LEARNING MATERIALS

SEMESTER : 4TH SEMESTER

BRANCH : MECHANICAL ENGINEERING

SUBJECT : THEORY OF MACHINE (TH-1)

FACULTIES : (1) ER. SUBODHAKANTA GARNAIK (LECT. IN MECH. ENGG.)
(2) ER. TARANISEN MOHANTY (LECT. IN MECH. ENGG.)

PURNA CHANDRA INSTITUTE OF ENGINEERING & TECHNOLOGY
AT/P.O.-CHHENDIPADA, DIST.-ANGUL.

[illegible]

- Thomas Bevan
- Ghosh & Malik
- Rattan (Amitabh Singh

Mechanics

- H.C Verma
- Beer & Johnson

[illegible] → Library . genesis

SOM

- Gere & Timoshenko
- Beer Hebbler
- G.H Ryder - [ES]

FM

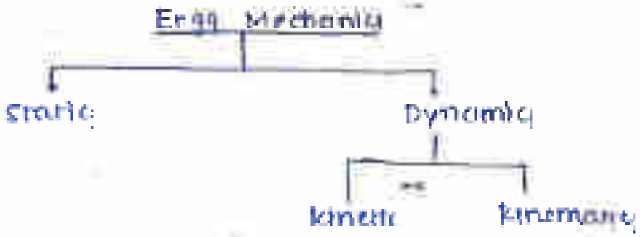
- Victor Streeter

T.O.M.

[10 - 12 Marks]

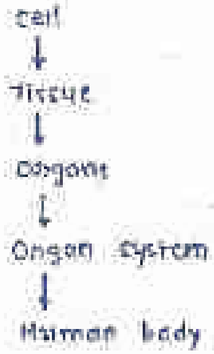
- Beer & Johnson (Vector Mechanics)
- J.L. Meriam (Dynamic
- Hebbler
- Grover mech. vib
- V.P Singh - mech. vib

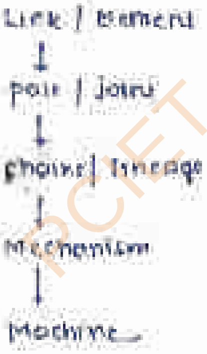
Powered by :-



- In Kinematics of Machines : we do the displacement : velocity & Acceleration analysis of different components of the m/c.
- We are not concerned with external forces acting on it
- In dynamics of machine we are concerned with all the forces that is external forces : restoring force , damping force etc acting on m/c

⇒ M/c :





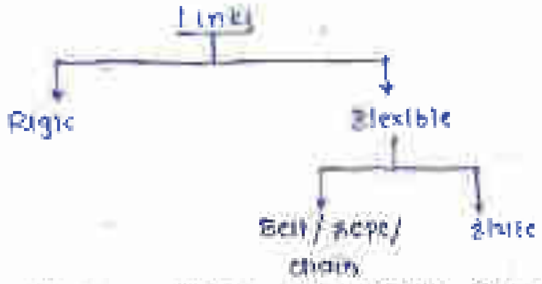
1

Link / Element → It is smallest unit of any machine

→ A link should be a resistant body (i.e. deformable) & need not to be rigid body always.

→ Link must be able to transfer the relative motion.

⇒ Types of link :-



UPSC : All the flexible links are having one directional rigidity that is they will work under a specific condition only.

Ex Belt / Rope will work as link when it is subjected to tension

where fluid will work as link when it is subjected to compression

— Spring follows Hooke's law

$\boxed{F_s \propto -x}$

& used for exerting force (restoring force)

UPSC :

→ Springs are mainly used to exert the restoring force therefore we can not considered spring as kinematics link

→ several parts manufactured separately but does not have relative motion between them will be considered as one link.

Ex Crank, crankshaft, flywheel, driven flange of clutch form one link only because all have same speed ω.

2

Pair / Joint → The inter connection between two or more links in such a manner that it permits the desired relative motion to get transferred will be known as kinematic pair.

① On the basis of degree of freedom
② On the basis of types of contact
③ On the basis of type of closure
④ On the basis of no. of links to be connected.

⊛ Degree of freedom (D.O.F)

— Total no. of independent co-ordinates that is fully various required to define any motion completely is known as degree of freedom

Translation (S)	Rotation (θ)
S_x	θ_x
S_y	θ_y
S_z	θ_z

$3T + 3R = 6$

$\boxed{\text{max dof} = 6 \big|_{\text{in 3D}}}$ is '6' not less "12"

In 2D :

planar :

Translation	Rotation
S_x	θ_y
S_z	

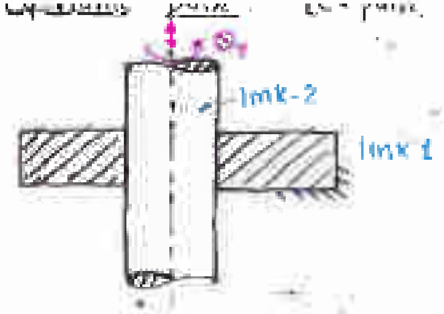
? Arrested / Restricted / constrained

T	R
S_y	θ_x
	θ_z

$\boxed{\text{max dof} = 3 \big|_{\text{in 2D}}}$

Actual dof = max possible dof − Restricted dof

i) Cylindrical pair (C - pair)



possible dof

T	R
S_y	θ_y

Arrested dof

S_x	θ_x
S_z	θ_z

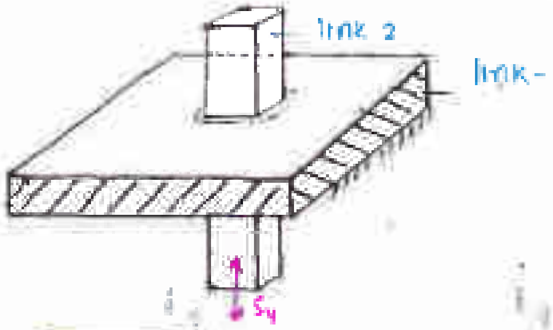
Act - dof = max' possible dof − arrested dof

= 6 − 4

$\boxed{dof = 2}$

Ex shaft in bearing

ii) Prismatic pair : [P - Pair]



$\boxed{dof = 1}$

Bending moment is represented half circle

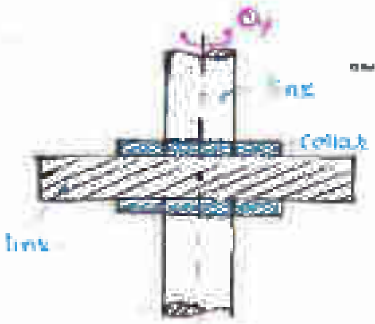
Torsional moment is ellipse shape

— If beam is hinged at both end then at both ends are revolute pair so that
$\boxed{BM = 0}$

Ex shaft in between piston - cylinder, crank + slotted box.

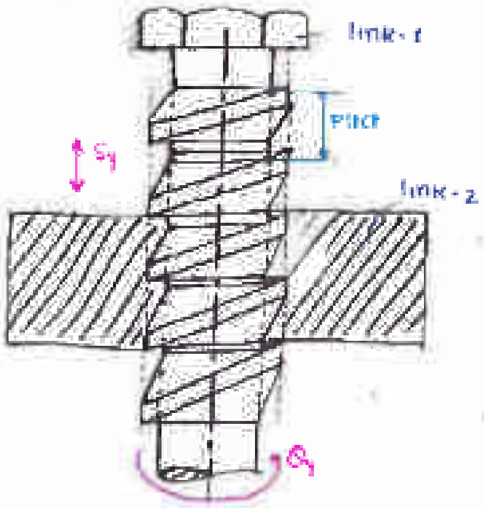
iii) Revolute pair [R - pair]

(Lower pair)



Ex Thrust bearing

$\boxed{dof = 1}$



$$\Delta S_y = f(\Delta\theta_y)$$

D.P.V I.P.V

$$\boxed{\frac{\Delta S_y}{\text{lead}} = \frac{\Delta\theta_y}{2\pi}}$$ GATE (SM)

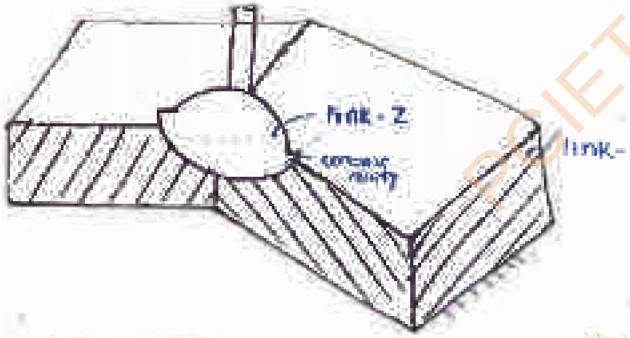
Ex: Nut & screw
power screw

Lead – The axial dis^n travelled by nut in complete rotation

pitch – Distance between two similar points on successive threads measured parallel to the pitch centroid axis

- Lead = pitch ⇒ single start thread
- Lead = 2×p ⇒ Double start thread

v) Spherical pair
(or) Globular pair (G - pair)



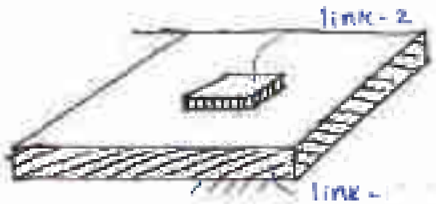
$\boxed{dof = 3}$

α_x
α_y
α_z

Ex: pendulum in common axis

Ex: Ball & socket joint
Joy stick

vi) Planar pair or Flat pair



$\boxed{dof = 3}$

Ex cube on surface

② on the basis of type of contact :

Kinematic pair

① Lower pair

- If there is area contact between the moving elements it is known as lower pairs.

Ex All above ex. are area contact

② Higher pair

- If there is point or line contact between the moving elements it comes under Higher pair

Kinematic Pair

Lower pairs

Linear motion pair

$\boxed{d.o.f = 1}$

Ex Revolute prismatic

Surface motion pairs

$\boxed{d.o.f > 1}$

Ex cylindrical spherical

Higher pairs

It

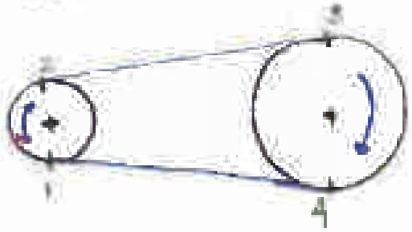
Wrapping pair

- If one link is wrapped over another link it is known as wrapping pair

$\boxed{d.o.f = 2}$

Ex Belt & pulley

NOTE :



- At every entry & exit to pulley Belt is doing rolling with slipping & hence it is examples of Higher pair.

