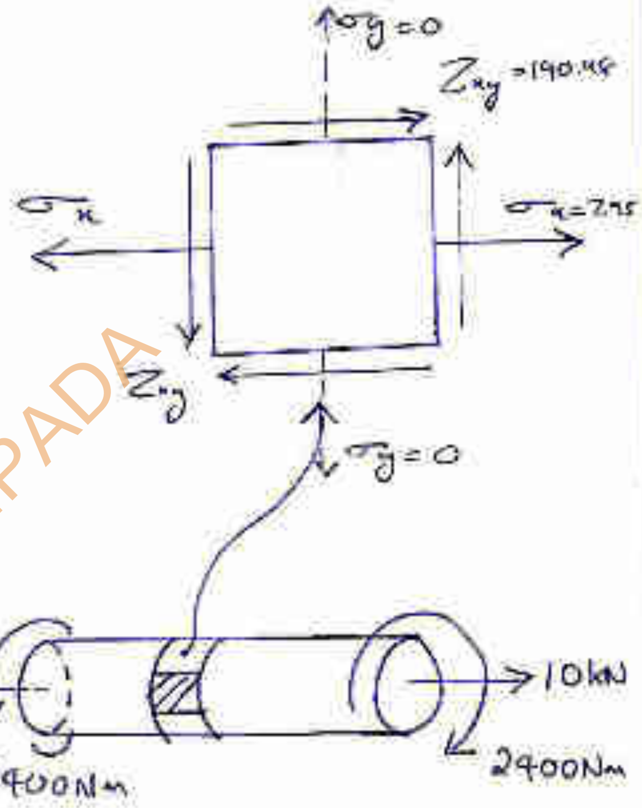
---

i) $\sigma_a = \frac{F}{A} = \frac{10\times1000}{\pi/4 \times 40^2}$

$\sigma_a = 7.95$ MPa

$\tau = \frac{16T}{\pi D^3} = \frac{16\times 2400\cancel{0}\times10^3}{\pi\times 40^3}$

$\tau = 190.98$ MPa



ii) $\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2} = \frac{7.95}{2} = 3.975$ MPa

$R = \left[\left(\frac{7.95-0}{2}\right)^2 + (190.98)^2\right]^{1/2} = 191.02$ MPa,

$\sigma_1 = \sigma_{avg}+R = 3.975+191.02 = 194.99$ MPa

$\sigma_2 = \sigma_{avg}-R = 3.975-191.02 = -187.045$ MPa

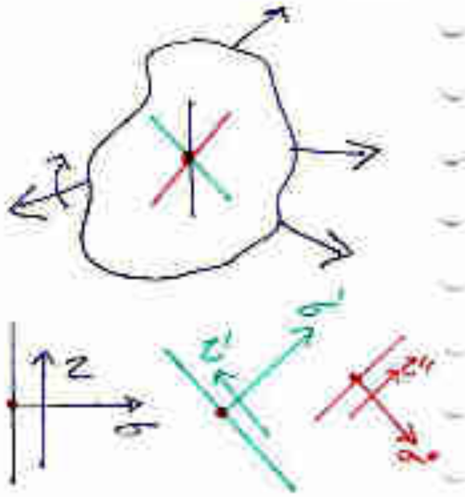
+ve twisting moment – –ve shear

–ve twisting moment – +ve shear

iii) $\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = 191.075$ MPa ; $\sigma_3=0$

$\tau_{max} = \underset{max}{} \left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_1-\sigma_3}{2}\right|, \left|\frac{\sigma_2-\sigma_3}{2}\right|\right]$ (check Always)

5.3 MOHR'S CIRCLE :-



[One body at one point, different planes, different stresses developed]

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Assume $\frac{\sigma_x+\sigma_y}{2}$ as σ_{avg}.

$\frac{\sigma_x-\sigma_y}{2}$ as $\bar{\sigma}$

$\sigma_{x'}$ be σ_θ

$$\sigma_\theta = \sigma_{avg} + \bar{\sigma}_{xy}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_\theta - \sigma_{avg} = \bar{\sigma}.\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_\theta = -\bar{\sigma}\sin 2\theta + \tau\cos 2\theta$$

$$\sigma_\theta - \sigma_a = \bar{\sigma}.\cos 2\theta + \tau\sin 2\theta \quad \text{—(i)}$$

$$\tau_\theta = -\bar{\sigma}.\sin 2\theta + \tau\cos 2\theta \quad \text{—(ii)}$$

Squaring and adding both →.

$$(\sigma_\theta - \sigma_a)^2 + (\tau_\theta)^2 = (\bar{\sigma})^2 + (\tau)^2.$$

i.e. $(\sigma_\theta - \sigma_a)^2 + (\tau_\theta)^2 = \overbrace{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}^{R}$

$$(\sigma_\theta - \sigma_a)^2 + (\tau_\theta)^2 = R^2$$

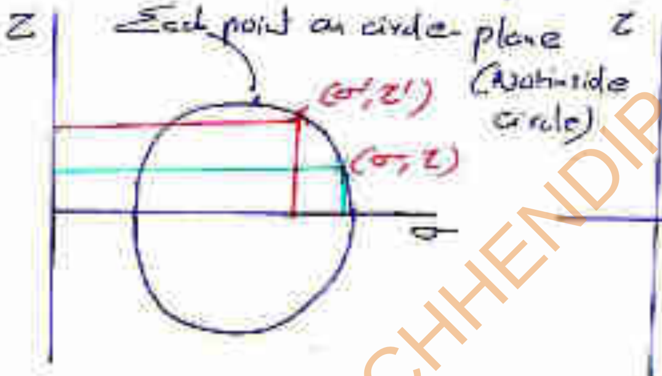
i.e. $(\sigma_\theta - \sigma_a)^2 + (\tau_\theta)^2 = R^2$ → Equ. of Circle.

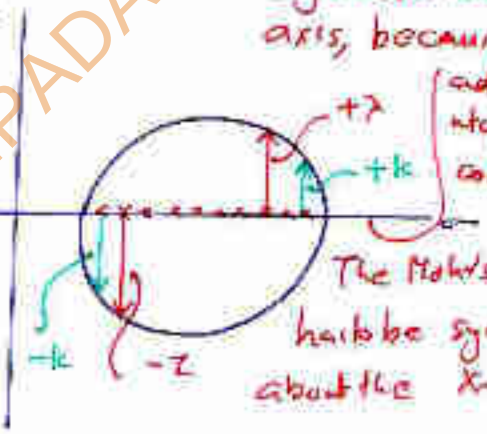
Prop. 1: σ, τ values of different orientations of an element, when plotted on a cartesian two dimensional plane, result into a circle known as Mohr's Circle. (Mohr's Stress Circle).

Prop. 2: A point on Mohr's Circle corresponds to a plane of certain orientation at element level. (one point - a plane);
X- coordinate represents the Normal stress of corresponding plane
y- coordinate represents the shear stress of corresponding plane.

Prop. 3: Centre = $(\sigma_{avg}, 0)$

"Radius = $\tau_{max} = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ "





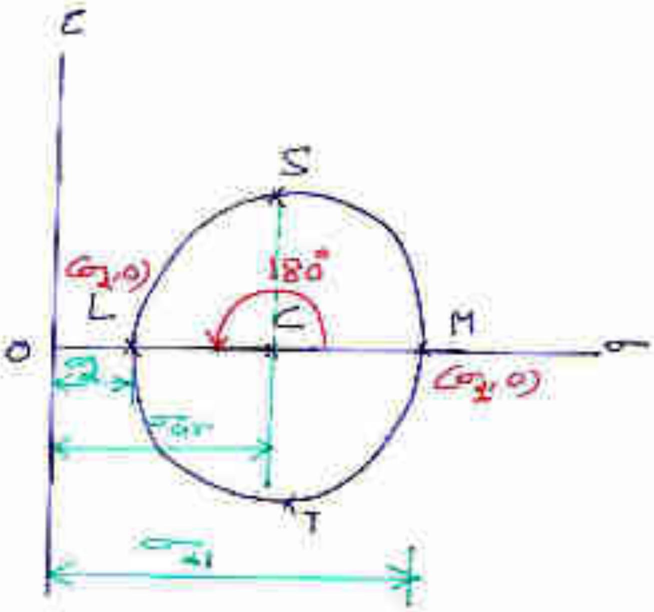
Symmetric about X-axis, because shear and complimentary shear are equal.

The Mohr's Circle has to be symmetric about the X-axis

Prop. 4: $\sigma_1 = OM = OC + CM = \sigma_{avg} + R$

$\sigma_2 = OL = OC - CL = \sigma_{avg} - R$

Prop 5: If the angle between 2 plane is θ, then the angle between corresponding on Mohr's circle will be 2θ.



Prop 6: Max. Shear plane at S and T.

Max Shear value CS

$CS = R = \tau_{max} = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ "

Normal Stress on plane of Max Shear = $\underline{\sigma_{avg}}$. "

5.4 GENERALISED HOOKES LAW:-

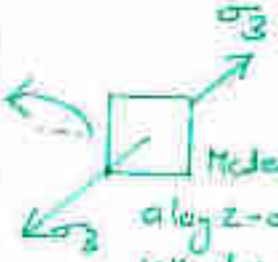
Stress applied on the 3D element. Material flows in x direction, decrease of material will occur in the y and z-direction.

Along X-direction.

Along the X-direction. : $\frac{+\sigma_1}{E}$ $-\frac{\nu\sigma_2}{E}$ $-\frac{\nu\sigma_3}{E}$

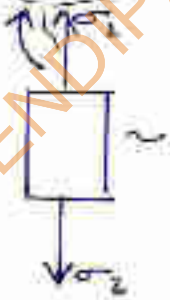
Along the Y-direction. : $-\frac{\nu\sigma_1}{E}$ $+\frac{\sigma_2}{E}$ $-\frac{\nu\sigma_3}{E}$

Along the Z-direction : $-\frac{\nu\sigma_1}{E}$ $-\frac{\nu\sigma_2}{E}$ $\frac{+\sigma_3}{E}$



Material along z-decrease in y and x. ($\frac{+\sigma_3}{E}$ $-\nu\sigma_3/E$)

Along the Y-direction, material will increase along y and decrease in x and z ($\frac{+\sigma_2}{E}$, $-\frac{\nu\sigma_2}{E}$, $\frac{\nu\sigma_2}{E}$)



~ Along Y-direction

Superimposing X: $\varepsilon_1 = \frac{+\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$

Y: $\varepsilon_2 = \frac{+\sigma_2}{E} - \frac{\nu\sigma_3}{E} - \frac{\nu\sigma_1}{E}$

Z: $\varepsilon_3 = \frac{+\sigma_3}{E} - \frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E}$

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{(\sigma_1+\sigma_2+\sigma_3)(1-2\nu)}{E}$$

Page No 22 Q.No 1

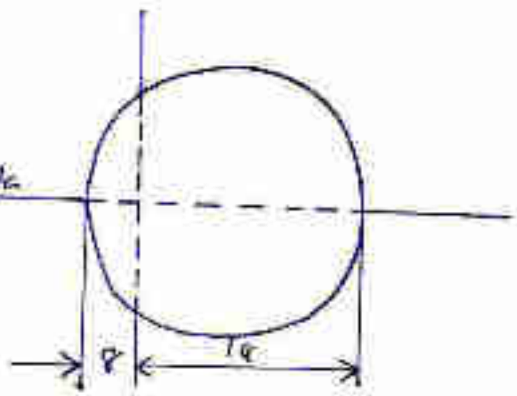
? Maximum principal stress = 18 MPa.

Minimum principal stress = −8 MPa

Max. Shear Stress = $R = \frac{18 - -8}{2} = 13\,MPa$

Normal stress on Max. shear plane

$= \frac{18 + -8}{2} = 5\,MPa$



Page No 23 Q.No 2

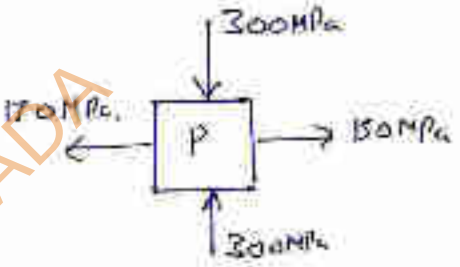
? $\sigma_1 - \sigma_2 = 2R$

$\sigma_2 = 10$; $R = 20$

$\sigma_1 = 50\,MPa$.

Page No 24 Q.No 3

? $E = 200\,GPa$; $\mu = 0.3$



$\sigma_3 = ?$ $\sigma_1 = 150$; $\sigma_2 = -300$

$\varepsilon_3 = \frac{\sigma_3}{E} - \frac{\upsilon\sigma_2}{E} - \frac{\upsilon\sigma_1}{E}$

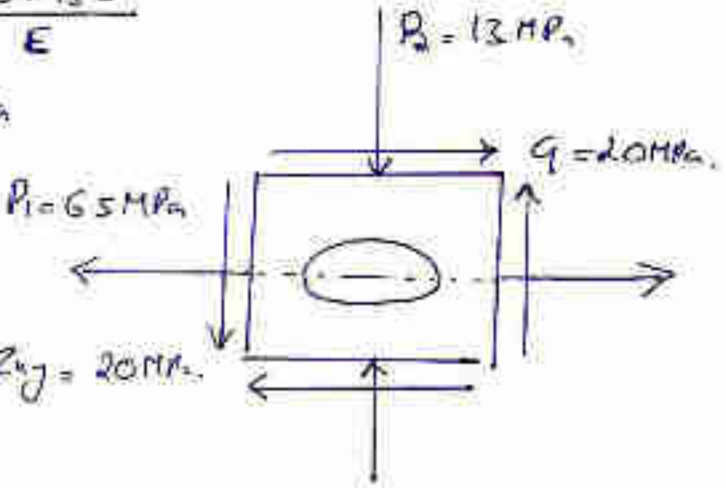
because Z increase in length of Z-direction so strain will be Zero.

$0 = \frac{\sigma_3}{E} - \frac{0.3 \cdot 300}{E} - \frac{\upsilon \times 150}{E}$

$\sigma_3 = -90 + 45 = 45\,MPa$

Page No 29 Q.No 4&5 ESE, CONVENTIONAL

? $\upsilon = 0.3$; $E = 200\,GPa$



$\sigma_x = 65$; $\sigma_y = -13\,MPa$; $\tau_{xy} = 20\,MPa$.

$\sigma_{avg} = 26$

$R = \left\{\left(\frac{65 - -13}{2}\right)^2 + 20^2\right\}^{1/2} = 43.82\,MPa$

$\sigma_1 = \sigma_{avg} + R = 26 + 43.82 = 69.82\,MPa$

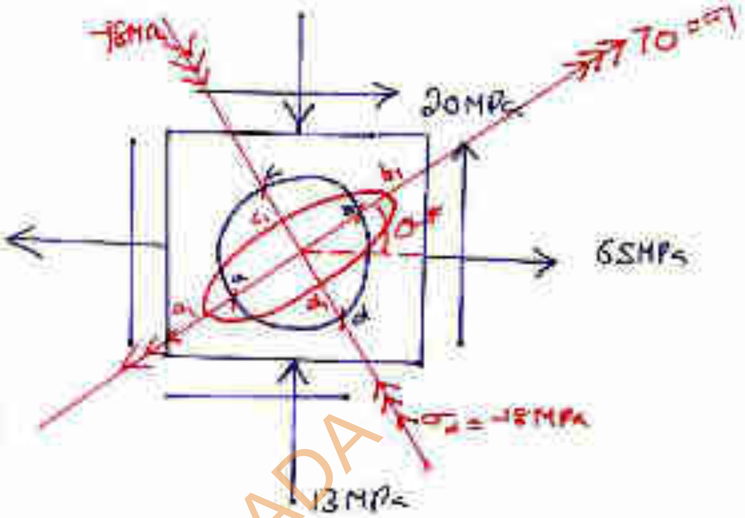
$\sigma_2 = \sigma_{avg} - R = 26 - 43.82 = -17.82\,MPa$

Initial diameter of circle = 300 mm ; $\sigma_1 = 70MPa$; $\sigma_2 = -18Pa$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$$

$$\varepsilon_2 = -\frac{\nu\sigma_1}{E} + \frac{\sigma_2}{E} - \frac{\nu\sigma_3}{E}$$

Only elongation and reduction in X-Y plane and $\sigma_3 = 0$.



$$\therefore \varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}.$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E}$$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}$$

$a_1b_1 = ab + \delta ab$ → ε_1

$a_1b_1 = ab\left[1 + \frac{\delta ab}{ab}\right]$

Similarly; $c_1d_1 = cd \, \boxed{+} \, \delta cd.$ → don't care, 'GHL' will give result automatically

$c_1d_1 = cd\left(1 + \frac{\delta cd}{cd}\right)$ → ε_2

→ Max. change along max. stress (σ_1)

$$\varepsilon_1 = \frac{70}{200\times10^2} - \frac{0.3\times-18}{200\times10^3}$$

$\varepsilon_1 =$ ~~27 mm~~ 3.77×10^{-4}

$$\varepsilon_2 = \frac{-18}{E} - \frac{\nu\times70}{E}$$

$$\varepsilon_2 = \frac{-18}{200\times10^3} - \frac{0.3\times70}{E}$$

$\varepsilon_2 = -1.95\times10^{-4}$

$\therefore a_1b_1 = ab\,[1 + \varepsilon_1]$

$a_1b_1 = 300\,[1 + 3.77\times10^{-4}]$

$a_1b_1 = \underline{\underline{300.1131}}$ mm.

$c_1d_1 = cd\,[1 + \varepsilon_2]$

$c_1d_1 = 300\,[1 - 1.95\times10^{-4}]$

$c_1d_1 = \underline{\underline{299.95mm}}$

Pg No. 79 / Q No. 8

Q. $\sigma_2 = 0$

$\sigma_{av} - R = 0$

$\sigma_{av}^2 = R^2$

$\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}$.

$(\sigma_x+\sigma_y)^2 = (\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2$

$4\sigma_x\sigma_y = 4.\tau_{xy}^2$.

$\tau_{xy} = \sqrt{\sigma_x.\sigma_y}$

Pg No. 83 / Q 12 → ESE

Q. $\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E} \xrightarrow{} 0$

$\varepsilon_2 = -\frac{\nu\sigma_1}{E} + \frac{\sigma_2}{E} - \frac{\nu\sigma_3}{E} \xrightarrow{} 0$

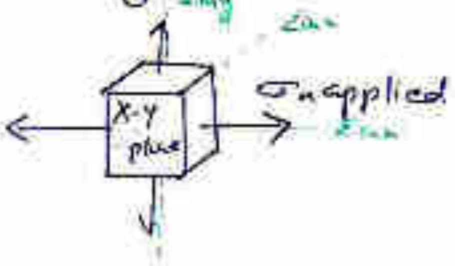
$\varepsilon_{zz} \neq$ ↓

$\varepsilon_3 = -\frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E} + \frac{\sigma_3}{E} \xrightarrow{} 0$

Inplane stress	and also
$\sigma_1 \neq 0$	$\varepsilon_1 \neq 0$
$\sigma_2 \neq 0$	$\varepsilon_2 \neq 0$
$\sigma_3 = 0$	$\varepsilon_3 \neq 0$

NOTE: Even stress in 1-d, The strain can be 3-dimensional, x y and z change

∴ A plane state of stress does not necessarily result into a plane state of strain

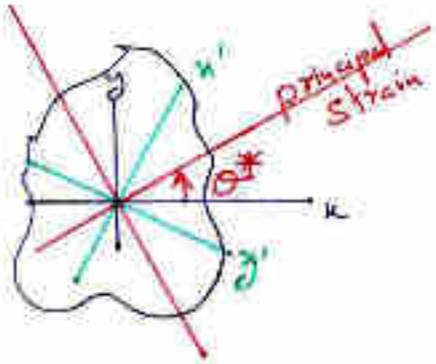


5.4 STRAIN-ANALYSIS:

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x-\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



$$\sigma_{x'}+\sigma_{y'} = \sigma_x+\sigma_y.$$

$$\sigma_{av} = \frac{\sigma_x+\sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma \longrightarrow \varepsilon \;;\; \tau \longrightarrow \frac{\gamma}{2}$$

$$\varepsilon_{x'} = \frac{\varepsilon_x+\varepsilon_y}{2} + \left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x+\varepsilon_y}{2} - \left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\gamma_{x'y'} = -\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{x'}+\varepsilon_{y'} = \varepsilon_x+\varepsilon_y \xrightarrow{\text{Analogy}} I_{x'}+I_{y'} = I_x+I_y$$

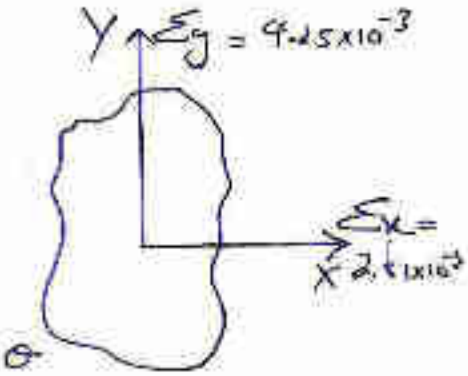
$$\varepsilon_{avg} = \frac{\varepsilon_x+\varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_1 = \varepsilon_{avg} + R$$

$$\varepsilon_2 = \varepsilon_{avg} - R$$

9 The state of strain at a point of loaded body is defined as ε at 0° 1 = 2.51×10^{-3}, ε at 90° = 4.25×10^{-3}, ε at 45° = 1.78×10^{-3} Determine principal strains at the point?



$\varepsilon_x = 2.5\times10^{-3}$; $\varepsilon_y = 4.25\times10^{-3}$

$\varepsilon_{45} = 1.78\times10^{-3}$.

$$\varepsilon_{x'} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\varepsilon_{x'} = 1.78\times10^{-3}$; $\theta = 45$

$$1.78\times10^{-3} = \left(\frac{2.5\times10^{-3}x + 4.25\times10^{-3}}{2}\right) + \overset{\to 0}{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 90} + \frac{\gamma_{xy}}{2}\sin 2\times45$$

$$1.78\times10^{-3} = 3.375\times10^{-3} + \frac{\gamma_{xy}}{2}\sin 90$$

$$\underline{\underline{\gamma = -3.2\times10^{-3}}}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{2.5\times10^{-3} + 4.25\times10^{-3}}{2} = 3.375\times10^{-3}$$

$$R = \left\{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2\right\}^{1/2}$$

$$R = \left\{\left(\frac{2.5\times10^{-3} - 4.25\times10^{-3}}{2}\right)^2 + \left(\frac{-3.2\times10^{-3}}{2}\right)^2\right\}^{1/2} = 1.82\times10^{-3}$$

$\varepsilon_1 = \varepsilon_{avg} + R.$

$\varepsilon_1 = 3.315\times10^{-3} + 1.82\times10^{-3}$

$\varepsilon_1 = \overset{5.195}{\cancel{1.555}}\times10^{-3}$

$\varepsilon_2 = \varepsilon_{avg} - R$

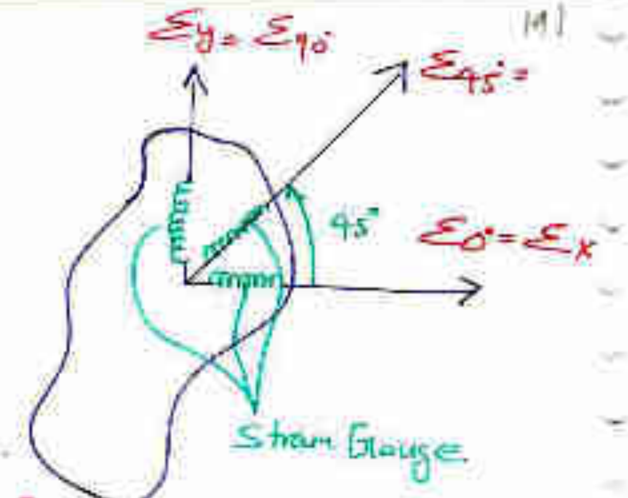
$\varepsilon_2 = 3.375\times10^{-3} - 1.82\times10^{-3}$

$\varepsilon_2 = 1.555\times10^{-2}$

b) STRAIN GAUGE:

i) Rossette readings → State of strain → → Principal Strains → Principal Stresses

Rossette readings: $\varepsilon_{0°}$, $\varepsilon_{90°}$, $\varepsilon_{45°}$ → State of strain: ε_x, ε_y, γ_{xy} → ε_{avg}, R



Rectangular Rossette.