$\boxed{\text{Total No. Higher pair} = 4}$

| Rolling = Translation + Rotation |

→ Higher pairs always restricted 1 D.O.F ← In plane total 3 = 2 tr, to Rotation = 1

→ All the surface motion pairs will violate Kutzbach eq^n

for Lower pair

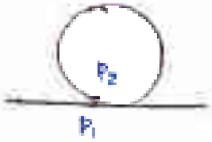
case-(a) link-1 is stationary
link-2 is moving
(pure translation)

$\boxed{K_{P_1} = \text{St. line}}$

case-(b) link-2 is stationary
link-2 is moving
(pure translation)

$\boxed{K_{P_1} = \text{St. line}}$

for Higher pair



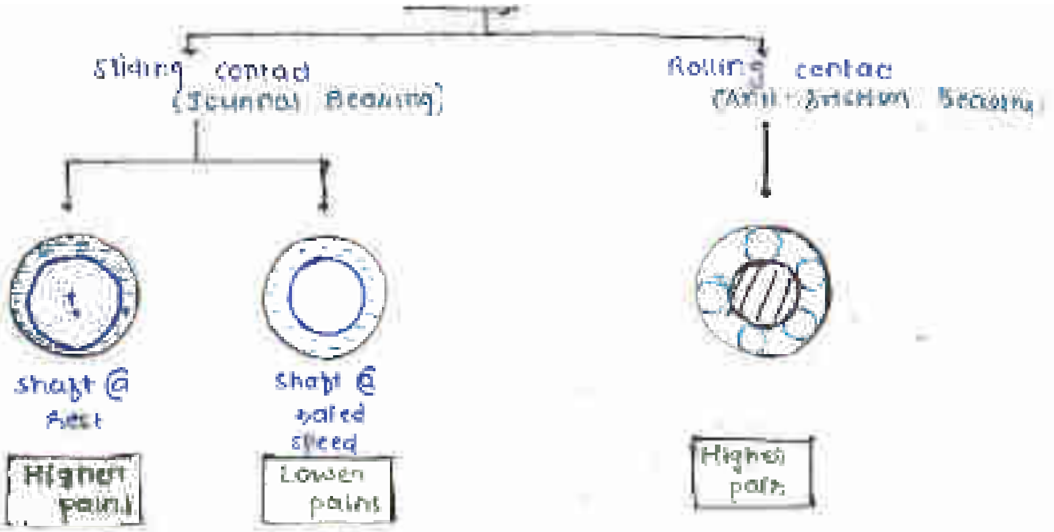
case-(a) : link-1 (st. line) is fixed
link-2 (circle) is moving
(pure rolling)

$\boxed{K_{P_2} = \text{cycloid} = K_{P_2}}$

case-(b) :- link-1 (circle) is fixed
link-2 (st. line) is moving
(pure rolling)

$\boxed{K_{P_1} = \text{involute} = K_{P_1}}$

	Lower pairs	Higher pairs
1)	Full joints	Half joints
2)	Area contact	point / Line contact
3)	can be inverted	can not be inverted
4)	More friction	Less friction
5)	Req^d more lubrication	less lubrication
6)	There will be more wear & tear due to friction	Higher pairs are subjected to more wear & tear under same max load
7)	Lower pair can withstand or hold more lubrication	Higher pair require less lubrication
		cam & follower



Note Bearings are not kinematic pair

- Bearings are mainly used to hold shaft in correct position & bear the load, it has nothing to do the transfer of relative motion. Hence Bearings are not kinematic pairs they are only pairs

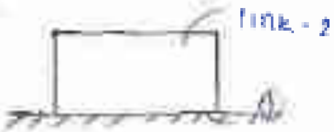
③ On the Basis of type of closure

(i) Closed pair

- If the link is completely entered in to another link
- The link which is inside another can not bring out without rupture of external link

(ii) Unclosed pair or open pair

- If pair is open in space



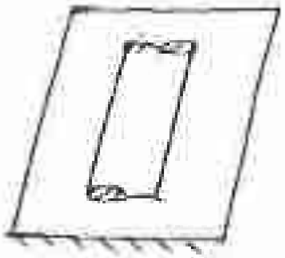
Ex

T	R
S_x	θ_x
S_z	θ_y
	θ_z

$\boxed{dof = 5}$

(iii) Slab on a surface :

Ex

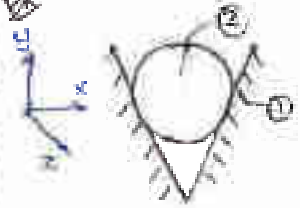


T	R
S_x	θ_y
S_z	θ_z

$\boxed{dof = 4}$

(iv) Cylindrical Bar in V-groove : (GATE)

Ex



Restricted

T	R
S_x	θ_x
S_y	θ_y

Restricted dof = 4

$\boxed{dof = 2}$

⇒ (*)

(i) Form closed pair

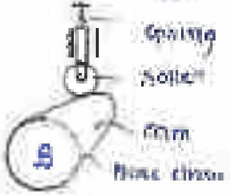
- If the contact that is formed between mating element due to geometrical specification the pair is known as formed closed pair.

Ex shaft & key
Nut & screw.

(ii) Force closed pair

- If the contact between mating element is due to some force either self wt of link or some external force (spring force) known as forced closed pair.

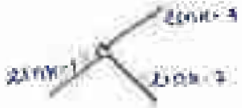
Ex cam & follower
(higher pair)



- Every link have minimum one or two nodes
- If two links are connected at one node it is known as binary joint

[: Binary Joint ;

- If three links are connected at one node is known as ternary joint



[Ternary Joint :

1 T Joint = 1 B + 1 B

1 T Joint ≡ 1 B Joint

- If four links are connecte

1 Q Joint ≡ 3 B Joint

NOTE

One the basis of motion between links, the kinematic classified pairs in 3 - categories

1) completely constrained pairs
- If the motion betn link is in unique dirn & and of unique type and does not depend on direction of force applied is an example of completely constrained pairs

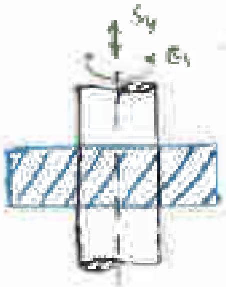
Ex: prismatic pair (P pair)

2) Incompletely constrained pairs
- If the motion is possible in more than one direction or more than one type it is known as incompleted constrained pairs

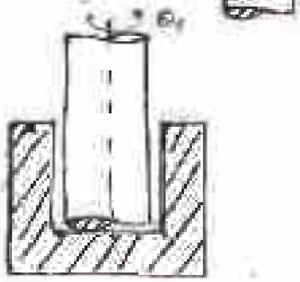
Ex: cylindrical pair (C - pair)

- If an incompletely constrained pair is converted into completely constrained pair either by applying some force or by changing the geometry or specification of mating element known as successfully constrained pair.

Eg - Rollers bearing
(Exclude pairs)
- piston - cylinder



shaft in radial bearing



foot step bearing

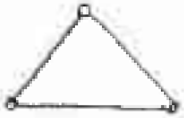
3

→ **Kinematic chain**

conditions :

1) 1st link should be connected to the last link directly or indirectly

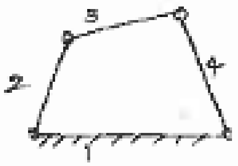
2) It should able to transform desired relative motion.



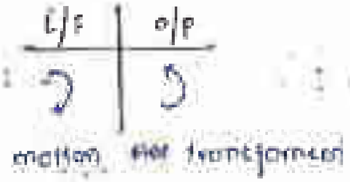
3 bar closed chain

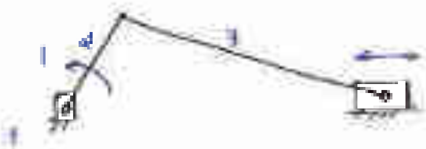
4 bar closed chain

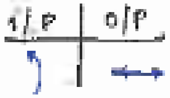
Mechanism :- If one link of chain is fixed and it is able to either transfer / transform f (or) both to the relative motion it is known as mechanism



four bar mechanism







Newton's 2nd Law
- Rate of change of momentum $F = \frac{d(mv)}{dt}$

5

→ **Machine**

- It is combination of various links and parts in such a manner that it is able to transfer / transform or both to the motion, force or power from some source to the load.

Ex I.C engine
Lathe M/C
Robots

Mechanism	vs Machine
- A mechanism is simple model for an complex machine It analogies to FBD (also for analysis in SOM - FBD Thermal system) - Several mechanism combined together may result in M/C - Ex clock (it not transform any energy only motion transform) Typewriter	- Machine consist of several mechanism. Hence we can say every machine considered as mechanism but every mechanism need not be M/C always

[illegible]

- 1 link is having 3 dof (in planar chain & Mechanism)
- If there are 'n' no. of links.

$$\boxed{\text{Total no. of dof} = 3n}$$ (for 'n' no. of links)

- Let us suppose, there are 'j' lower pairs (linear motion pair $\boxed{dof = 1}$)
  ↳ equivalent no. of binary joints

→ Restricted d.o.f. due to linear motion pair = 2 (binary joints)

$$\boxed{\text{total restricted dof} = 2j}$$

Actual dof = max^m possible dof − max^m restricted dof

$$\boxed{dof = 3n - 2j}$$

⇒ **Effect of higher pairs**

- each higher pair restricted one dof. let there is 'h' no of higher pairs
- Hence dof restricted by 'h' higher pairs = h

$$\boxed{dof = 3n - 2j - h} \Leftarrow \text{Chain}$$

In Mechanism

In mechanism one link is fixed / ground / frame

$$dof = 3n - 2j - h - 3$$

$$\boxed{dof = 3(n-1) - 2j - h} \Leftarrow \text{for Mechanism}$$

It is "Kutzbach Equation".

dof < 0 → super structure / indeterminate structure

dof = 0 → structure / frame / Truss

dof ≥ 1
- d.o.f = 1 ⇒ kinematic / constrained mechanism
- d.o.f > 1 ⇒ Unconstrained Mechanism

→ **physical Interpretation d.o.f :-**

- Degree of freedom predicts No. of path variable or No. of equations required between input & output motion.
- Degree of freedom predicts about the No. of links that should be controlled by input (or No. of path variable that should be controlled) in order to have constrained Mechanism.

Grubler's criterion

$\boxed{dof = 1}$
$\boxed{h = 0}$ ← constrained mechanism

$\therefore 3(n-1) - 2j - 0 = 1$
$3n - 3 - 2j = 1$
$\boxed{3n = 2j + 4}$

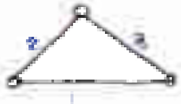
$\boxed{n_{min} = 4}$

$3n$ = even no
$2j$ = even
4 = even

n = 1 ×
n = 2 ×
n = 3 (odd)
n = 4 ✓

- According grubler's criterion to make a mechanism consist of all the lower pair having dof = 1 is actual 4.