Principal Strains $\varepsilon_x = \varepsilon_{0°}$; $\varepsilon_y = \varepsilon_{90°}$

$\varepsilon_{45°}$

$\overline{\varepsilon_1} = \varepsilon_{avg} + R$

$\varepsilon_2 = \varepsilon_{avg} - R$

Principal Stresses

$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}$

$\varepsilon_2 = \frac{-\nu\sigma_1}{E} + \frac{\sigma_2}{E}$

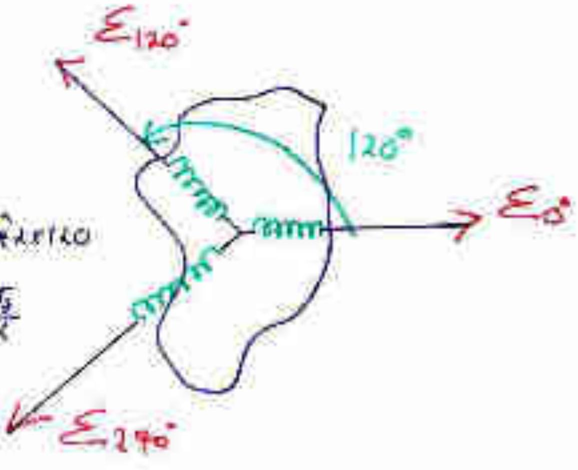
$$\varepsilon_{45°} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\times 45 + \left(\frac{\gamma_{xy}}{2}\right)\sin 2\times 45$$

$$\varepsilon_{45°} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\cancelto{0}{\frac{\varepsilon_x - \varepsilon_y}{2}}\right)\cos 90 + \left(\frac{\gamma_{xy}}{2}\right)\sin 90°$$

$$2\varepsilon_{45°} = (\varepsilon_x + \varepsilon_y) + \gamma_{xy}$$

$$\gamma_{xy} = 2\varepsilon_{45°} - (\varepsilon_0 + \varepsilon_{90°})$$

ii) STAR ROSSETTE:



$\varepsilon_0 = \varepsilon_x$

$$\varepsilon_{120} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\overset{\cos}{\sin}2\times120 + \frac{\gamma_{xy}}{2}\overset{\sin}{\cos}2\times120$$

$$\varepsilon_{120} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\times -\frac{1}{2} + \frac{\gamma_{xy}}{2}\times\frac{-\sqrt{3}}{2}$$

$$\varepsilon_{120} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) - \frac{\varepsilon_x}{4} + \frac{\varepsilon_y}{4} - \frac{\gamma_{xy}\sqrt{3}}{4}$$

$$\varepsilon_{120} = \frac{\varepsilon_x}{4} + \frac{3}{4}\varepsilon_y - \frac{\sqrt{3}\gamma_{xy}}{4} \quad \text{--- (i)}$$

$$\varepsilon_{240} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\times240 + \frac{\gamma_{xy}}{2}\sin 2\times240$$

$$\varepsilon_{240} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\times -\frac{1}{2} + \frac{\gamma_{xy}}{2}\times\frac{\sqrt{3}}{2}$$

$$\varepsilon_{240} = \frac{\varepsilon_x}{4} + \frac{3}{4}\varepsilon_y + \frac{\sqrt{3}\gamma_{xy}}{4} \quad \text{--- (ii)}$$

(i) + (ii) ⇒

$$\varepsilon_{120} + \varepsilon_{240} = \frac{\varepsilon_0 + 3\varepsilon_y}{2}$$

$$2(\varepsilon_{120} + \varepsilon_{240}) = \varepsilon_0 + 3\varepsilon_y$$

$$2(\varepsilon_{120} + \varepsilon_{240}) - \varepsilon_0 = 3\varepsilon_y.$$

$$\varepsilon_y = \frac{2(\varepsilon_{120} + \varepsilon_{240}) - \varepsilon_0}{3}$$

Sub. ε_y in (1) $\Rightarrow$.

$$\varepsilon_{120} = \frac{\varepsilon_0}{4} + \frac{\cancel{3}}{4} \times \frac{2(\varepsilon_{120} + \varepsilon_{240}) - \varepsilon_0}{4\,\cancel{3}} - \frac{\sqrt{3}}{4}\gamma_{xy}.$$

$$\varepsilon_{120} = \frac{\cancel{\varepsilon_0} + 2\varepsilon_{120} + 2\varepsilon_{240} - \cancel{\varepsilon_0}}{4} - \frac{\sqrt{3}}{4}\gamma_{xy}.$$

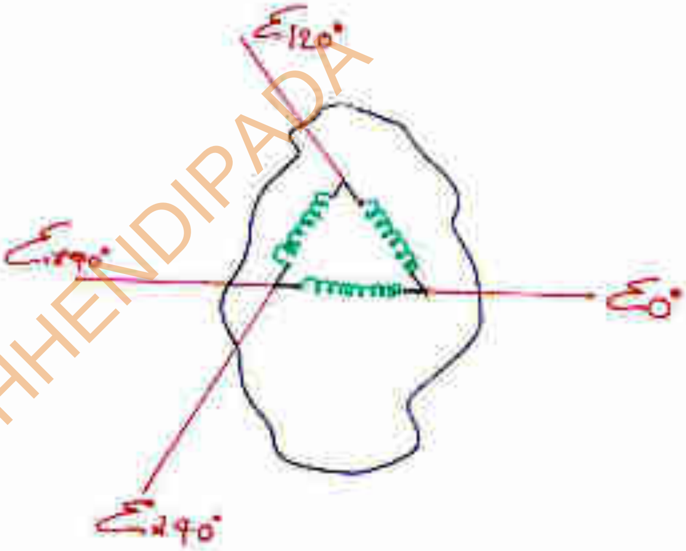
$$\varepsilon_{120} = \frac{2\varepsilon_{120} + 2\varepsilon_{240}}{4} - \frac{\sqrt{3}}{4}\gamma_{xy}.$$

$$\frac{\sqrt{3}}{4}\gamma_{xy} = \frac{2\varepsilon_{120} + 2\varepsilon_{240}}{4} - \varepsilon_{120}\cdot\frac{4}{4}$$

$$\frac{\sqrt{3}}{4}\gamma_{xy} = \frac{2\varepsilon_{240} - 2\varepsilon_{120}}{4}$$

$$\gamma_{xy} = \frac{2(\varepsilon_{240} - \varepsilon_{120})}{\sqrt{3}}$$

(ii) △ ROSSETTE:



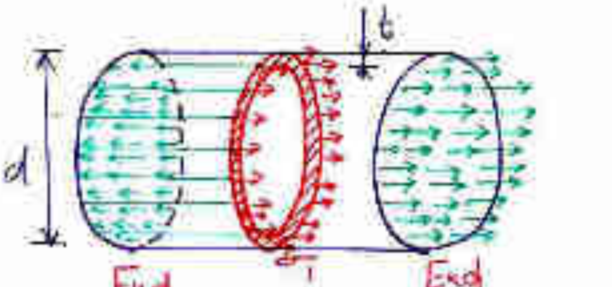
Equations same as Star rossette.

6. THIN & THICK VESSELS:

6.1 THIN CYLINDRICAL SHELLS: [$t <<< d$]

a) Stress:-

Fluid pressurised inside cylinder. If internal & external dia are given σ_{avg} has to be taken.



(N.B - Don't consider effect of pressure here around edges)

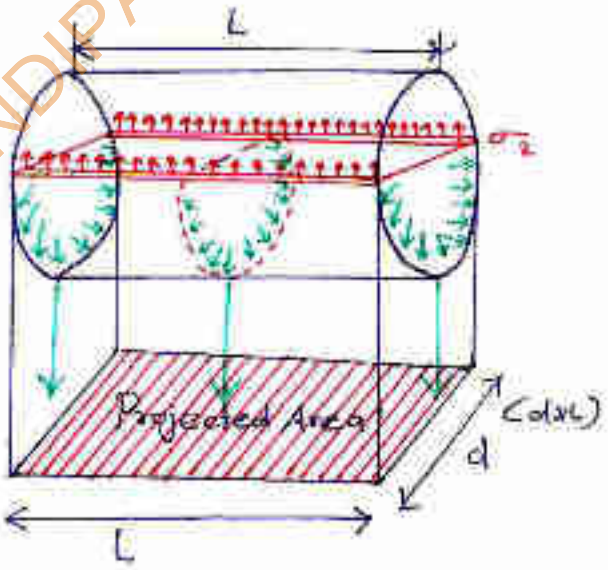
Equating the forces on both sides ⟶

$$\sigma_1 \times \pi d t = P \times \frac{\pi d^2}{4}$$

$$\sigma_1 = \frac{Pd}{4t}$$

$$\rightarrow \quad \sigma_1 = \frac{Pd}{4t}$$

σ_1 - Longitudinal Stress



Equating the forces on both sides.

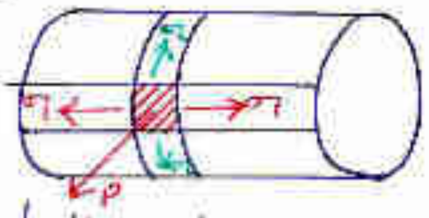
$$\sigma_2 \times 2t\cancel{L} = p.d\cancel{L}$$

$$\sigma_2 = \frac{P.d}{2t}$$

$$\rightarrow \quad \sigma_2 = \frac{P.d}{2t}$$

σ_2 - Circumferential Stress or Hoop Stress.

(due to pressure the surface will become concave)



Similar in Surface tension and balloon. Neighbouring particles will resist outward movement.

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{\cancel{2}t}$$

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}$$ → Applicable for cylinder, surface tension

at $r_1 = \infty$; $r_2 = R$

$$\frac{\sigma_1}{\infty} + \frac{\sigma_2}{R} = \frac{p}{t}$$

$$\frac{\sigma_2}{D/2} = \frac{p}{t}$$

$$\rightarrow \sigma_2 = \frac{P.D}{2.t}$$

$$\sigma_1 = \sigma_L = \frac{Pd}{4t} \; ; \sigma_3 = P$$

$$\sigma_2 = \sigma_H = \frac{Pd}{2t} \; ; \; \tau_{max} = \frac{Pd}{4t}$$

$$\tau_{max} = \underset{Max}{} \left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right| \right]$$

* → Because $t \lll d$; $d/t \ggg 1$

So neglecting it, So it's called as thin cylinder. $\therefore \sigma_3 \approx 0$

$$\tau_{max} = \underset{max}{} \left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - 0}{2}\right|, \left|\frac{\overset{0}{\cancel{\sigma_3}} - \sigma_1}{2}\right| \right]$$

i.e. $\tau_{max} = \frac{Pd}{4t}$

b) Shear stress (Max):-

$$\tau_{max} = \frac{Pd}{4t}$$

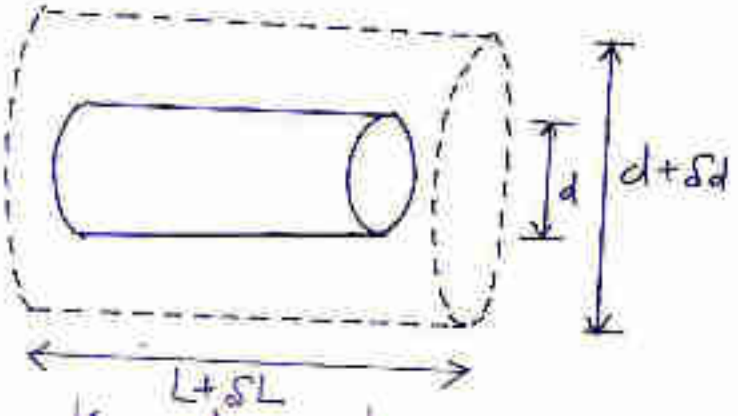
c~~d~~) Strain:-

LONGITUDINAL : $\frac{\delta L}{L} = \varepsilon_L = \frac{\sigma_1 - \upsilon\sigma_2}{E}$

CIRCUMFERENTIAL : $\frac{\delta c}{c} = \varepsilon_c = \frac{\sigma_2 - \upsilon\sigma_1}{E} = \frac{\delta \pi d}{\pi d}$

d) Volumetric Strain:

Changed volume →



$$\varepsilon_V = \frac{\delta V}{V} = \frac{\cancel{\pi/4}(d+\delta d)^2 (L+\delta L) - \cancel{\pi/4} d^2 L}{\cancel{\pi/4}\, d^2 L}$$

$$\varepsilon_V = \frac{[d^2 + \overset{\to 0}{\delta d^2} + 2d.\delta d]\,[L+\delta L] - d^2 L}{d^2 L}$$

$$\varepsilon_V = \frac{[\cancel{d^2 L} + 2d.L.\delta d + d^2.\delta L + \overset{\to 0}{\overline{2d.\delta d.\delta L}}] - \cancel{d^2 L}}{d^2 L}$$

$$\varepsilon_V = \frac{2d.L.\delta d + d^2.\delta L}{d^2 L}$$

$$\varepsilon_V = \frac{\cancel{d^2}\,\delta L}{\cancel{d^2} L} + \frac{2d.L\ \delta d}{d^2 L}$$

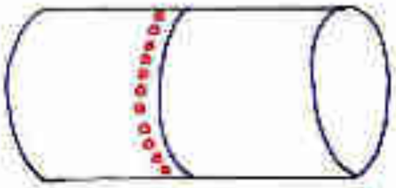
$$\varepsilon_V = \frac{\delta L}{L} + 2\left(\frac{\delta.\pi.d}{\pi.d}\right)$$

$$\varepsilon_V = \varepsilon_L + 2\,\varepsilon_c$$

Cylinder can be made by transverse joints and also by along the length. Therefore the due to the thickness of the rivets or joints the area decreases so the longitudinal stress will become larger. Due to circumferential joint longitudinal stress will be way more.