Ex. 1

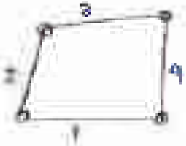


$f = 3(n-1) - 2j - h$
$= 3(3-1) - 2(3)$
$= 0$

- If is chain (not a mechanism) is one link is fixed then become struc.

n = 3
j = 3
h = 0
$\boxed{dof = 3n - 2j - h}$ (chain)
$= 9 - 6$
$= 3$

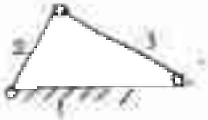
Ex. 2



$f = 3(4) - 2(4)$
$\boxed{f = 4}$

n = 4
j = 4
h = 0

3



- Frame one link is fixed so it mechanism after is

$dof = 3(n-1) - 2j - h$

$dof = 3(3-1) - 2(3) - 0$
$\boxed{dof = 0}$

n = 3
h = 0
j = 3

It is structure / truss (dof = 0) / frame used to transfer a load

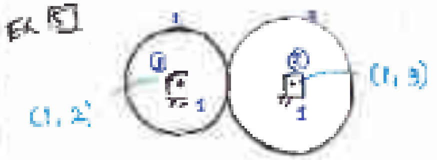
$j = 4$
$h = 0$

$\boxed{d.o.f = 1}$ ⇒ It is kinematic mechanism

NOTE
- It is a constraint mechanism
- d.o.f = 1 ⇒ only one eqn required betn input & output
- d.o.f = 1 ⇒ only one link must controlled by input in order to have constrained mechanism.

NOTE :
- If '1' is subtracted from the d.o.f freedom from any chain & any answer comes '1' then only that chain could be called as "kinematic chain"

Ex. 5



$n = 3$
$j = 2$
$h = 1$

$3(3-1) - 2(2) -$
$6 - 4 - 1$

$f = 3(n-1) - 2j - h$
$= 3(3-1) - 2(2) - 1$
$\boxed{f = 1}$ ← constraint &
seqn & eqn

Ex 6

Belt & Rope

$n = 4$
$j = 2$
$h = 4$
$f = 3(4-1) - 2(2) - 4$
$\boxed{f = 1}$

Ex. 7

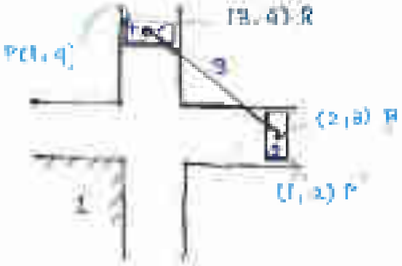
single slider

(1, 4)
prismatic

$n = 4$
$j = 3R + 1P = 4$
$h = 0$

$f = 3(4-1) - 2(4) - 0$
$\boxed{f = 1}$

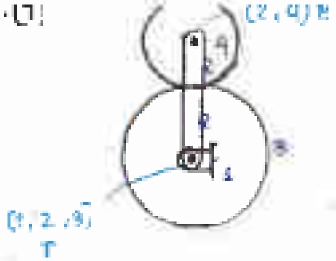
Ex. 8



Double Slider

$n = 4$
$j = 2P + 2R = 4$
$h = 0$
$f = 3(4-1) - 2(4) - 0$
$\boxed{f = 1}$

Ex. 17



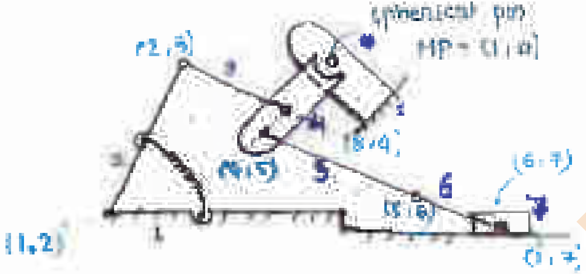
j = [illegible] + LT
= 1 [illegible] + 1 6 + 1 [illegible] = 3
h = 1
dof = 3 (4 − 1) − 2(3) − 1
$\boxed{dof = 2}$

It is an unconstrained mechanism
d.o.f = 2 ⇒ Therefore two eq^n are required between input & output that is two links are working as output link

	i/p	o/p
unconstrained d.o.f = 2	3	2, 4
constrained d.o.f = 1	3	either 2 or 4

⇒ In an epicyclic gear train, arm will always be connected either with some input or some output

Ex. 15



n = 8

n = 7
j = 7
h = 1
d.o.f = 3(7 − 1) − 2(7) −
$\boxed{dof = 3}$
3 eq^n required

⇒ Kinematic Diagrams of various link

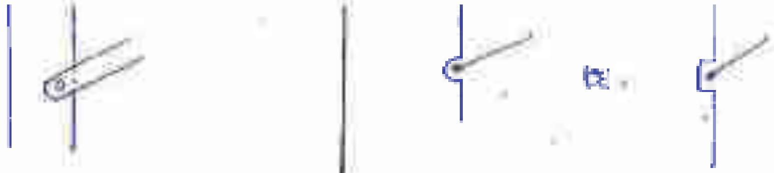
Actual | Kinematic

(a)

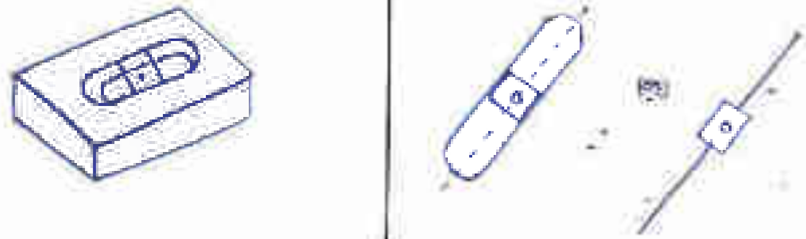
(b)

(c)

c)



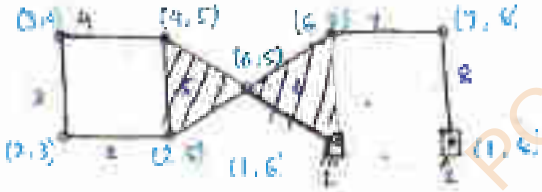
d)



e)

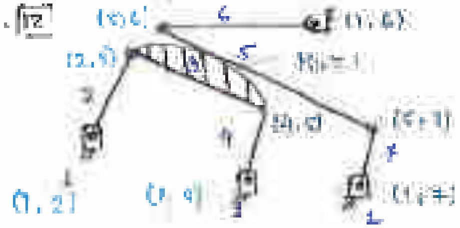
f)

Ex. 11



n = 8
j = 9
3(8 − 1) − 2(9)
21 − 18
$\boxed{d.o.f = 3}$

Ex. 12



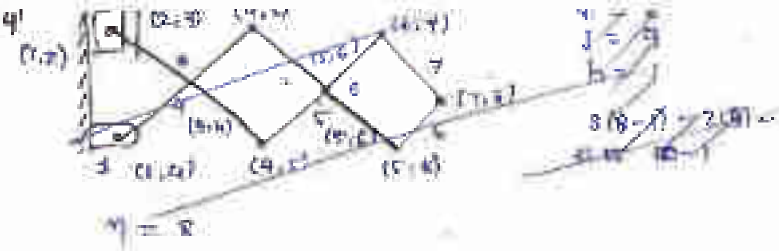
n = 7
j = 8
hp = 1
3(7 − 1) − 2(8) − 1
18 − 16 − 1
$\boxed{d.o.f = 1}$

Ex. 13

n = 4
j = 3
hp = 1
9 − 6 − 1
$\boxed{f = 2}$

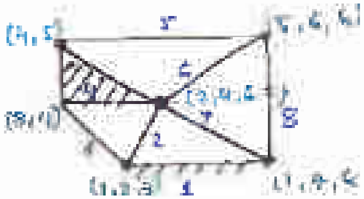
3(4 − 1) − 2(3)
9 − 6 − 1

14'



14

15



n = 8
j = 2B + 3T + 1Q
= 2 + 6 + 3
= 11
F = 21 − 22 = $\boxed{-1}$

⇒ Exceptions to the Kutzbach equations

$$\boxed{D.O.F = 3(n-1) - 2j - h_p}$$

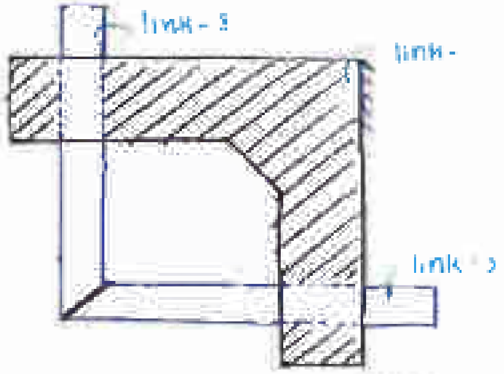
- Kutzbach equation is only valid for planer mechanism that is in which different point of different link move in parallel planes & consist of mainly Revolute pair and Prismatic pair.
- There are some exceptions when Kutzbach equation get violated, in this cases we should use modified Kutzbach equation

★ Modified Kutzbach Equation :

$$\boxed{D.O.F = 3[n - n_r - 1] - 2[j - j_r] - h - F_r}$$

Where :
n = total no. of links
n_r = total no. of redundant link
j = total no. of binary pair joint
j_r = total no. of redundant joint
h = no of h.p
F_r = redundant dof

in all the mechanism, it consist of 3 links [illegible] having lower pairs only



prismatic pairs - 3

$n = 3$

$j = 3$

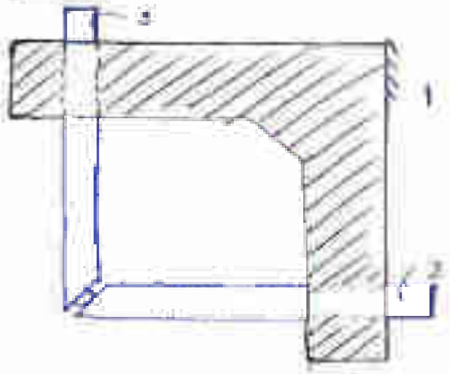
$h = 0$

$f = 3(n-1) - 2j - h$

$= 3(3-1) - 2(3) - 0$

$\boxed{f = 0}$

- It means it is a structure, none of the given linkage is able to transfer the relative motion from link - 1 to link - 2



$n = 3$

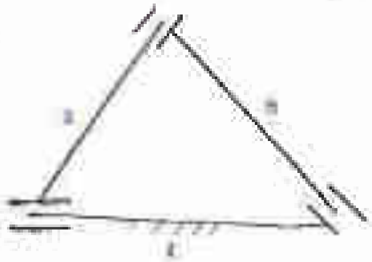
$j = 2$

$h_p = 1$

$f = 3(n-1) - 2j - h_p$

$= 3(3-1) - 2(2) - 1$

$\boxed{f = 1}$



$\boxed{f = 1}$

Case-(ii): If a mechanism consist of redundant link:

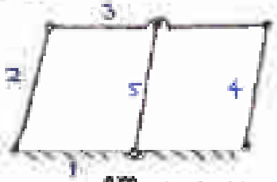
parallelogram linkage

$l_1 = l_3$ and $l_1 \parallel l_3$

$l_2 = l_4$ and $l_2 \parallel l_4$

critical position or uncertainty configuration

(i) All the links will become co-linear which leads to decrement in rigidity & chance of failure will be maximum corresponding to it.
In order to prevent the failure; corresponding to uncertainty configuration; we use redundant link in mechanism & it should be connected to parallel same link.



Double ||gm Linkage

simple Kutzback eq

$n = 5$
$j = 6$
$h = 0$

$f = 3(5-1) - 2(6) - 0$

$\boxed{f = 0}$

Modified Kutzback

$D.O.F = 3[n - n_r - 1] - 2(j - j_r] - h - f_r$

$= 3[5-1-1] - 2[6-2] - 0 - 0$

$\boxed{f = 1}$

→

$\boxed{D.O.f = 1}$

NOTE:

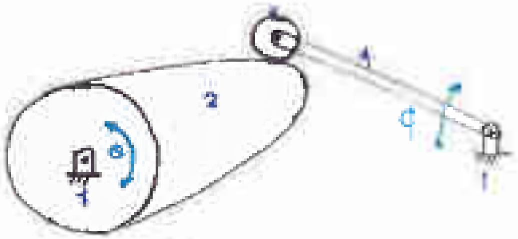
E/m linkage having $\boxed{f = 1}$

→

$\boxed{f = 0}$

because link '5' is not ||' to 2 & 4

Case-(iii): The mechanism which consist of redundant pair



Oscillating cam & follower mechanism

$n = 4$
$j = 3$
$h = 1$

$f = 9 - 6 - 1$

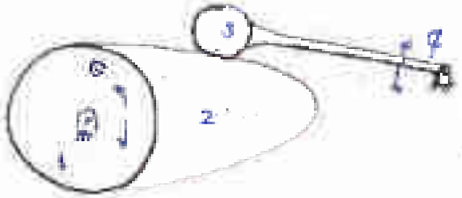
$\boxed{f = 2}$

$\theta = f(\phi)$ { θ = [illegible] of cam
{ ϕ = oscillation of follower

Hence, we require only one equation between input & output therefore d.o.f for cam & follower is actually '1'.

Explanation (a):

- If we welded the follower's joint 3



$f = 3(3 - 1) - 2(2) - 1$

$\boxed{f = 1}$

- D.o.f by modified Kutzback eq'

$f = 3[n - n_L - 1] - 2[j - j_R] - h - F_R$

$= 3[4 - 1 - 1] - 2[3 - 1] - 1$

$\boxed{f = 1}$

Explanation (b):

- Mechanism consist of redundant degree of freedom. If pair would be redundant like '3' but not joint redundant

$f = 3[n - n_L - 1] - 2[j - j_R] - h - F_R$

$= 3[4 - 0 - 1] - 2[3 - 0] - 1 - 1$

$\boxed{f = 1}$

- The spinning motion of roller is redundant, hence cam & follower mechanism consist of one redundant d.o.f.
- cam & follower are constraint mechanism.

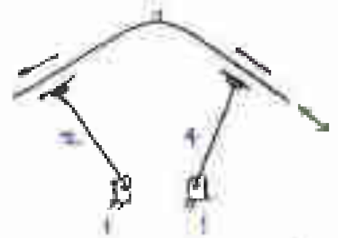
Case-(iv): Mechanism consisting cylindrical motion pair (or) redundant d.o.f.

cylindrical motion is d.o.f > 1

$d.o.f = 3[n - n_L - 1] - 2[j - j_R] - h - F_R$

$= 3[4 - 0 - 1] - 2[4 - 0] - 0 - 1$

$\boxed{f = 0}$



d.o.f =

Note:

(i) According to Grubler's criteria min. no. of links required to make a mini mechanism is '4'

(ii) A mechanism which consist of all the '3' prismatic pair is possible (case-(i))

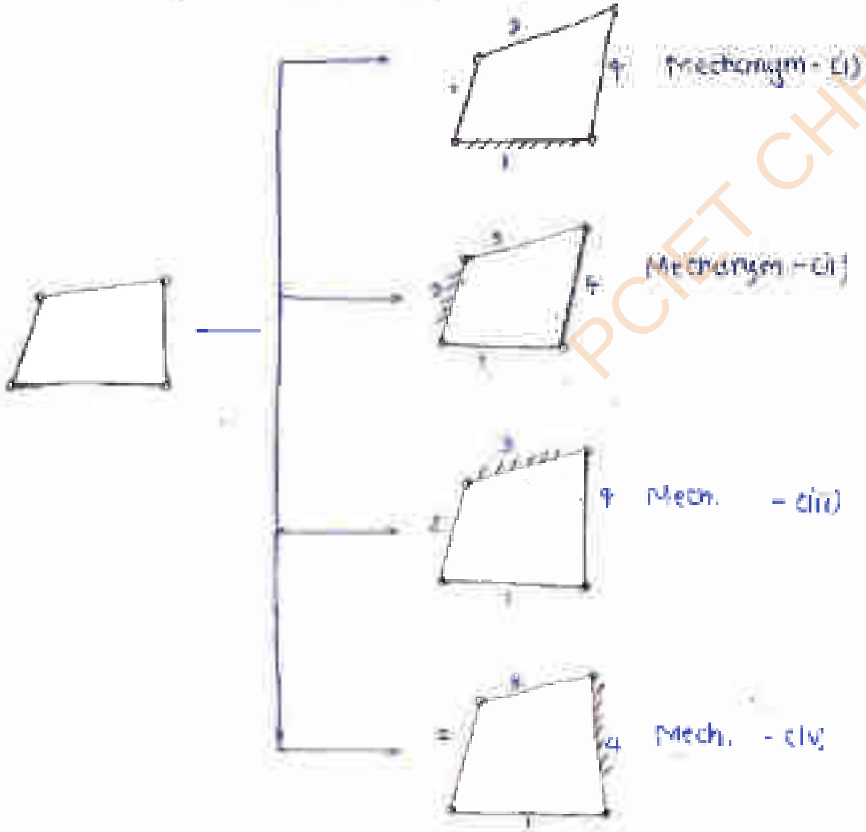
(iii) Minimum '3' links are required to make a mechanism consisting of atleast one higher pair.

Eg Gear & pinion mechanism

⟹ Inversion of a Mechanism:

(purpose of inversion is to analyse easily the mechanism problem)

→ The process of fixing different links of a mechanism is known as inversion of mechanism



- If there are 'n' no. of links then possible inversion will also be n.
- Inversion of mechanism does not change the relative motion between their links it is the characteristic of parent kinematic chain but inversion do affect the absolute motion various links.
- Inversion are used to make the problem simple in some c
  Eg. cam & follower mechanism
  sun & planet gear train
  → Higher pair can not be inverted.

⇒ Range of Movement :-
  (a) Grashof's Law :-

- On the basis of type of movement the links are classified as followed.
  i) Frame / fixed link
  → which does not move.
  ii) Crank :
  → The link which is able to execute full circular motion or which can rotate completely
  iii) Rocker / Lever :
  → The link which can not rotate completely that is the one wh is oscillate.
  iv) Coupler
  → the link which is opposite to frame of the link which connects input to the output
- On the basis of relation between dimensions of various links 4-bar mechanisms are classified in 3 categories

⇒ class - I Linkage

$$\boxed{l_{min} + l_{max} < p + q}$$ Grashof's Linkage

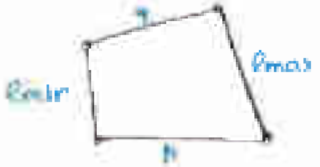
⇒ class - II Linkage

$$\boxed{l_{min} + l_{max} > p + q}$$ Non Grashof's Linkage (NGL)

⇒ class - III Linkage

$$\boxed{l_{min} + l_{max} = p + q}$$ Transition linkage or special Grashof's linkage

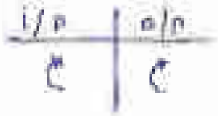
- The position of shortest link is always decide the type of inversion in Grashof linkage.



Kinematic
~~pair~~
chain

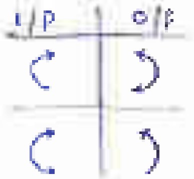
→ Inversion - (a) : If shortest (lmin) link is fixed

- The input & output both will be able to execute full circular motion.



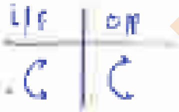
Crank - crank
(or)
Double crank
(or)
Drag link mechanism

→ Inversion - (b) : If shortest (lmin) is adjacent to fixed



Crank Rocker
(or)
Rocker crank

→ Inversion - (c) : If shortest (lmin) link is opposite to the fixed (or) lmin is coupler.



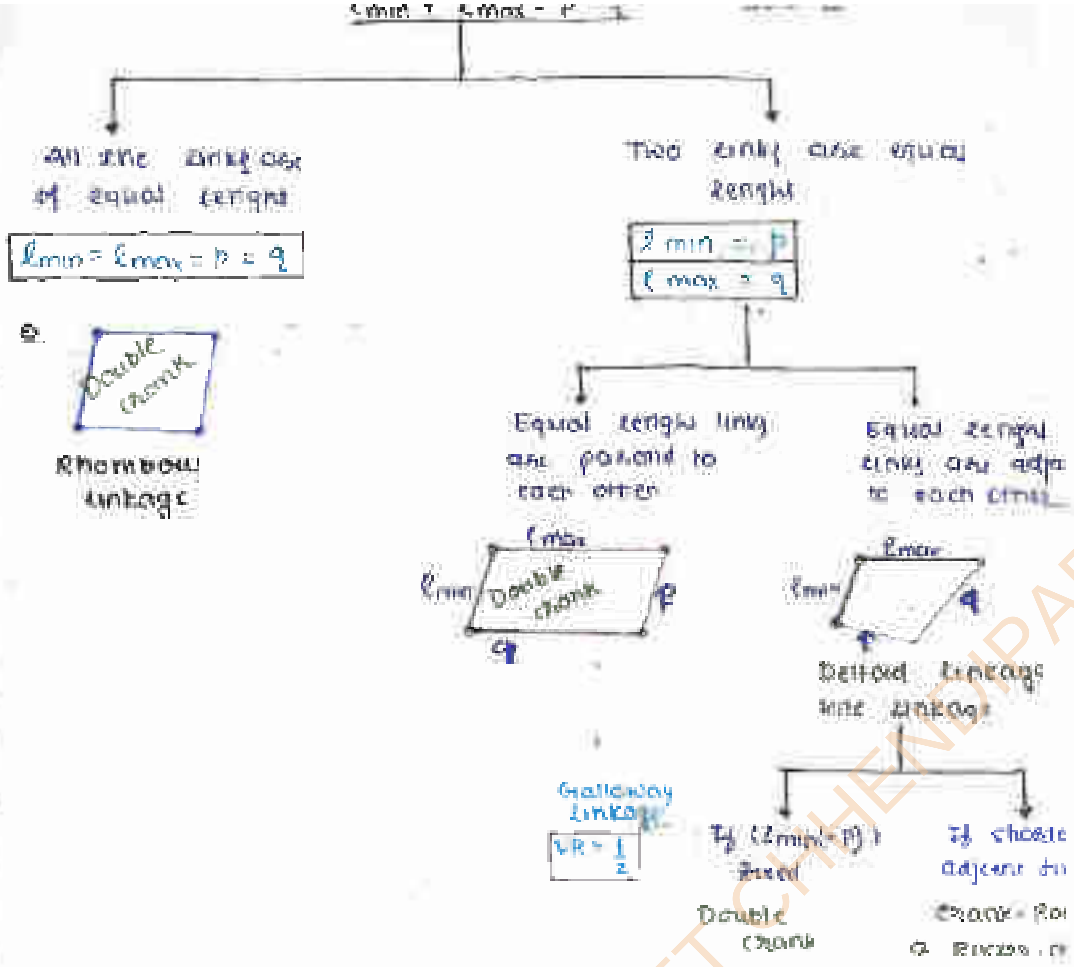
Rocker Rocker
(or)
Double Rocker
(or)
Lever mechanism

⇒ Inversion of class - II linkage :

- All the possible inversion of non grashof's linkage double rocker only

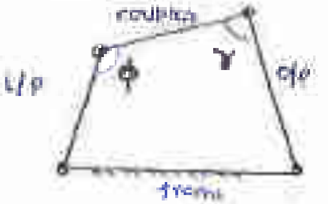
⇒ Inversion of class - III linkage

- inversion of transmission linkage will follow the inversion of Grashoff linkage.