$(\sigma_{log})_{actual} >>> Pd/4t$

Efficiency of circumferential joint, η_c



Circumferential joint.

$$\eta_c = \frac{(Pd)/4t}{(\sigma_l)_{actual}}$$

$$(\sigma_l)_{actual} = \frac{Pd}{4t.\eta_c}$$

Similarly For longitudinal cylinder. longitudinal rivets are done.

$$\eta_L = \frac{(Pd)/2t}{(\sigma_2)_{actual}}$$

$$(\sigma_2)_{actual} = \frac{Pd}{2t\times\eta_L}$$

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Q1.02

Q $L=2m$; $d=900mm$; $P=1.6MPa$; $t=12m.$; $\nu=0.3$

$$\sigma_1 = \frac{Pd}{4t} = \frac{1.6\times900}{\cancel{2000}\,12\times4} = 30MPa$$

$$\sigma_2 = \frac{Pd}{2t} = \frac{1.6\times900}{2\times12} = 60MPa.$$

$$\tau_{max} = \frac{Pd}{4t} = 30MPa.$$

$$\epsilon_v = \epsilon_l + 2\epsilon_c$$

$$\epsilon_l = \frac{\sigma_1 - \nu\sigma_2}{G} = \frac{30 - \nu\times60}{2\times10^5} = 6\times10^{-5}$$

$$\epsilon_c = \frac{\sigma_2 - \nu\sigma_1}{G} = \frac{60 - 0.3\times\sigma_1}{2\times10^5} = 2.55\times10^{\cancel{-5}-4}$$

$$\frac{\delta V}{V} = 6\times10^{-5} + 2\times2.55\times10^{-4} = \underline{\underline{5.7\times10^{-4}}}$$

$$\frac{\delta V}{V} = 5.7\times10^{-9}$$

$$\delta V = 5.7\times10^{-9}\left(\frac{\pi}{4}\times900^2\times2000\right)$$

$$\delta V = 725236.66\,mm^3 \approx 725.236\,cm^3$$

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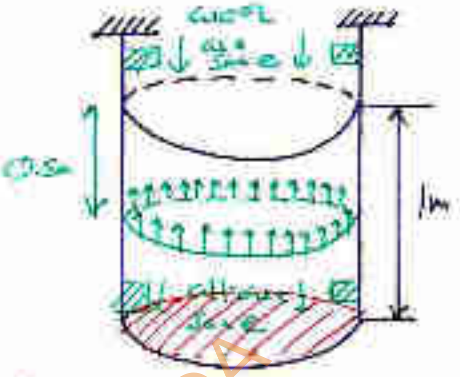
Q.546

? $P_{at\,mid} = \rho h g$.

$h = 0.5\times1000 = 500\,mm$.

$P_{mid} = \rho g(500)$

$P_{mid} = \frac{\rho v g}{v}(500)$



$\frac{\rho v}{v} = 10\,kN/m^3$; $\frac{\rho v}{v}$ = weight density

$\rho v = 10\,kN/m^3$

$P_{mid} = 10\times\cancel{9.81}\times0.5 = 5\,kPa$

σ_L - caused due to end pressure

$P_{mid} = 5\times10^{-3}\,MPa$

Note :-

$\rightarrow \sigma_1 = \frac{P_1 d}{4t} = \frac{\overset{10}{\cancel{5}}\times10^{-3}\times1000}{4\times1} = 1.25\,MPa \times2 = 2.5\,MPa$ (because the pressure will be at end face so h will be 1m)

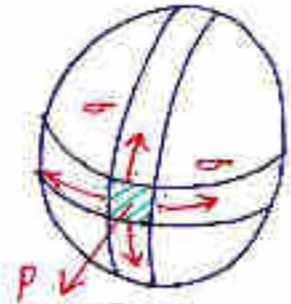
$\sigma_2 = \frac{Pd}{2t} = \frac{5\times10^{-3}\times1000}{2\times t} = 2.5\,MPa$

$E = 100\times10^3\,MPa$; $\nu = 0.3$

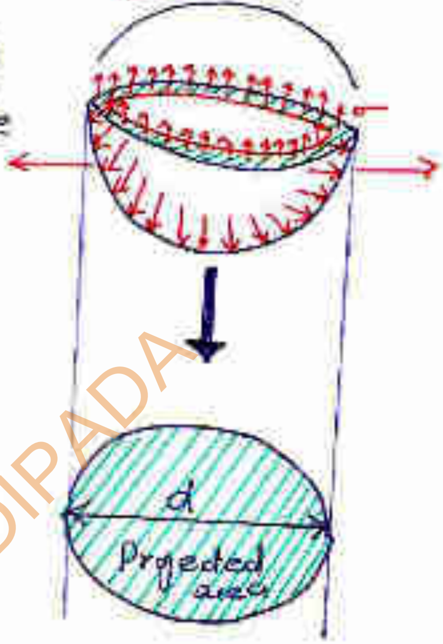
→ Note - end pressure has to be taken for σ_1 or σ_L

6.2 THIN SPHERICAL SHELL:

a) STRESS:



Element is trying to go out but the neighbouring particle holds it back.
Cut the spherical tank.



Downward portion has a bursting tendency and it is balanced by the Sphere material.

Equilibrium of force:

$$\sigma \times \pi d \times t = p \times \pi d^2/4$$

→ $\sigma = \frac{pd}{4t}$ (only circumferential stress)

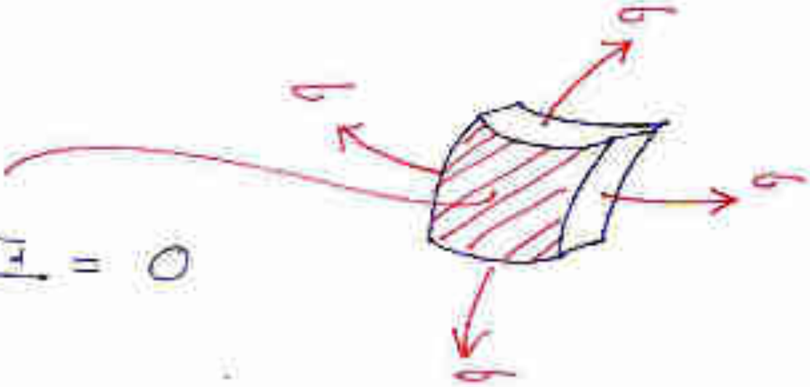
$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{p}{t}.$$

$$\frac{\sigma}{R} + \frac{\sigma}{R} = \frac{p}{t}$$

$$\frac{2\sigma}{R} = \frac{p}{t}$$

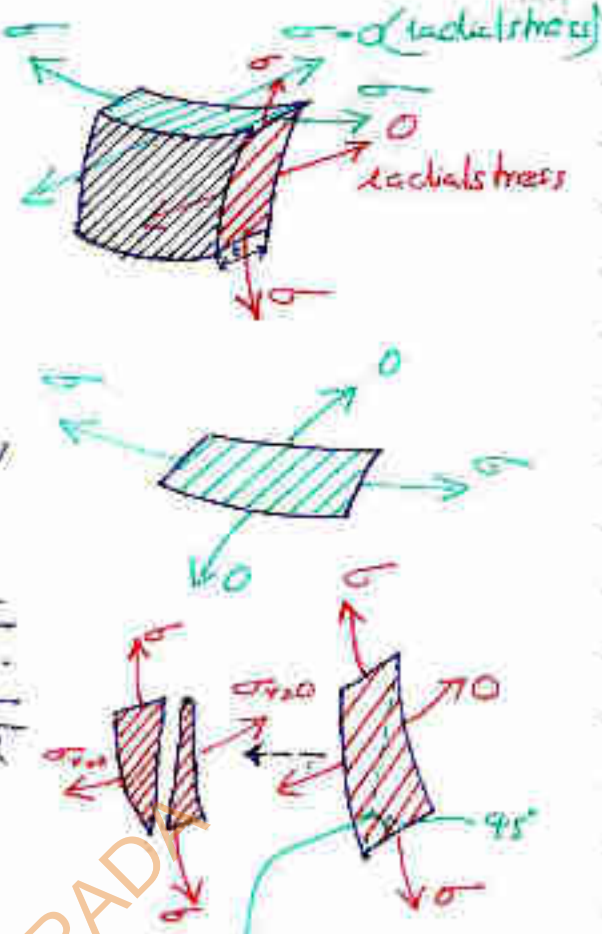
→ $\sigma_c = \frac{pR}{2t} = \frac{pd}{4t}$

Inplane shear will be zero. Plane with thickness.



$$\frac{\sigma_1 - \sigma_2}{2} = 0$$

b) MAX. SHEAR STRESS:



In the blue plane $\left|\frac{\sigma_1 - \sigma_3}{2}\right| = 0$ because in plane shear stress is zero. But the crack generates at red and green plane because due to the σ_c (circumferential stress) and radial stress will be zero.

$\therefore$ At red plane $\tau = \left|\frac{\sigma - 0}{2}\right| = \frac{\sigma}{2}$.

Sly at green plane, $\tau = \left|\frac{\sigma - 0}{2}\right| = \frac{\sigma}{2}$

$$\therefore \tau_{max} = \frac{\sigma}{2} = \frac{Pd}{8t}$$

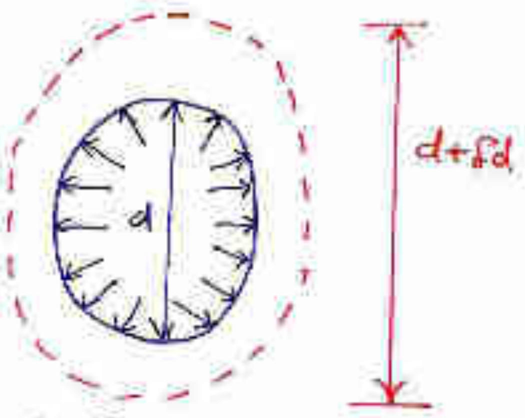
Shear will develop, Crack will propagate due to shear. Fracture will occur.

c) CIRCUMFERENTIAL STRAIN:

$$\varepsilon_c = \frac{\sigma - \nu\sigma}{E} = \frac{\delta(\pi d)}{\pi d}$$

$$\varepsilon_c = \frac{Pd(1-\nu)}{4tE}$$

d) VOLUMETRIC STRAIN:



$$\varepsilon_v = \frac{V_f - V_i}{V_i} \quad ; v = \frac{\pi}{6} d^3$$

$$\varepsilon_v = \frac{\frac{\pi}{6}(d+\delta d)^3 - \frac{\pi}{6} d^3}{\frac{\pi}{6} d^3}$$

$$\varepsilon_v = \frac{\frac{\pi}{6}(\cancel{d^3} + 3d^2 \cdot \delta d + \overset{0}{\cancel{3d\,\delta d^2}} + \overset{0}{\cancel{\delta d^3}}) - \cancel{\frac{\pi}{6} d^3}}{\frac{\pi}{6} d^3}$$

$$\underline{\varepsilon_v} = \frac{3\cancel{d^2} \cdot \delta d}{d^{\cancel{3}}}$$

$$\varepsilon_v = \frac{3 \cdot \pi \cdot \delta d}{\pi d} \quad ; \varepsilon_v = 3\varepsilon_c$$

e) EFFICIENCY:

$$(\sigma)_{actual} = \frac{Pd}{4t\eta_c}$$

For Fluids: $\frac{\delta V}{V} = \frac{-p}{K}$; K- Bulk Mod. of Elasticity

$\frac{\delta V}{V} = \left|\frac{-p}{K}\right|$. ; p- pressure.

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Q 3 d 4
?

$d = 800mm$; $t = 4mm$

$\delta V = 50cm^3$

$E = 2\times10^5$ MPa.