⇒ Transmission angle [γ]

- It is the parameter to indicate the effectiveness of lower pair mechanism
- It is acute angle (0 < 90) formed between coupler & output link

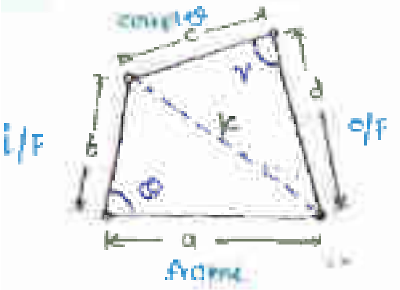


⇒ Pressure angle (ϕ)

→ It is the angle between input & coupler.

→ pressure angle is mainly used as a parameter of effectiveness in higher pair mechanism

here ε follows_



$$k^2 = a^2 + b^2 - 2ab\cos\theta$$

$$k^2 = c^2 + d^2 - 2cd\cos\gamma$$

$$a^2 + b^2 - 2ab\cos\theta = c^2 + d^2 - 2cd\cos\gamma$$

$$\boxed{2cd\cos\gamma = c^2 + d^2 - a^2 - b^2 + 2ab\cos\theta}$$

$$\gamma = f(\underbrace{a,b,c,d}_{const}, \theta)$$

→ $\gamma = g(\theta)$

for max^n or min^n

$$\frac{d\gamma}{d\theta} = 0$$

⇒ $2cd(-\sin\gamma)\frac{d\gamma}{d\theta} = 0 + 2ab(-\sin\theta)$

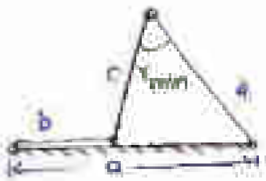
$$\boxed{\frac{d\gamma}{d\theta} = \frac{ab}{cd}\cdot\frac{\sin\theta}{\sin\gamma}}$$

→ $\frac{ab}{cd}\frac{\sin\theta}{\sin\gamma} = 0$

⇒ $\frac{\sin\theta}{\sin\gamma} = 0$

$\boxed{\sin\theta = 0}$ ⇒ $\boxed{\theta = 0^\circ}$ $\boxed{\theta = 180^\circ}$

for $\theta = 0^\circ$:



$$(a-b)^2 = c^2 + d^2 - 2cd\cos\gamma_{min}$$

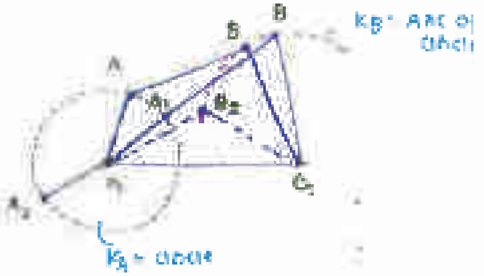
for $\theta = 180^\circ$

$$(a+b)^2 = c^2 + d^2 - 2cd\cos\gamma_{max}$$

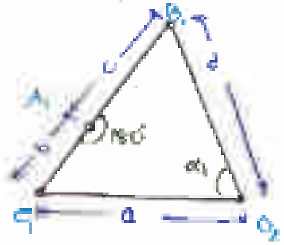
Note :

(i) $\theta = 0^\circ$ & 180° leads to min. transmission angle and it is possible either in double crank mechanism or crank rocker mechanism

toggle position



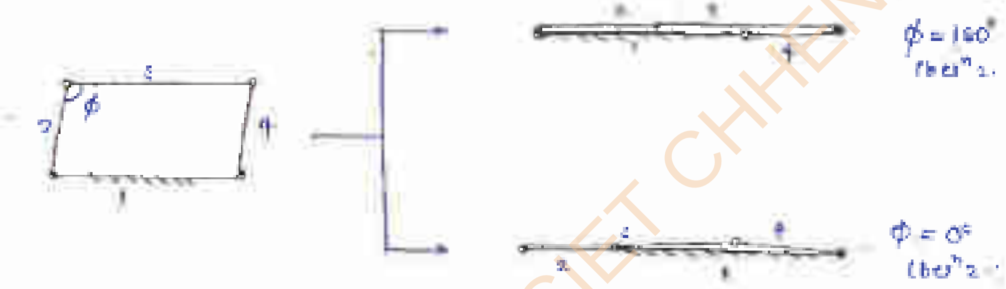
for $\phi = 180°$



for $\phi = 0°$

NOTE:

In parallelogram linkage



Mechanical Advantages [MA]

- It is analogous to efficiency of the engine
- Mechanical advantages are defined of the ratio of torque at output link to the input link torque

$$MA = \frac{\text{Torque @ o/p}}{\text{Torque @ i/p}}$$

$$MA = \frac{T_4}{T_2}$$

- If there is no power loss ;

power @ i/p = power @ o/p

$$T_{in}\,\omega_{in} = T_{out}\,\omega_{out}$$

$$\boxed{\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = MA}$$

MA ω_{in}

$\boxed{VR = \frac{\omega_{out}}{\omega_{in}} \quad <}$ ⇒ $\boxed{MA = \frac{1}{VR}}$

→ $MA = f\left[\frac{\sin\gamma}{\sin\phi}\right]$

→ Corresponding to toggle position $MA = \infty$

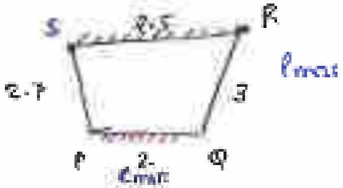
Hook joints (IES)

→ It is universal joint
→ It is spatial mechanism (3D) is
→ Non parallel - non planar)

CRO-4)

$\boxed{VR = 1}$

(5)



It is class - (2) problem

$l_{min} + l_{max} = 2 + 3 = 5$

$p + q = 2.7 + 3 = 5.7$

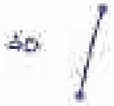
$l_{min} + l_{max} < p + q$

Shortest link

$\boxed{RS}$

(6)

$l_f = 20$ ←
$l_{in} = 40$
$l_o = 50$
$l_{ow} = 60$



$l_{min} + l_{max}$ □ $p + q$

$20 + 60 < 50 + 40$

Class - I problem.

Shortest link will decide ⇒ full circle

10)

$\ell_c = 20$ ✓

$\ell_{max} = 40$

$\ell_{max} = 60$ ✓

$K_{min} + K_{max} \square P + Q$

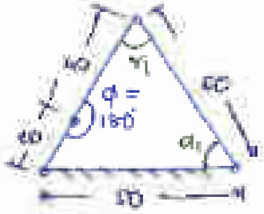
$40 + 60 \square 50 + 40$

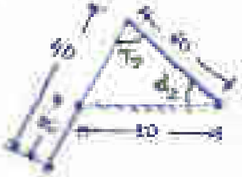
$60 < 90$

class-①

→ Fixed link → extreme position

$\phi = 0^\circ$ or $\phi = 180^\circ$





$\phi = 180^\circ$

$(20+40)^2 = (50)^2 + (60)^2 - 2(50)(60)\cos\alpha_1$

$\boxed{\alpha_1 = 65.3^\circ}$

$(40-20)^2 = (50)^2 + (60)^2 - 2(50)(60)\cos\alpha_2$

$\boxed{\alpha_2 = 18.19^\circ}$

max transmission angle

$\gamma_1 = (?)$

$(50)^2 = (20+40)^2 + (60)^2 - 2(60)(60)\cos\gamma_1$

$\boxed{\gamma_1 = 49.346^\circ}$

for $\gamma_2 = (?)$

~~$(40-20)^2 - 50^2$~~ $+ 60^2$

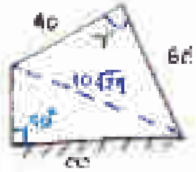
$50^2 = (40-20)^2 + 60^2 - 2(20)(60)\cos\gamma_2$

$\boxed{\gamma_2 = 51.8^\circ}$

for maximum angle rock to oscillating

$\alpha_1 - \alpha_2 = 47.18'$

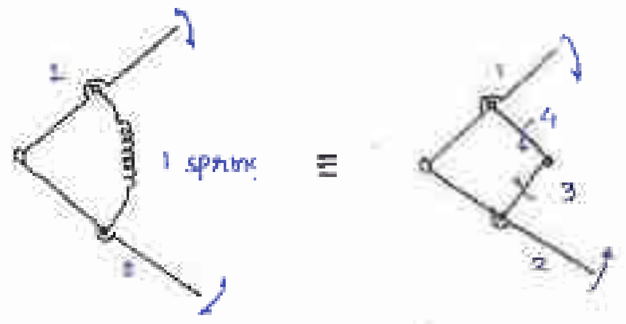
total dist^n travell by rocker = $2(\alpha_1 - \alpha_2)$



$(10\sqrt{29})^2 = (40)^2 + (60)^2 - 2(40)(60)\cos\gamma$

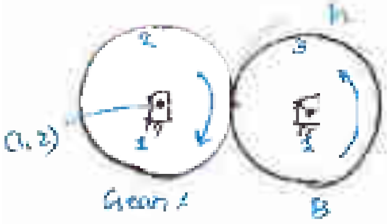
$\boxed{\gamma = 61.34}$

[illegible]



1 spring = 2 Binary link

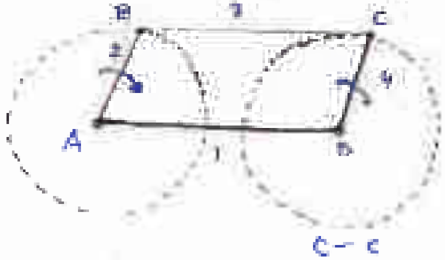
→ Higher pairs

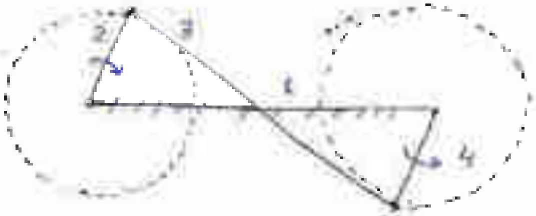


n = 3
j = 2
h = 1

Double - crank
or
Drag link mechanism

→ convert in parallelogram

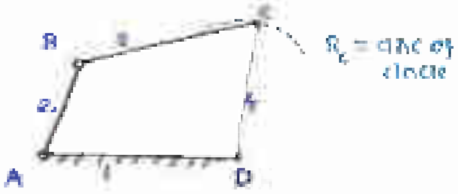




n = 4
j = 4
h = 0

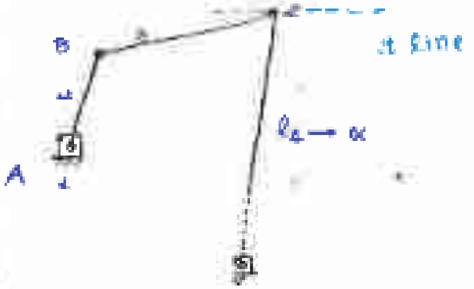
1 pair = 1 link + 2 Binary joint

1 higher pair = 2 lower pair



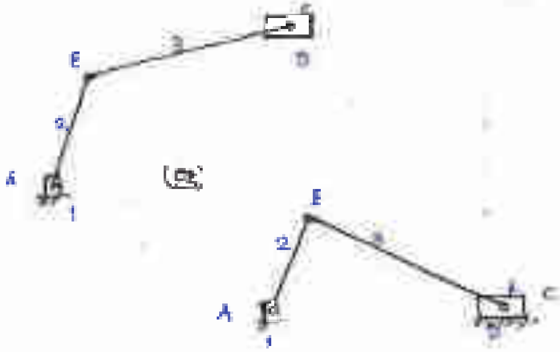
R_c = size of circle

↓ C will oscillate & for making it straight line



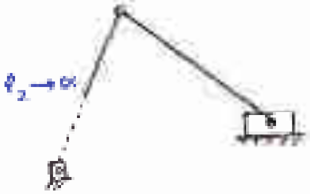
st line

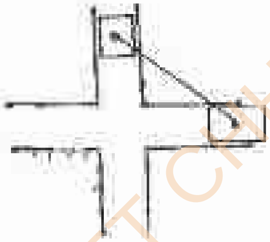
$l_4 \to \infty$



→ since straight line is a part of circle whose center lies at infinite.