$\nu = 0.3$

$$\frac{\delta v}{V} = 3\times \varepsilon_c$$

$$\frac{\delta v}{V} = 3\times\left(\frac{Pd(1-\nu)}{4tE}\right)$$

$$\frac{50\times10^3}{\pi\times 800^3/6} = 3\left(\frac{P\times 800(1-0.3)}{4\times4\times2\times10^5}\right)$$

$$1.865\times10^{-4} = 5.25\times10^{-4}p$$

$$P = \underline{\underline{0.355MPa}}$$

$$\sigma = \frac{Pd}{4t}$$

$$\sigma = \frac{0.355\times800}{4\times4}$$

$$\sigma = 17.77\ MPa$$

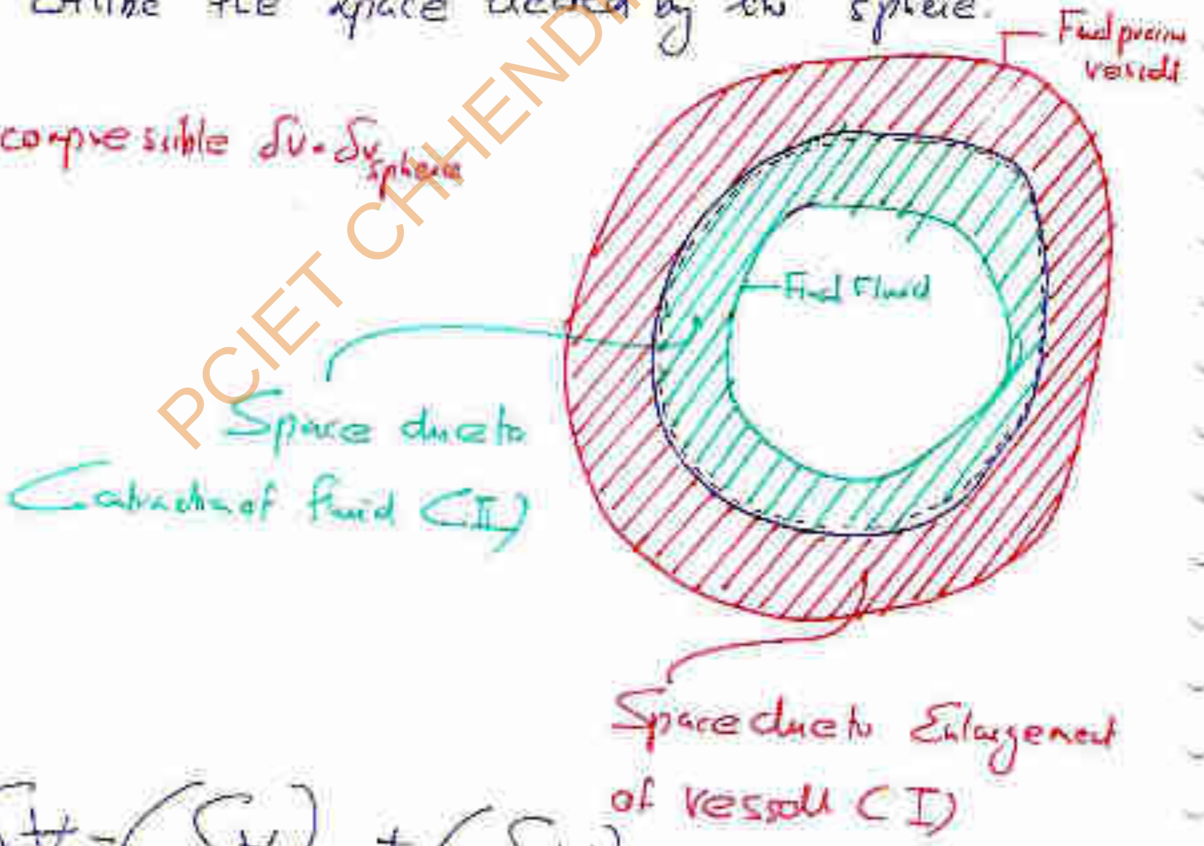
ESG → ? A Spherical tank of dia. 900mm is initially just filled at atmosphere pressure by water. Thickness is 10mm. Determine the additional volume of water that must be pumped into the tank to raise the internal pressure to 1.5 MPa? For sphere E = 100 GPa, ν = 0.3. For water k = 2 GPa.

→ Fluid incompressible k will be ~~zero~~ infinite. No change in volume.

If fluid is compressible so k is given. Fluid also contract. At higher pressure the vessel expands and fluid will contract. (Because of Bulk modulus of Elasticity)

So II will be the volume of fluid created due to contract and I will be the space created by the sphere.

For Incompressible $\delta v = \delta v_{sphere}$



$$\therefore \underline{\delta \forall} = (\delta \forall)_{\text{Sphere}} + (\delta V)_{\text{Fluid}}$$

$$\frac{\delta\forall}{\forall} = 3\varepsilon_c + \frac{p}{k}.$$

$$\delta\forall = (3\varepsilon_c \times \forall) + \left(\frac{p}{k} \times \forall\right)$$

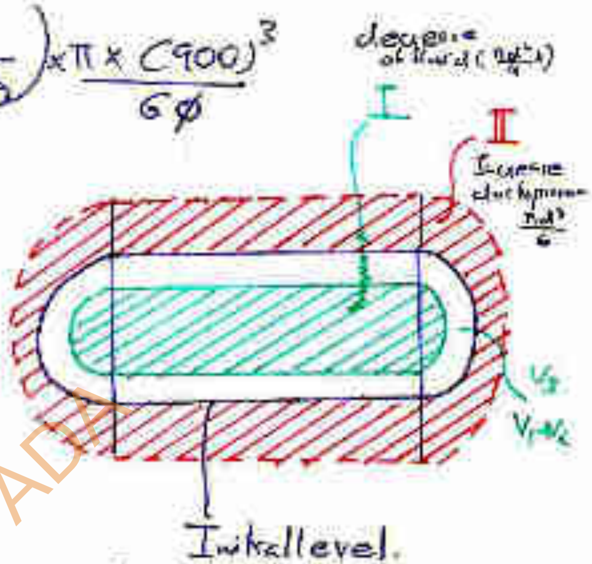
$$\delta\forall = \left(\frac{3pd(1-\nu)}{4t.E} + \frac{p}{k}\right) \times \frac{\pi d^3}{6\phi}$$

→ C.H. RYDER-139

$$\delta V = \left(\frac{3\times1.5\times900(1-0.3)}{4\times10\times100000} + \frac{1.5}{2000}\right)\times\frac{\pi\times(900)^3}{6\phi}$$

$$\delta\forall = \underline{\underline{5.56\times10^5\ mm^3}}$$

$\delta\forall = \delta\forall_1 + \delta\forall_2 + \delta\forall_3$



$E_{cy} = E_{sph} = 200\ GPa$

$\nu_{cy} = \nu_{sph} = 0.3$

$K = 2\times10^{5} MPa\ (?GPa)$

$P = 1\ MPa$

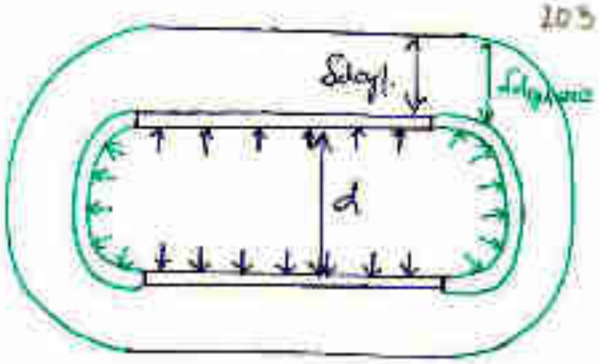
E.SE → Q. A cylindrical shell has to be closed by hemispherical ends. Thickness of cylinder tc and sphere ts. Same material & will have same internal dia. which is following true.

a) $t_c > t_s$ → $t_{cylinder} > t_{sphere}$

b) $t_s > t_c$

c) $t_s = t_c$

d) Can't be done.



Circumferential Strain of cylinder =

Circumferential Strain of sphere.

But $\delta d_{cy} = \delta d_{sphere}$.

$$(\epsilon_c)_{cy} = (\epsilon_c)_{sphere}.$$

$$\frac{-\nu\sigma_1 + \sigma_2}{\cancel{E}} = \frac{Pd(1-\nu)}{4t\cancel{E}}.$$

$$-\nu\frac{\cancel{Pd}}{4t_c} + \frac{\cancel{Pd}}{2t_c} = \frac{\cancel{Pd}(1-\nu)}{4t_s}$$

$$\frac{-\nu}{4t_c} + \frac{2}{4t_c} = \frac{(1-\nu)}{4t_s}.$$

$$\frac{(2-\nu)}{4t_c} = \frac{(1-\nu)}{4t_s}$$

$$\frac{t_c}{t_s} = \frac{(2-\nu)}{(1-\nu)}$$

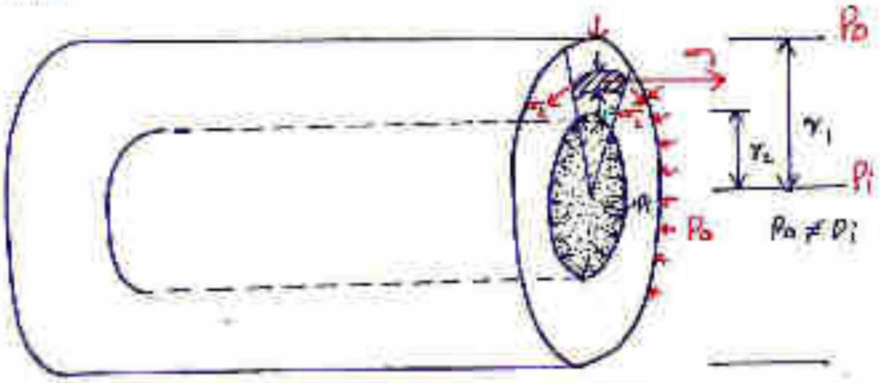
put $\nu = 0.3$.

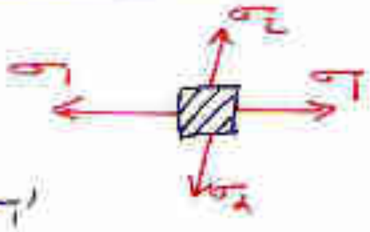
$t_c = 2.4\,t_s$

$\therefore t_c > t_s$

6.3 THICK CYLINDAR:

Radial pressure/ Stress must decrease as move away from the centre. Radial stresses are not negligible.

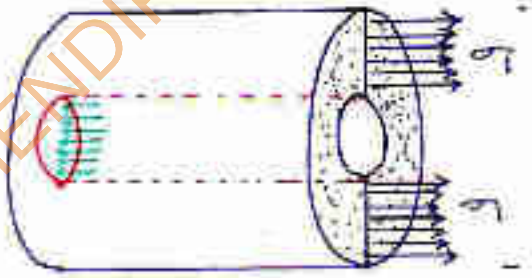




To determine the longitudinal stress σ_1, Cut the cylinder section.

a) STRESS :-

$$\sigma_1 \times \frac{\pi(d_o^2 - d_i^2)}{4} = \frac{P d_i^2}{4}$$



Constant - Not a variable $\quad \sigma_L = \sigma_1 = \frac{P d_i^2}{(d_o^2 - d_i^2)}$

$\sigma_3 = -p = -P(r) \longrightarrow$ Function of 'r'. (pressure decrease cause -ve)

$E\varepsilon_1 = \sigma_1 - \nu\sigma_2 - \nu(-p)$

Radial stresses are considered.

$E\varepsilon_1 = \sigma_1 - \nu\sigma_2 + \nu p$

$E\varepsilon_1 - \sigma_1 = -\nu(\sigma_2 - p)$

$\sigma_2 - p = \left(\frac{E\varepsilon_1 - \sigma_1}{-\nu}\right)$

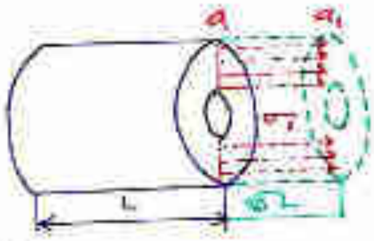
Because σ_1 constant, elongation of cylinder will be constant.

$\therefore \sigma_1$ is Constant.

$\sigma_1 \rightarrow$ constant.

E.S.E.

$\longrightarrow$ N.B. "Plane Section remains plane,"
All points same amount axially, $\sigma_1 = c$.



$E\varepsilon_1 = \sigma_1 - \nu\sigma_2 + \nu p$

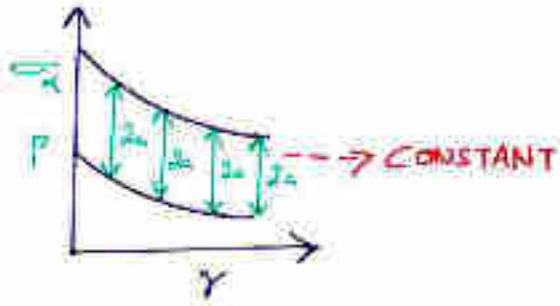
→ $\sigma_2 - p = \underbrace{\frac{E\varepsilon_1 - \sigma_1}{-\nu}} \rightarrow 2a$ $\qquad \sigma_2 - p = \frac{E\varepsilon_1 - \sigma_1}{-\nu} = 2a$ .....(i)

$\sigma_2 - p = 2a \rightarrow$ constant.

because p is a function of 'r' then $\sigma_2 - p =$ constant, σ_2 has also be a function of 'R'. Since the diff. has to be constant.

$\sigma_2 > p$

i.e. $\sigma_2 = p + 2a$



→ $p = f(r)$.