→ **Double slider mechanism :-**





[*] Inversion single slider mechanism :

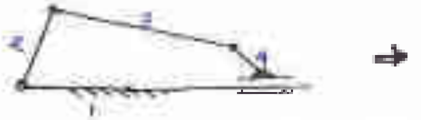
Kinematic chain

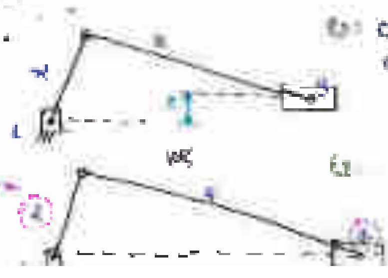
$l_{min} + l_{max}$ □ $p + q$

$l_2 + \infty$ [<] $l_3 + \infty$

so it belong's class I mechanism

Inversion-(I) Link 1 is fixed



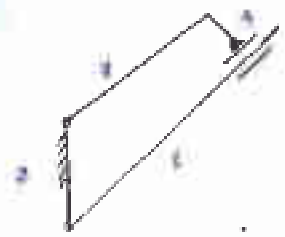


(1) offset slider crank chain

(2) IC engine, Reciprocating compressor

→ If input link 2 it becomes compressor

or



I/P	O/P
ζ	ζ

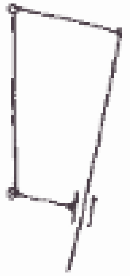
Ex: With worth Quick Return mechn

Inversion - (iii) If link - 3 is fixed i.e. CR. is fixed

crank - rocker

→

Ex: oscillating cylinder engine mechanism



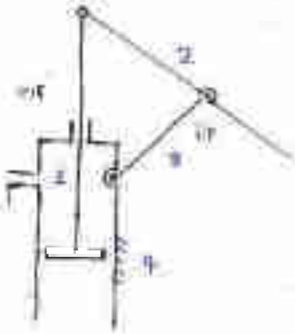
(or

Ex: crank & slotted bar QRM

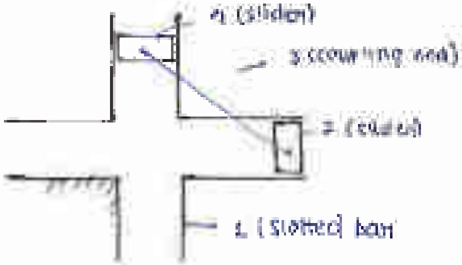
Inversion - (iv) If slider fixed i.e. link 4 is fixed

Rocker - Rocker

⇒

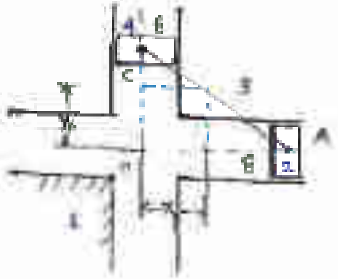


Ex: Hand pump



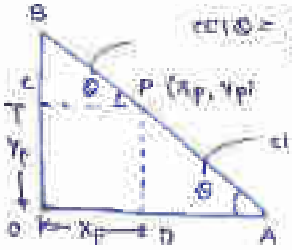
$$l_{min} + l_{max} < p + q$$
$$l_{min} + \infty < \infty + \infty$$

Inversion - (I) Link-1 i.e., slotted bar is fixed



Rocker - Rocker / Lever mechanism

Here Rocker 1's opposite link-3 is fixed becomes Rocker - Rocker



$\cos\theta = \frac{PC}{BP} \Rightarrow \cos\theta = \frac{x_p}{BP}$

$\sin\theta = \frac{PD}{PA} \Rightarrow \sin\theta = \frac{y_p}{AP}$

$$\boxed{\left(\frac{x_p}{BP}\right)^2 + \left(\frac{y_p}{AP}\right)^2 = 1}$$

→ Locus of P = ellipse

→ Elliptical trammel

→ Special case

[A]

K_1 = Ellipse

→ K_2 = ellipse

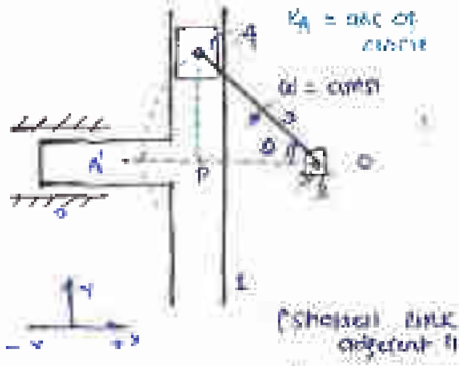
[B] If P is midpoint of AB :

$AP = BP \Rightarrow \frac{x_p^2}{(AP)^2} + \frac{y_p^2}{(AP)^2} = \Rightarrow \boxed{x_p^2 + y_p^2 = (AP)^2}$

→ circle

→ Kp − straight line

→ inversion (ii): If slider is fixed. (Scotch-yoke mechanism)



K_A = acc of crank

ω = const

(slotted link adjacent link)

displacement of yoke bar

$$x = PA'$$
$$= OA' - OP$$
$$= OA - OA\cos\theta$$
$$\boxed{x = OA(1-\cos\theta)}$$

→ crank - rocker (link-4 is fixed link-1 is adjacent to fixed is crank rocker)

→ velocity (v):

$$v = \frac{dx}{dt} = \frac{d}{dt}\{OA(1-\cos\theta)\}$$
$$= OA\left[0-(-\sin\theta)\frac{d\theta}{dt}\right]$$
$$\boxed{v = OA\cdot\omega\cdot\sin\theta}$$

→ Accⁿ (a).

$$a = \frac{dv}{dt} = OA\ \omega\cdot\cos\theta\ \frac{d\theta}{dt}$$

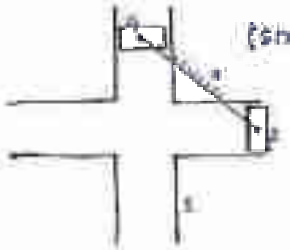
~~$a = OA\cdot\omega^2\sin\theta$~~

$$\boxed{a = OA\cdot\omega^2\cdot\cos\theta}$$

Scotch yoke mechanism

$$\boxed{\text{SHM} \Leftrightarrow \text{oscillation}}$$

[illegible]



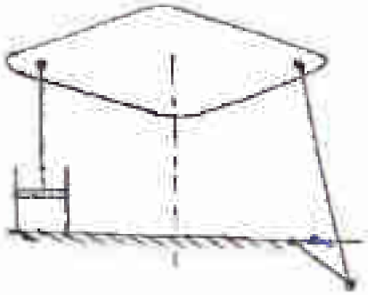
Oldham coupling

→ Double crank

- Oldham's coupling is used for connect two shaft which are having parallel misalignment

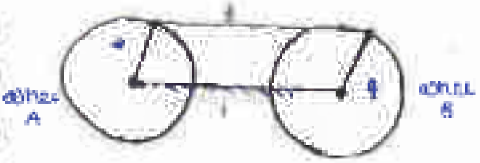
⟹ Inversion of simple four bar mechanism

Inversion - (a)



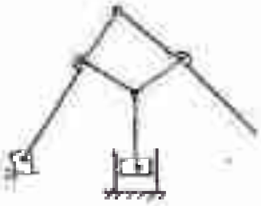
Ex Beam engine (or) watt engine

Inversion - (b)



coupling rod of locomotive

Inversion - (c)



watt's indicator diagram

- A QRM is mechanism in which return stroke consumes less time than cutting stroke since return stroke is ~~[illegible]~~ idle & it should occur as fast as possible where cutting is main working stroke & maximum energy consumption occurs during this stroke.
- For all QRM we define a quick return ratio as follows

$$\boxed{Q.R.R. = \frac{\text{time required in cutting stroke}}{\text{time required in return stroke}}}$$

→ If angular speed of driver is constant

i.e. $\theta = \omega t \rightarrow \boxed{\theta \propto t}$

$$QRR = \frac{\text{Angular dis}^n \text{ travelled in cutting stroke } (\alpha)}{\text{Angular dis}^n \text{ travelled in return stroke } (\beta)}$$

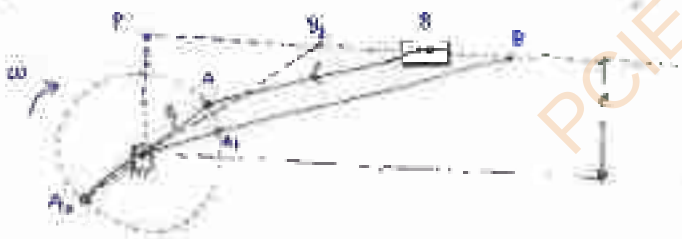
$$\boxed{QRR = \frac{\alpha}{\beta} > 1}$$

Note :

If QRR given less than <1

then $\boxed{QRR = \frac{\beta}{\alpha} < 1}$

Offset slider crank quick return mechanism :



Stroke length $= B_1B_2$

$= PB_1 - PB_2$

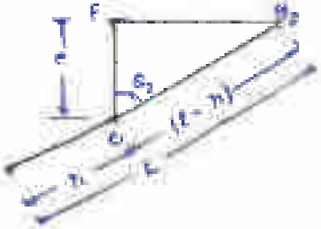
In ΔOB_1P

$\sin\theta_1 = \frac{B_1P}{OB_1} \Rightarrow B_1P = OB_1 \sin\theta_1$

$\boxed{B_1P = (l+r)\sin\theta_1}$

$\cos\theta_1 = \frac{OP}{OB_1}$ ≈

$\boxed{\cos\theta_1 = \frac{e}{l+r}}$



$\sin\theta_2 = \frac{PB_2}{OB_2} = \frac{PB_2}{l-r}$

$$\boxed{PB_2 = (l-r)\sin\theta_2}$$

$$\boxed{\cos\theta_2 = \frac{e}{l-r}}$$

$\Rightarrow$ $\boxed{\text{stroke length} = (l+r)\sin\theta_1 - (l-r)\sin\theta_2}$

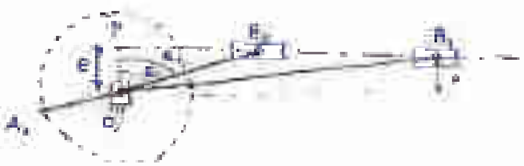
→ $QRR = \frac{\text{angle turned in cutting stroke}}{\text{angle turned in return stroke}}$

$$\boxed{QRR = \frac{180° + \phi}{180° - \phi}}$$

where; $\phi = \theta_1 - \theta_2$

PRO - 10)

$r = 20$ cm
$l = 40$ cm
$e = 10$ cm



$\cos\theta_1 = \frac{e}{l+r} = \frac{10}{20+40}$

$$\boxed{\theta_1 = 80.40°}$$

$PB_1 = (l+r)\sin\theta_1$
$= (20+40)\sin 80.40$
$PB_1 = 59.16$ cm

$\cos\theta_2 = \frac{e}{l-r} = \frac{10}{40-20}$

$$\boxed{\theta_2 = 60°}$$

$PB_2 = (l-r)\sin\theta_2$
$= (40-20)\sin 60$
$= 17.32$

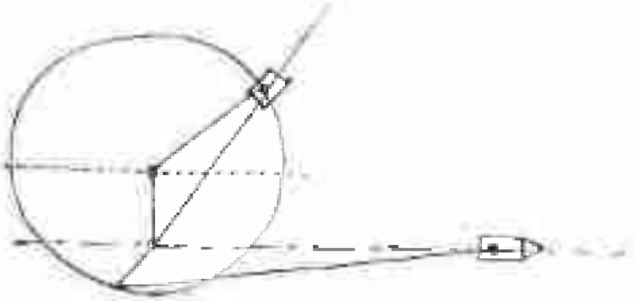
$\text{stroke} = PB_1 - PB_2$

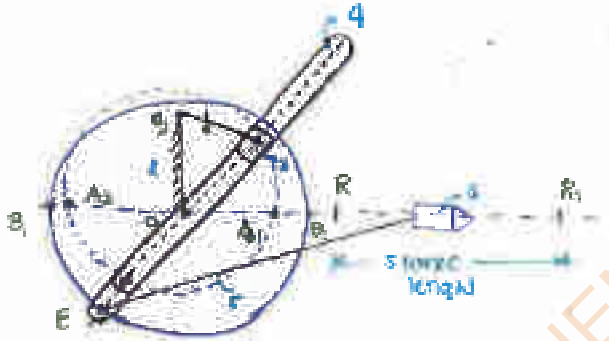
$$\boxed{\text{stroke} = 41.84}$$

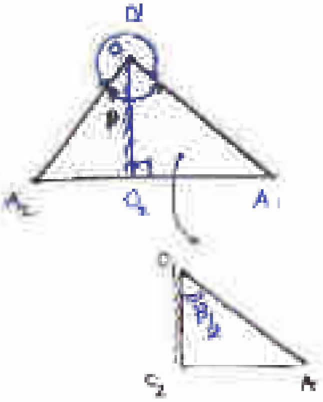
— $QRR = \frac{180° + \phi}{180° - \phi} = \frac{180° + 20.405}{180° - 20.405}$
$= \frac{200.405}{159.595}$

$$\boxed{QRR = 1.25}$$

$\phi = \theta_1 - \theta_2$
$= 80.40° - 60$







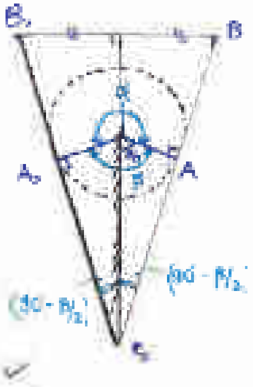
$QRR = \frac{\alpha}{\beta} \ (>1)$

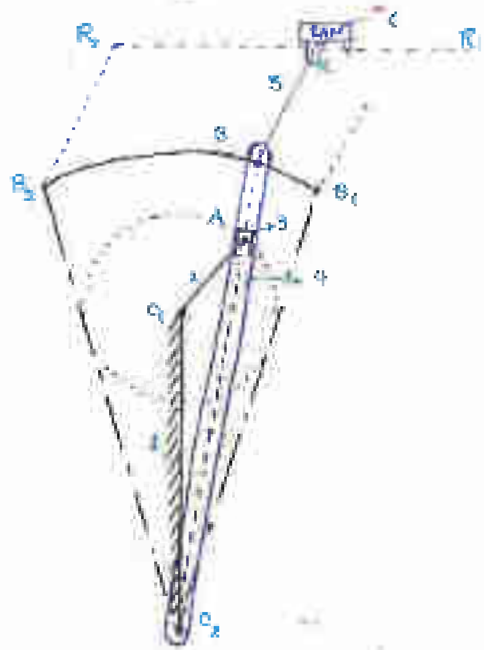
$\alpha + \beta = 360$

$\cos\frac{\beta}{2} = \frac{O_1O_2}{O_1A}$

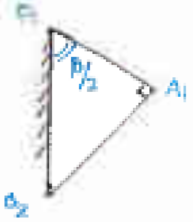
$$\boxed{\cos\frac{\beta}{2} = \frac{\text{Fixed link length}}{\text{driving crank length}}}$$

Stroke length = R_1R_2
= B_1B_2





→ Extreme position



$\cos \beta/2 = \frac{O_1A_1}{O_2A_1} = \frac{O_1A_1}{O_1O_2}$

$$\boxed{\cos \frac{\beta}{2} = \frac{\text{crank length}}{\text{fixed link length}}}$$

$$\boxed{QRR = \frac{\alpha}{\beta} > 1}$$

; α + β = 360

→ Stroke length :

stroke length $= R_1R_2$

$= B_1B_2$

$= B_1P + B_2P$

$= 2B_1P$

$= 2O_2B_1 \sin(90 - \beta/2)$

$=$ ~~$2 \cdot O_2B_1 \sin$~~ $\beta/2$

$= 2 \cdot O_2B_1 \cdot \cos \beta/2$

$$\boxed{\text{stroke length} = 2 \times \frac{\text{length of slotted bar} \times \text{crank length}}{\text{fixed link length}}}$$

✓ → Use — shaper M/C

$l = 40$ cm

$\cos \beta/2 = \frac{\text{crank length}}{\text{fixed length}} = \frac{20}{40}$

$\cos \beta/2 = 20/40 = 1/2$

$\frac{\beta}{2} = 60 \Rightarrow \boxed{\beta = 120°} \Rightarrow \boxed{\alpha = 240}$

$QRR = \frac{\alpha}{\beta} = \frac{240°}{120°} \Rightarrow \boxed{QRR = 2}$

$\boxed{46}$ $QRR = \frac{1}{2} = \beta/\alpha \Rightarrow \alpha + \beta = 360$

$\cos \beta/2 = 1/2 =$

NOTE :

→ If QRR = 2 : 1 (or) 1 : 2 crank length always be half of of the fixed link length

(i) Analytical Approach
- Vector Algebra
- Complex No

(ii) Graphical Approach
- Instantaneous centres of rotation [I - centres] velocity
- Velo^n & Acc^n diagrams

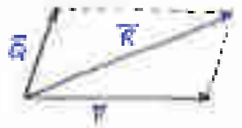
★ **Vectors**

$\vec{a} = \boxed{|\vec{a}|} \cdot \hat{a}$

$|\vec{a}|$ → magnitude

$\hat{a}$ → unit vector (direction)

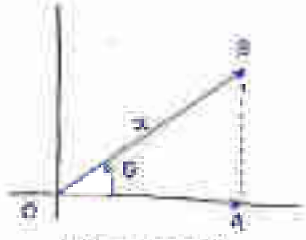
→ $\vec{P} + \vec{Q} = \vec{R}$



$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

→ $\vec{AB} \neq \vec{BA}$

but $\vec{AB} = -\vec{BA}$



$OA = x\cos\theta$

$OB = x\sin\theta$

$\vec{OB} = \vec{OA} + \vec{AB}$

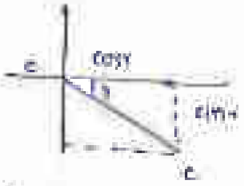
$|\vec{OB}| \cdot \hat{OB} = |\vec{OA}| \cdot \hat{OA} + |\vec{AB}| \hat{AB}$

$x \cdot \hat{OB} = (x\cos\theta)\hat{i} + (x\sin\theta)\hat{j}$

$\boxed{\hat{OB} = \hat{i}\cos\theta + \hat{j}\sin\theta}$

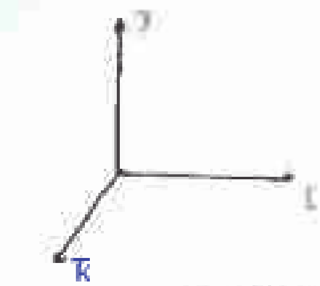
$\hat{OA} = \hat{i}\cos\alpha + \hat{j}\sin\alpha$

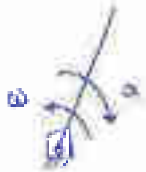
$\hat{OB} = -\hat{i}\cos\beta + \hat{j}\sin\beta$



$\hat{OC} = \hat{i}\cos\gamma - \hat{j}\sin\gamma$

→ [illegible]





$\vec{\omega} = \omega\,\hat{k}$ (following right hand rule)

$\vec{\alpha} = \alpha[-\hat{k}]$

indicate direction k

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

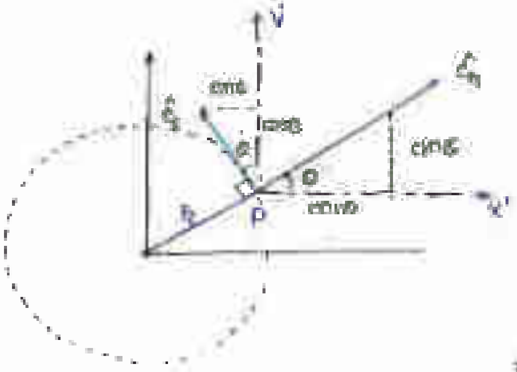
$\hat{k} \times \hat{i} = \hat{j}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}$

(Reverse cycle product give (-ve) results)