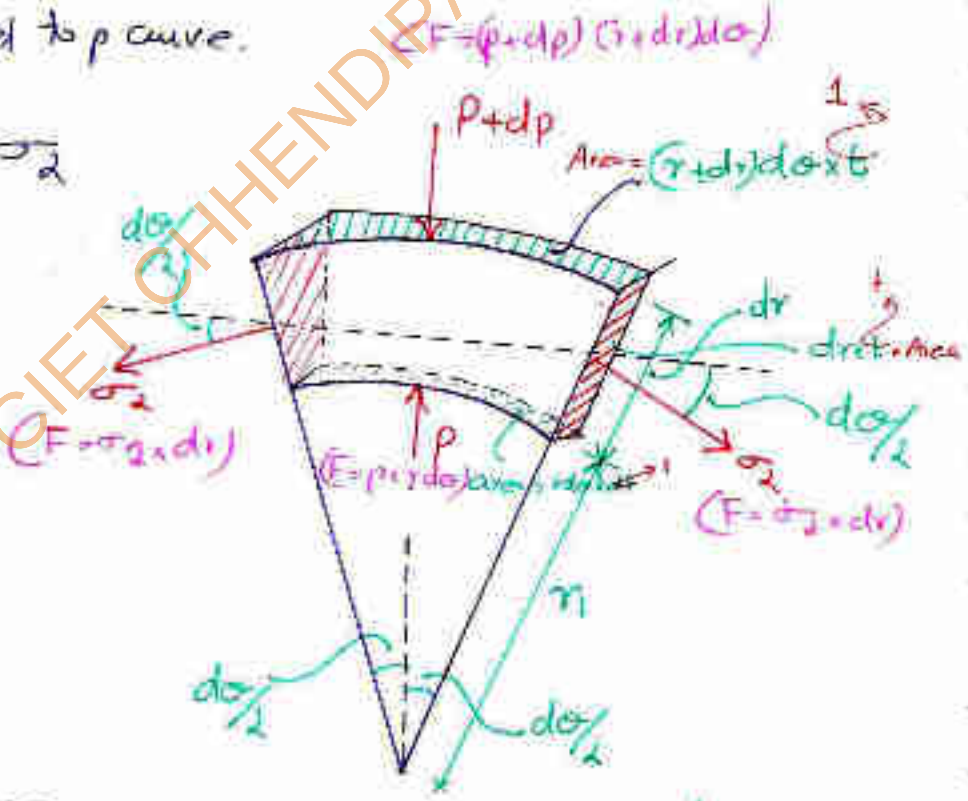
→ $\sigma_2 - p = 2a$

→ $\sigma_2 = \sigma_2(r)$

→ $\sigma_2 = p + 2a$

→ σ_2 curve parallel to p curve.

To determine σ_2



$\Sigma F_r = 0$ (Radially)

$p\,d\theta \times r = (p+dp)(r+dr)d\theta + 2\sigma_2\, dr \times \overset{d\theta/2}{\cancel{\sin\frac{d\theta}{2}}}$

$\cancel{p \times r d\theta} = \cancel{p r d\theta} + p\,dr\,\cancel{d\theta} + dp\, r\,\cancel{d\theta} + dp.\,dr\,\cancel{d\theta} + \cancel{2}\sigma_2\, dr.\, \frac{\cancel{d\theta}}{\cancel{2}}$

$0 = p.dr + \overset{0}{\cancel{dp.dr}} + \sigma_2 dr + r dp$

$\sigma_2 + p + \frac{r\,dp}{dr} = 0$ .....(ii)

$(i) + (ii) \longrightarrow$

$$p + 2a + p + r\frac{dp}{dr} = 0$$

$$2p + r.\frac{dp}{dr} = -2a$$

$\times$ly by $r \longrightarrow$.

$$2pr + r^2\frac{dp}{dr} = -2ar$$

$$\frac{d}{dr}(pr^2) = -2ar.$$

$$\int d(pr^2) = \int -2ar.dr$$

$$pr^2 = \frac{-2ar^2}{2} + B$$

$$pr^2 = -ar^2 + B.$$

$$pr^2 + ar^2 = B$$

$$pr^2 + ar^2 = B$$

$$\frac{pr^2}{r} = \frac{-ar^2}{r} + \frac{B}{r^2}$$

$$p = \frac{B}{r^2} - a$$

$$p = -a + \frac{B}{d^2/4}\ {}^{4B}$$

$$p = -a + \frac{b}{d^2} \longrightarrow 1^{st} \text{ Lame's Equation}$$

a, b are constant, Radial pressure at any location.

$$\sigma_2 = a + \frac{b}{d^2}$$

$$\sigma_2 - p = 2a$$

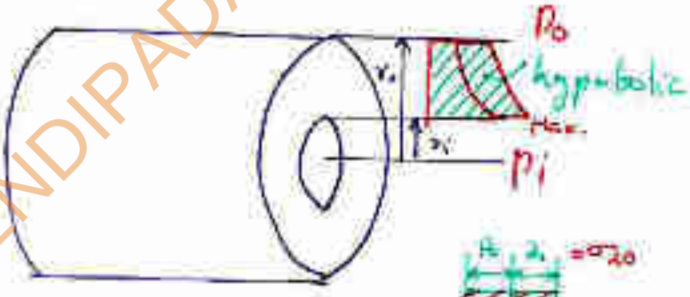
$$\sigma_2 - \left(-a + \frac{b}{d^2}\right) = 2a$$

$$\sigma_2 = a + \frac{b}{d^2}$$

$$\sigma_2 = a + \frac{b}{d^2}$$

$\therefore\ p \propto 1/r$ (1st lame's equation)

At and inner radius $p = p_i$



Max value σ_{2i} @ circumferential radius of inner radius.

$$(i)_R\ \sigma_3 = p = -a + \frac{b}{d^2}$$

$$(ii)_H\ \sigma_2 = a + \frac{b}{d^2}$$

$\Big\}$ LAME'S EQN.

N.B: For thick cylinder subjected to internal pressure, the point at the inner most circumference will be critical point. with max tensile stress equal to circumferential stress at inner most radius.

9. The maximum tensile stress permitted in a thick cylinder, inner and outer ~~radius~~ 8cm and 12cm respectively, ~~The~~ is 20 MPa. The external pressure is 6 MPa. What Internal pressure can be applied? Plot curves showing the variation of hoop and radial stresses through the material

—

D_i = ~~18~~ 16cm ; D_o = ~~12cm × 2~~ = ~~24cm~~ 24cm P_o = 6 MPa.

at $D = D_o$ = ~~240~~ 240mm ; P_o = 6 MPa C. Newa put -ve sign

$$6 = -a + \frac{b}{(240)^2} \text{ ——— (1)}$$

at $D = D_i$ = 160mm ; σ_2 Max occurs.

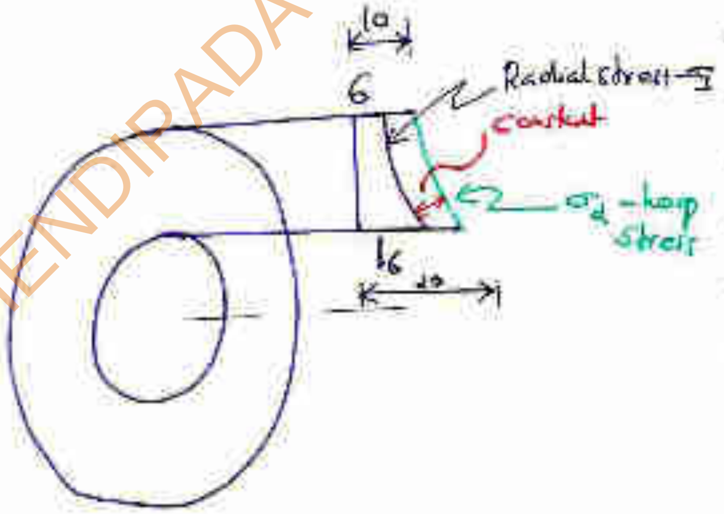
$$20 = a + \frac{b}{(160)^2} \text{ ——— (1)}$$

$$26 = \frac{b}{(240)^2} + \frac{b}{(160)^2}$$

$$\underline{\underline{b = 460800}}$$

$$\therefore 20 = a + \frac{b}{(160)^2}$$

$$a = 20 - 18 = \underline{\underline{2}}$$



Put $D = D_i$ = 160mm

$$P = -a + \frac{b}{(160)^2}$$

$$P = -2 + \frac{460800}{(160)^2}$$

$$\underline{\underline{P = 16 \text{ MPa}}} = \sigma_2$$

90 The cylinder of a hydraulic ram is 6cm internal diameter. Find the thickness required to withstand an internal pressure of 40MPa if the maximum tensile stress is limited to 60MPa and max shear stress is limited to 50MPa?

$P_i = 40MPa$; $D_i = 60mm$; $\sigma_2 = 60MPa$ (at inner dia) (Max)

at $D = D_i = 60mm$; $P_i = 40MPa = \sigma_3$.

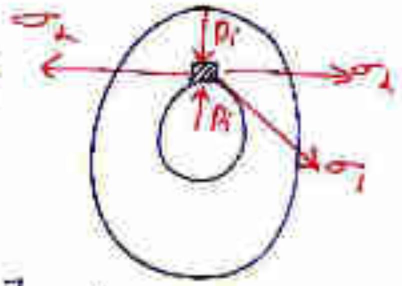
$$\sigma_3 = -a + \frac{b}{d^2}.$$

$$40 = -a + \frac{b}{60^2}. \text{ —— (i)}$$

$$(\sigma_2)_{Max} = a + \frac{b}{d^2}. \quad (\text{Max will be at internal dia})$$

$$(\sigma_1)_{Max} \; \cancel{60} = a + \frac{b}{60^2} \leq 60 \text{ —— (ii)}$$

$$100 = \frac{2b}{60^2} \qquad a + \frac{b}{60^2} \leq 60 \text{ —— (ii)}$$



Critical point at inner radius

$$\tau_{max} = Max\left[\left(\frac{\sigma_1 - \sigma_3}{2}\right), \left(\frac{\sigma_2 - \sigma_3}{2}\right), \left(\frac{\sigma_3 - \sigma_1}{2}\right)\right] \quad \text{here } \sigma_2 >>> \sigma_1$$

$$\tau_{max} = \left|\frac{\sigma_2 - \sigma_3}{2}\right|$$

$\tau_{max} = 60 MPa$

$$\frac{\sigma_{2i} + P_i}{2} \leq 50.$$

$$\frac{a + \frac{b}{60^2} + -a + \frac{b}{60^2}}{2} \leq 50$$

$$\frac{b}{60^2} \leq 50 \text{ —— (iii)}$$

$40 = -a + \frac{b}{60^2} \longrightarrow (i)$

@ $\sigma_{Max} = a + \frac{b}{60^2} \le 60 \longrightarrow (ii)$

$\frac{b}{60^2} \le 50 \longrightarrow (iii)$

Solving (i) & (ii).

$100 = \frac{2b}{60^2}$

$b = 180000$

$60 = a + \frac{180000}{60^2}$

$a = 10$

N.B- Po Not given take it as Zero.

Check in (iii).

$\frac{180000}{60^2} \doteq 50 = 50$ ✓

Finished.

at $d = d_0$; $\sigma_3 = -a + \frac{b}{d^2}$.

$\sigma_3 = P_o = -10 + \frac{180000}{(60+T)^2}$

Outer pressure not given → Take atmosphere $P_o = 0$

$0 = -10 + \frac{180000}{(60+T)^2}$

$10(60+T)^2 = 180000$

$(60+T)^2 = 180000/10$

$d_0 = 134.16$ m..

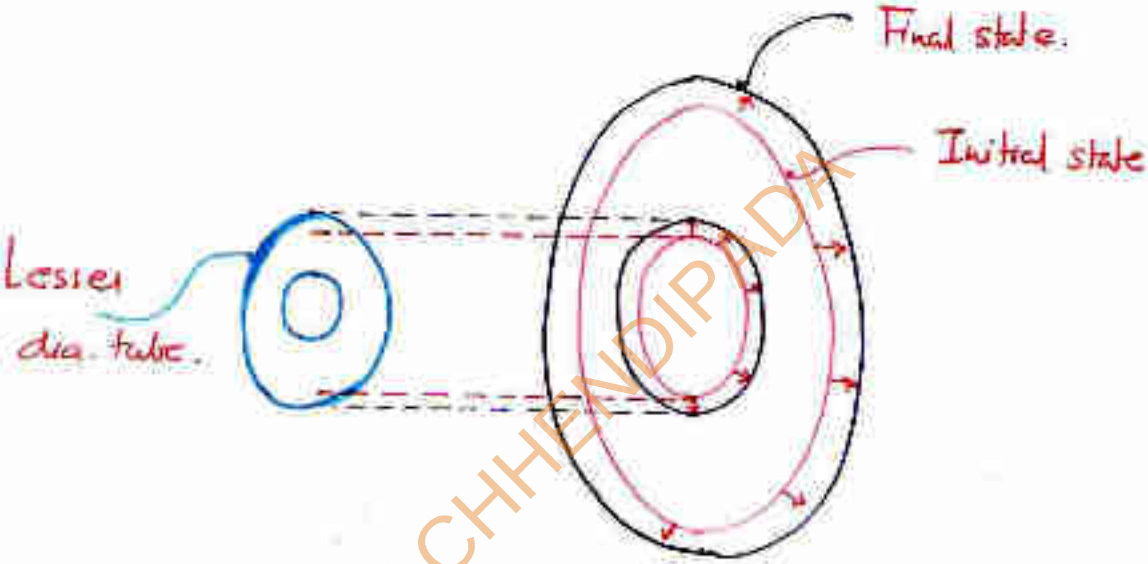
$t = \frac{134.16 - 60}{2}$

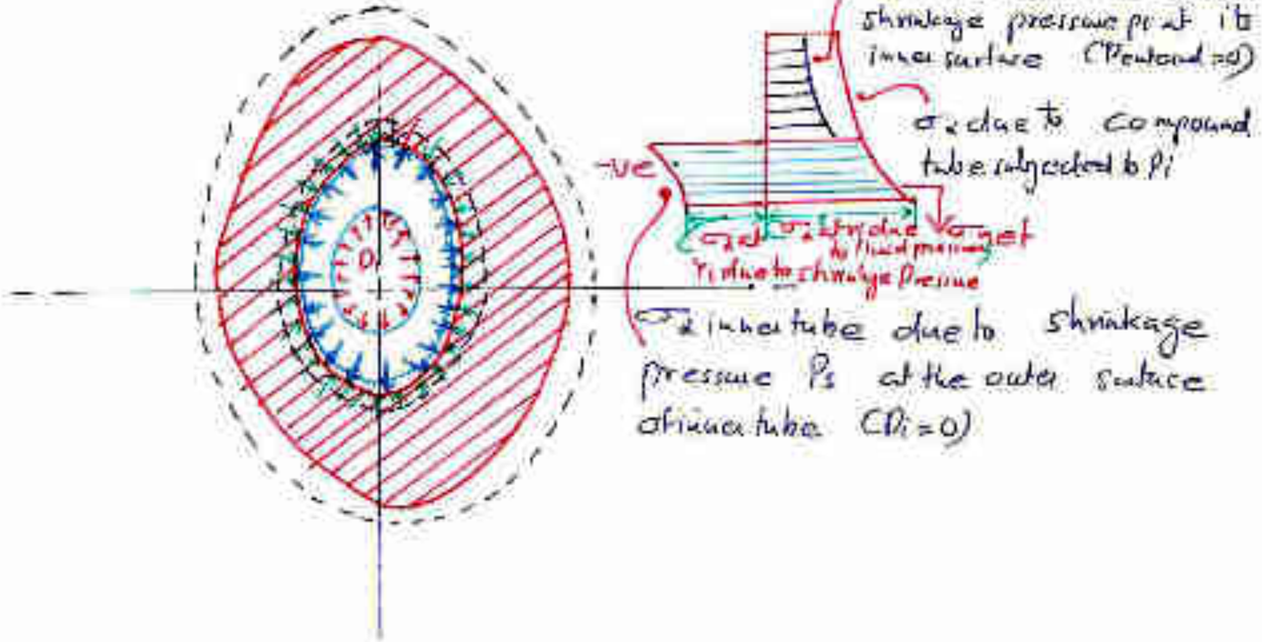
$t = 37.08$ mm

ESE → MAIN

6.4 COMPOUND TUBES: (SHRINK FIT):

Two cylinder and one cylinder is, having lesser diameter. During the rise of temperature the fit occurs between the cylinder due to expansion. After fitting the shrinking occurs and compress the inner tubes. Cannot fully recovered. Only upto a limit. The tubes produced called compound tubes and the method is called **shrink fit.**





9. A tube 8cm 4cm inside and 6cm outside dia. is to be reinforced by shrinking on a second tube of 8cm outside dia. The compound is to withstand an internal pressure of 50MPa. If the final max. stress in each tube has to be same, Draw the diagram of variation of hoop stress in various tubes and also find the maximum stress?

Inner dia. of inner tube = 40mm

Outer dia of outer tube = 80mm

Compound tube due to internal fluid pressure:

(Entire Cylinder of 40mm D_i and 80mm D_o)

Internal fluid pressure - 50MPa (@ 40mm)

At d = 80mm the pressure P_o = 0MPa

Applying Lames Equation.

$$\sigma_3 = -a + \frac{b}{d^2}$$

$$50 = -a + \frac{b}{40^2}. \quad (@ D = D_i = 40;\ P = 50)$$

$$0 = -a + \frac{b}{80^2} \quad (@ D = D_o = 80;\ P = 0)$$

Solvig both.

$$50^{*} = -a + a + \frac{-b}{80^2} + \frac{b}{40^2}$$

$$\underline{\underline{b = 106666.67}}$$

$$a = 16.67$$

$$(\sigma_2)_{at\,40} = 16.67 + \frac{106666.67}{40^2} = 83.34 \text{ MPa}$$

$$(\sigma_2)_{at\,60} = 16.67 + \frac{106666.67}{60^2} = 46.29 \text{ MPa}$$

$$(\sigma_2)_{at\,80} = 16.67 + \frac{106666.67}{80^2} = 33.33 \text{ MPa}$$

Hooper stress are developed and Calculated.

Consider tube individually. (Black tube under compression)

#1. Inner tube subjected to Compression, Shrinkage pressure (P_s) which is acting at the outer surface causes compression at outer surface. *($P_1 = 0$) → Assumed (Separately treating)

→ at D = 40 mm ; $\sigma_3 = 0$ (Considered) ↓ Not taken earlier value

$$\therefore \quad 0 = -a' + \frac{b'}{d^2}$$

$$0 = -a' + \frac{b'}{40^2} \quad \text{or} \quad 0 = -a_1 + \frac{b_1}{40^2} \quad \text{——(iii)}$$

→ at D = 60mm ; $\sigma_3 = P_s = ?$ (At junction $P_s = \sigma_3$)

$$P_s = -a_1 + \frac{b_1}{60^2} \quad \text{——(iv)}$$

Solve (iii) and (iv).

$$0 - P_s = -a_1 + a_1 + \frac{b_1}{40^2} - \frac{b_1}{60^2}$$

$$-P_s = \frac{b_1}{2880} \quad ; \quad b_1 = \frac{-2880 P_s}{P_s}$$

$$0 = -a_1 - \frac{2880}{P_s \times 40^2} \quad \therefore \quad a_1 = -1.8 P_s$$

$a_1 = -1.8 P_s$; $b_1 = -2880 P_s$ [Innertube = 40mm → 60mm]

$\sigma_{d \text{ at } 40mm} = -1.8 P_s - \frac{2880 P_s}{40^2}$.

σ_d at 40mm $= \underline{\underline{-3.6 P_s}}$ (Compressive)

σ_d at 60mm $= -1.8 P_s - \frac{2880 P_s}{60^2}$

σ_d at 60mm $= \underline{\underline{-2.6 P_s}}$ (Compressive)

Max value at Inner most Surface, of inner tube

$\sigma_d = 83.34 + -3.6 P_s$

Outer tube is subjected into shrinkage pressure at inner radius. Due to Newton's law the same force is subjected to outer tube.

#2. Outer tube subjected to P_s at inner surface. (External pressure at outside will be zero)

At d = 60m; $P = P_s$.

$\sigma_d = -a_2 + b/d^2$

$P_s = -a_2 + \frac{b_2}{60^2}$

At d = 80mm; $P = 0$.

$0 = -a_2 + \frac{b_2}{80^2}$.

$P_s = \frac{b_2}{60^2} - \frac{b_2}{80^2}$

$\underline{\underline{b_2 = 8228 P_s}}$

$0 = -a_2 + \frac{8228 P_s}{80^2}$

$\underline{\underline{a_2 = 1.28 P_s}}$

$\sigma_d = a_2 + \frac{b_2}{d^2}$

$\sigma_d = 1.28 P_s + \frac{8228 P_s}{d^2}$

at α = 60° ;

$$\sigma_2 = 1.28P_s + \frac{8228P_s}{60^2}$$

$$(\sigma_2)_{at60} = 3.56P_s$$ (Tensile)

at α = 80 ;

$$(\sigma_1)_{at80} = 1.28P_s + \frac{8228P_s}{80^2}$$

$$(\sigma_1)_{at80} = 2.56P_s$$ (Tensile)

Max value for outer tube = 46.29 + 3.56Ps

Max value at outer tube = Max value at inner tube.

$$83.34 - 3.6P_s = 46.29 + 3.57P_s$$

$$\therefore \underline{\underline{P_s = 5.26MPa}}$$

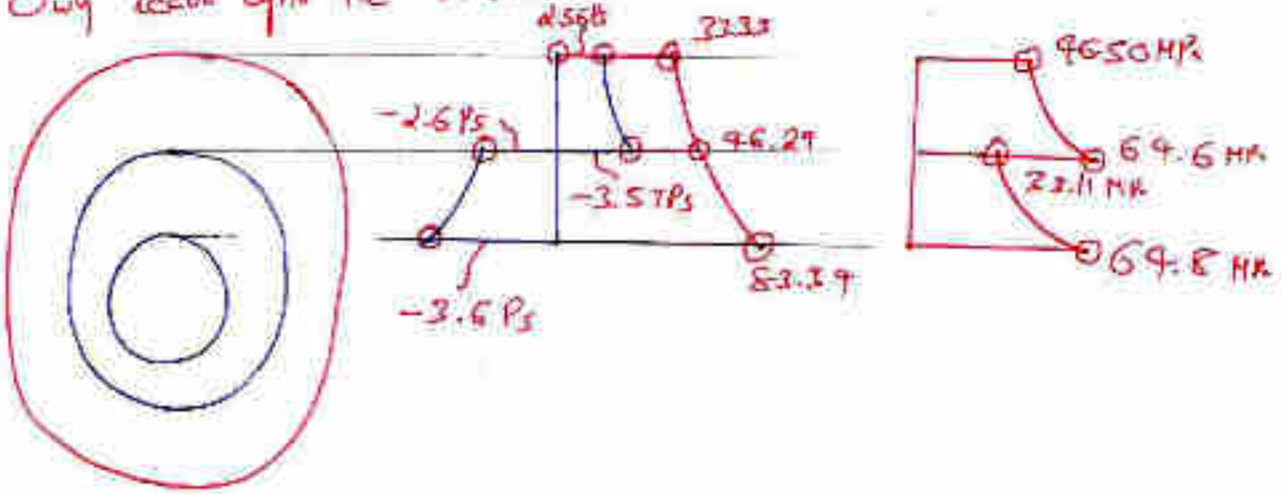
At inner tube at 40mm = 83.39 − 3.6×5.26 = 64.764 MPa

At inner tube at 60mm = $\overset{46.29}{\cancel{83.34}}$ − 2.6 × 5.26 = 32.8 MPa.

At outer tube at the 60mm = 46.29 + 3.57 × 5.26 = 64.76 MPa.

At the outer tube at 80mm = 33.33 + 2.56Ps = 46.79 MPa

N.B: More stress by less area given the same resisting force as the less stress and more area. (Under utilisation can be avoided) Most of materials greater value of stresses. So Area can be reduced. ⟵ ↑ $\underline{A}$↓ (necessity) Only used upto the Allowable value.



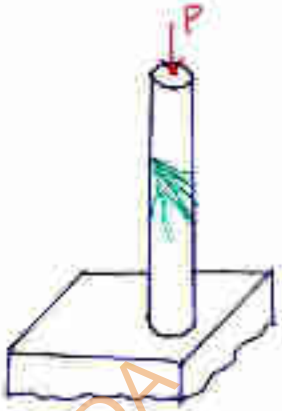
7. COLOUMNS & STRUTS:-

7.1: MODES OF FAILURES:

"Component that subjected to Compressive load."

i) Crushing occurs due to shearing at any angle. Max. compressive stress must be less than the allowable value which the Function of material.

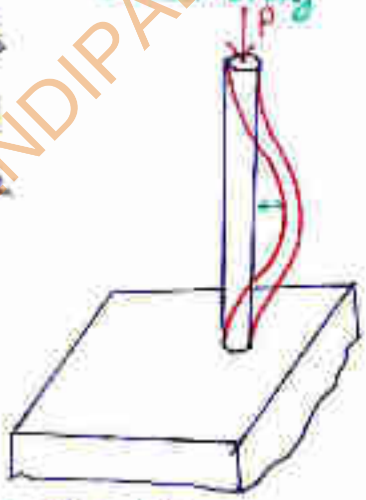
$$(\sigma_{allowable/comp}) > \sigma_{comp}$$



(i) Crushing

N.B. Bending occurs due to couple. Buckle due to comp. load. Sudden

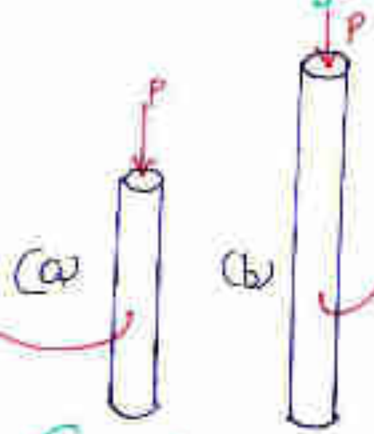
ii) Buckling – Consider colum A and B Same cross sectional area and made of same material and length will be not equal.



(ii) Buckling

$$(\sigma_{Buck})_A > (\sigma_{Buck})_B$$

$$\hat{\sigma}_{Buck.} \propto \frac{1}{L}$$



(a) will require more stress to buckle

less stress needed to buckle.

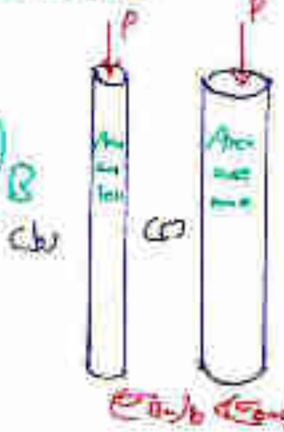
$$(\sigma_{Buck})_A > (\sigma_{Buck})_B$$

For colum B and C.

$$\hat{\sigma}_{Buck} \propto A\,K$$

$$\therefore (\sigma_{Buck})_B < (\sigma_{Buck})_C$$

$$\hat{\sigma} \propto \frac{A\,K}{L}$$



$(\sigma_{Buck})_b < (\sigma_{Buck})_c$

$\therefore \hat{\sigma} = K/\lambda$

K - distance at which area is distributed.

K - Radius of Gyration.

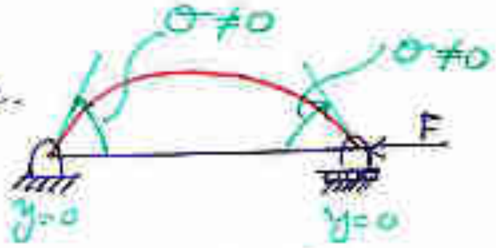
$\hat{\sigma} = \frac{1}{l/k}$

7.2 EULER'S THEORY:-

$\frac{A}{F}$

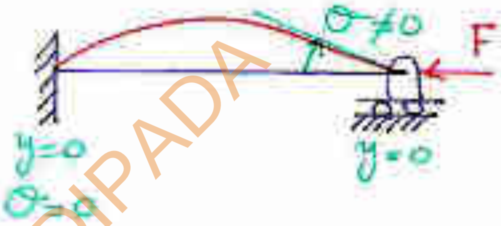
Max. Force F applied at the hinge.

(i) Both ends are hinged.



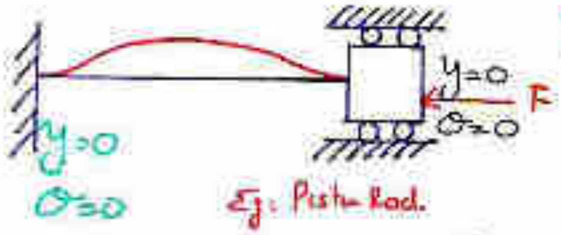
$\frac{\pi^2 E I_{min}}{L^2}$

(ii) One end fixed and other end hinged. [One end displacement is constrained and other end displacement and rotation is constrained]



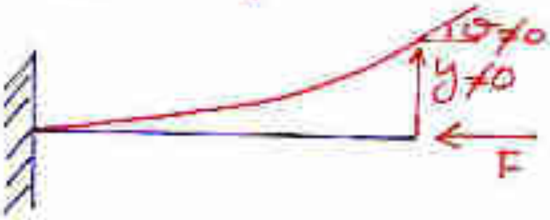
$\frac{2\pi^2 E I_{min}}{L^2}$

(iii) Columns Fixed at both ends [Rotation constrained on the both the ends]



$\frac{4\pi^2 E I_{min}}{L^2}$

(iv) One end Fixed and one end free.



$\frac{1}{4}\frac{\pi^2 E I_{min}}{L^2}$

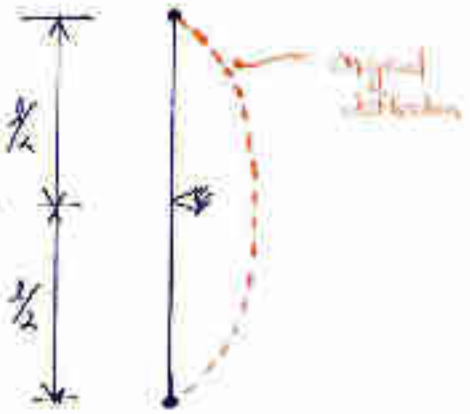
Page No. 83
Q. No. 1

Q hinged at two ends laterally supported.

Since they are being connected in series, Strength will be minimum of both.

$\hat{F} = \text{Min}(F_1, F_2)$ here $F_1 = F_2$

$$\hat{F} = \frac{\pi^2 E I}{(L/2)^2} = \frac{4\pi^2 E I}{L^2}$$



Pg No. 83
Q. No. 2

Q $L_1 = 2L$; $L_2 = L$

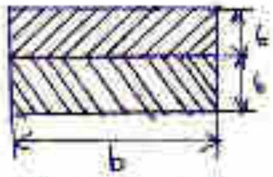
$$\frac{\hat{F}_1}{\hat{F}_2} = \frac{\frac{\pi^2 E I}{(2L)^2}}{\frac{\pi^2 E I}{L^2}} = 1/2^2 = 1/4$$

min. I $(I_1 << I_2)$

Pg No. 84
Q. No. 3

Q
$$\frac{\hat{F}_{\text{with bond}}}{\hat{F}_{\text{w/o bond}}} = \frac{\cancel{\pi^2} \cancel{E} \left(\frac{b \times (2t)^3}{12}\right)}{\cancel{\pi^2} \cancel{E} \left(\frac{b t^3}{6}\right)}$$

$$\frac{\hat{F} \text{ with bond}}{\hat{F} \text{ w/o bond}} = \frac{8}{2} = \frac{4}{1}$$



w/ bond $I = \frac{b \times (2t)^3}{12}$, min

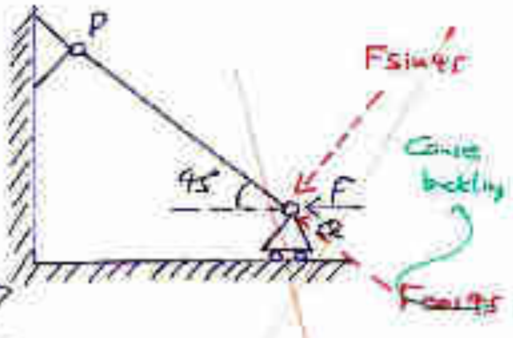
without bond, two separate columns side by side. Total strength will be the sum of individual columns.

$\frac{bt^3}{12} + \frac{bt^3}{12} = \frac{2bt^3}{12} = \frac{bt^3}{6}$ → Min.

GATE: Pg No. 83
Q. No. 4

$$\frac{F}{\sqrt{2}} \le \frac{\pi^2 E I}{L^2}$$

$$F = \frac{\sqrt{2}\, \pi^2 E I}{L^2}$$



→ Draw free body diagram:-

Consider free body diagram and Q →

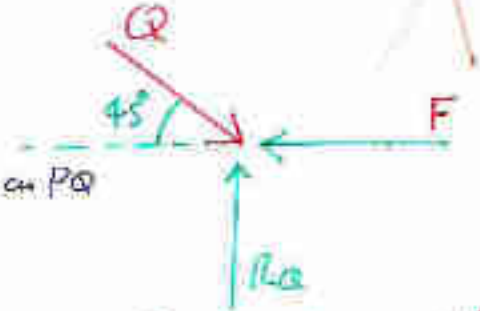
$\Sigma F_x = 0$

∴ $Q \cos 45 = F$.

$Q = F\sqrt{2}$ → Axial force on PQ

$$F\sqrt{2} \le \frac{\pi^2 E I}{L^2}$$

$$F = \frac{\pi^2 E I}{\sqrt{2} \cdot L^2}$$



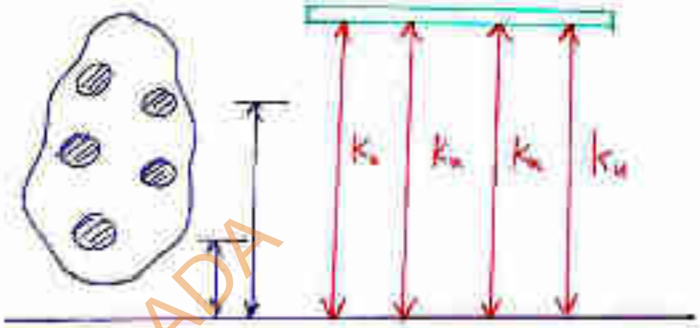
(F must balanced by R_Q)

→ MOMENT OF INERTIA:

The distance at which entire area can be assumed to be concentrated is known as Radius of gyration.

“The single hypothetical common distance at which entire area to be assumed located such that it produces the same moment of inertia as that of actual distributed area is known as radius of gyration.”

$$I_1 = A K_n^2$$



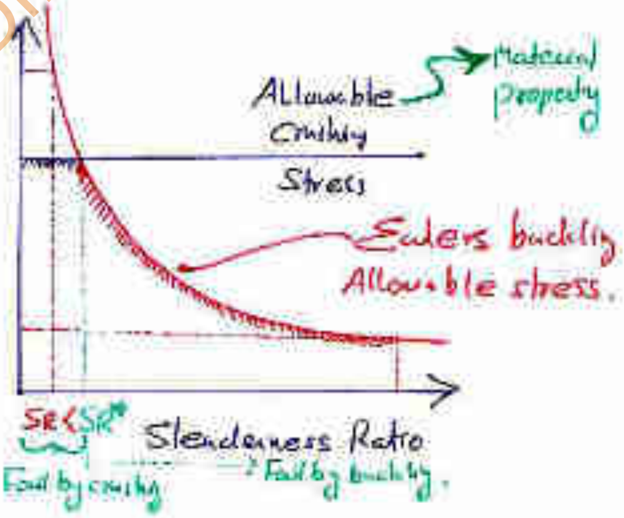
Crushing : $\hat{\sigma} = f(\text{material})$

Buckling : $\hat{\sigma} = \frac{\hat{F}}{A} = \frac{\pi^2 E\, I_{min}}{A L^2}$

$$\hat{\sigma} = \frac{\pi^2 E\, k^2}{L^2}$$

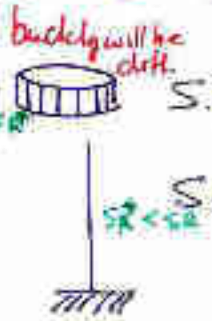
$$\hat{\sigma} = \frac{\pi^2 E}{L^2/k^2}$$

$$\hat{\sigma} = \frac{\pi^2 E}{(S.R)^2}$$



→ $\hat{\sigma} \propto \frac{1}{(S.R)^2} = f(\text{material}, S.R)$

Can be done with crushing →



$S.R = 0\,;\quad \hat{\sigma} \rightarrow \infty$

$S.R = \infty\,;\quad \hat{\sigma} \rightarrow 0$

→ If SR < SR*, Crushing more critical, Short Coloumns

→ If SR > SR*, buckling more critical, long coloumns

9 For $\hat{\sigma} = 300 MPa$; $E = 200\ GPa$, determine S.R?

$$\hat{\sigma} = \frac{\pi^2 . E}{(S.R)^2}$$

$$300 = \frac{\pi^2 \times 200 \times 10^3}{(S.R)^2}$$

$$S.R. = \underline{\underline{81.11}} \longrightarrow \text{For Steel}$$

$$\frac{\delta V}{V} = \varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{\sigma_1 + \sigma_2 + \sigma_3 (1-2\upsilon)}{E}$$

→ Incompressible - deformation can't be allowed. $\delta V = 0$

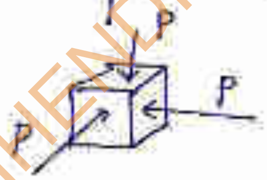
$\sigma_1 + \sigma_2 + \sigma_3 \neq 0$; $(1-2\upsilon) = 0$

$\upsilon = \frac{1}{2}$

→ Compressible - $\upsilon < \frac{1}{2}$

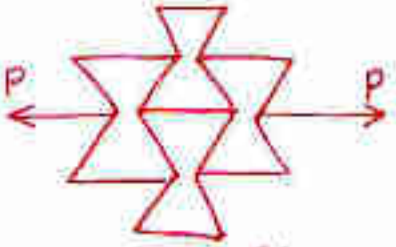
$0 < \upsilon < 0.5$ → metals

→ $\varepsilon_V = \frac{-3P(1-2\upsilon)}{E}$



$$\varepsilon_V = \frac{-P}{\boxed{E/3(1-2\upsilon)}}$$

→ Bulk Mod of Elasticity



→ $\frac{E}{3(1-2\upsilon)} = K$

$\upsilon = -\frac{\varepsilon_1}{\varepsilon_2}$ {because elongation ...}

$-1 < \upsilon < 0.5$

$E = 3K(1-2\upsilon)$

↓

Metal are stronger 1-D loading i.e $E \overset{>}{\cancel{<}} K$

i.e $E/K \overset{>}{\cancel{<}} 1$

Isotropic - 2 elastic constants
Anisotropic - 21 "
Orthotropic - 9 "

$3(1-2\upsilon) \overset{>}{\cancel{<}} 1$

$1-2\upsilon \overset{>}{\cancel{<}} \frac{1}{3}$

$2\upsilon - 1 \overset{<}{\cancel{>}} -\frac{1}{3}$

$2\upsilon < 1 - \frac{1}{3}$

$\upsilon < \frac{1}{3}$