→



$$\boxed{\hat{e}_n = \hat{i}\cos\theta + \hat{j}\sin\theta}$$

$$\boxed{\hat{e}_t = \hat{j}\cos\theta - \hat{i}\sin\theta}$$

Displacement eqⁿ

$\vec{OP} = |\vec{OP}|\cdot\hat{OP}$

$$\boxed{\vec{OP} = r_c \cdot \hat{e}_n}$$

$\vec{OP} = r_c[\hat{i}\cos\theta + \hat{j}\sin\theta]$

⇒ <u>Velocity Equation</u>

$$\vec{v} = \frac{d\vec{OP}}{dt} = \frac{d}{dt}[r_c[\hat{i}\cos\theta + \hat{j}\sin\theta]]$$

$$= r_c\frac{d}{dt}[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

$$= r_c\left[\hat{i}(-\sin\theta)\frac{d\theta}{dt} + \hat{j}(\cos\theta)\frac{d\theta}{dt}\right]$$

$$\boxed{\vec{v} = r_c\omega[-\hat{i}\sin\theta + \hat{j}\cos\theta]}$$

$$\boxed{\vec{v} = (r_c\omega)\,\hat{e}_t}$$

mathematically :

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r_c}}$$

$V_A = (OA \cdot \omega)\, \hat{e}_t$

Note

① Relative velocity have only component

② The component will always be perpendicular to the line

★ Acceleration Eq^n

vary vary

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[r\omega(-\hat{i}\sin\theta + \hat{j}\cos\theta)]$$

$$= r\omega \frac{d}{dt}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

$$= r\omega\left[-\hat{i}\cos\theta \frac{d\theta}{dt} + \hat{j}\left(-\sin\theta \frac{d\theta}{dt}\right)\right]$$

$$+ (-\hat{i}\sin\theta + \hat{j}\cos\theta)\cdot\frac{d}{dt}(r\omega)$$

$$= r\omega^2(-\hat{i}\cos\theta - \hat{j}\sin\theta) + (-\hat{i}\sin\theta + \hat{j}\cos\theta)\left[r\frac{d\omega}{dt}\right]$$

$$\boxed{\vec{a} = r\omega^2(-\hat{e}_r) + r\alpha(\hat{e}_t)}$$

→ Acceleration has got two components

① normal

② tangential

$\vec{a}_{normal} = r\omega^2(-\hat{e}_r)$ (Radial)

$\vec{a}_n = \vec{\omega}\times(\vec{\omega}\times\vec{r})$

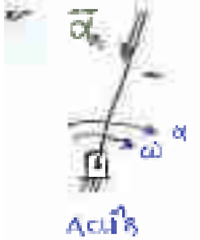
→ Direction of normal acc^n will always be towards the centre of rotation.

$\vec{a}_{tangential} = r\alpha(\hat{e}_t)$

$\vec{a}_t = \vec{\alpha}\times\vec{r}$

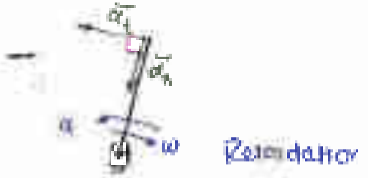
tangential acceleration is always take along the tangent.

→ Normal acc^n is always perpendicular to tangential



$a = a_{radial} + a_{tangential}$

$\vec{a} = OA\,\omega^2(-\hat{e}_r) + OA\,\alpha(\hat{e}_t)$



→ $\vec{a}_{rad}$ & $\vec{a}_{tan}$ is always perpendicular so

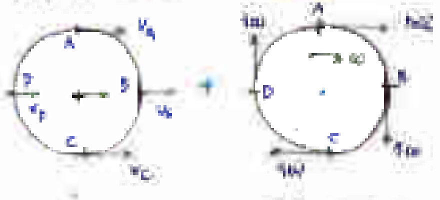
$$|\vec{a}_R| = \sqrt{[|\vec{a}_n|]^2 + [|\vec{a}_t|]^2}$$

$$\boxed{a_R = \sqrt{a_n^2 + a_t^2}}$$

→ **Rolling Motion**

Rolling = Translation + Rotation

for velocity :

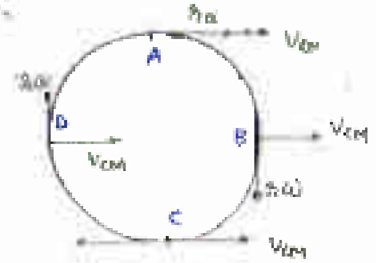


mass distribution is not important

Distribution of mass important

$$\boxed{v_A = v_B = v_C = v_D = v_{CM}}$$

Resultant :



@ point C

$V_{CM} = R\omega$

$$\boxed{V_{point\ C} = 0}$$

→ pure rolling

→ Rolling without slipping

$V_{CM} \neq R\omega$

Skidding

$$\boxed{V_{CM} > R\omega}$$

→ striking on ice road

forward slipping

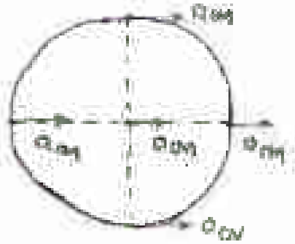
Slipping

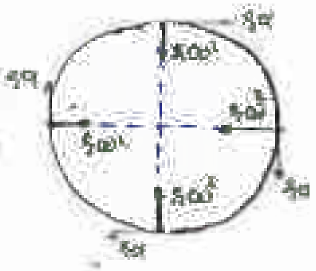
$$\boxed{V_{CM} < R\omega}$$

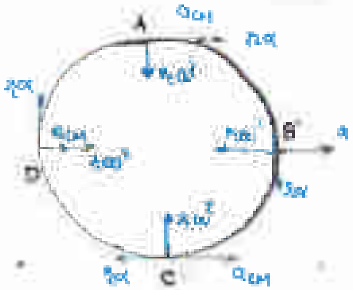
→ Back the wheel stuck in mud

Back ward slipping

[illegible] Acceleration







at point C

$a_{CM} = R\alpha$

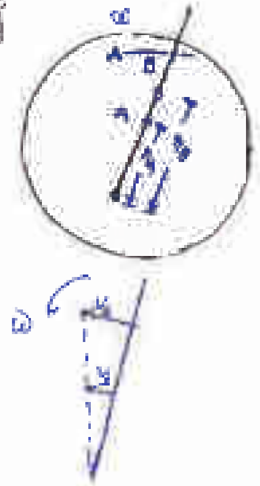
$a_{point\,C} = R\omega^2$ (accn)

→ Pure rolling

Rolling without slipping

Q - HS

& $\boxed{KG}$



$|\vec{V}_{B/A}| = \vec{V}_B - \vec{V}_A$

$= |\vec{V}_B| - |\vec{V}_A|$

$= r_B\omega - r_A\omega$

$= (r_B - r_A)\omega$ (dirⁿ of comp of dirⁿ of p

$|\vec{a}_{BA}| = r_B\omega^2 - r_A\omega^2$

$= (r_B - r_A)\omega^2 \; (-\hat{r})$ (towards centre o

→ property = f(space, time)

unsteady → prop. = f(time)

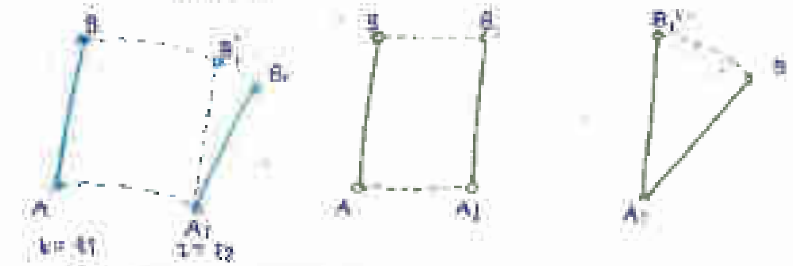
steady → prop ≠ f(time)

prop = f(space) → Non uniform

prop ≠ f(space) → Uniform

[illegible] Centre of Rotation or I-centre

General motion = Translation + Rotation



— In pure rotation there will be some finite cr of rotation

— Since straight line is part of circle; whose centre lies at infinite, this definition enables us to define translation as an example of rotation whose rotation centre is at infinite

— Hence we can conclude that every general motion is a kind of rotation and centre of rotation will be known as instantaneous centre or I-centre.

$$\overline{V_a} = \overline{V_o} + \overline{V_{A/O}} \qquad \overline{V_{A/O}} = \overline{V_A} - \overline{V_O}$$

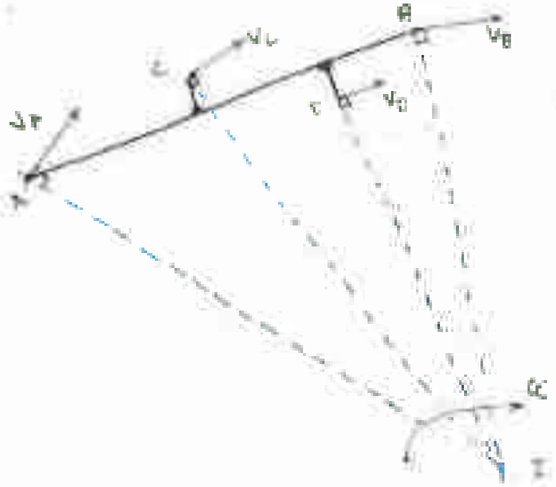
$$= 0 + OA\cdot\omega\cdot\hat{e}_t$$

$$\boxed{\overline{V_A} = (OA\cdot\omega)\,\hat{e}_t}$$

$|\overline{V_A}| = CA\ \omega$ → ang. speed of link on which A exist

center of rotation

the point whose velo^n is to be calculated



$$V_A = IA\ \omega_{AB}$$

$$V_B = IB\cdot\omega_{AB}$$

$$\boxed{\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IC} = \frac{V_D}{ID} = \dots = \omega_{AB} = \text{const}}$$

— If two links are connected with revolute pair the R-pair
ⓐ will itself become I-center.

— If two links are connected with prismatic pair the I-centre
ⓑ will itself at infinite

I_{12} @ ∞

ⓒ If a link is in pure rolling motion over another link, then point of contact will become an I-centre.

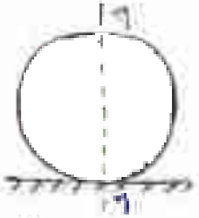
— If we consider Rotation + translation centre of mass rotation, centre of mass of disc & mass moment inertia should be considered about centre of mass.

$$K.E = \text{Rot } K.E$$
$$= \frac{1}{2} I_P \omega^2$$
$$I_P = I_c + mr^2$$
$$= \frac{mr^2}{2} + mr^2 = \frac{3}{2} mr^2 \text{ (Disc)}$$
$$= \frac{1}{2} \times \frac{3}{2} mr^2 \omega^2$$
$$\boxed{K.E = \frac{3}{4} mr^2 \omega^2}$$
(Translation only)

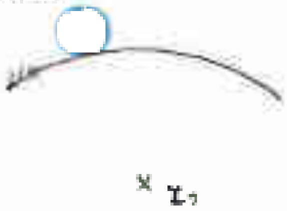
$$K.E = \text{Trans. } K.E + \text{Rot } K.E$$
$$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$
$$= \frac{1}{2} m (r\omega)^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\omega^2$$
$$\left\{ I = \frac{mr^2}{2} \text{ (Disc)} \right.$$
$$= mr^2\omega^2\left[\frac{1}{2} + \frac{1}{4}\right]$$
$$\boxed{K.E = \frac{3}{4} mr^2 \omega^2}$$
(Translation & Rotational both)

③ I-centre will lie somewhere along the common normal at the point of contact.

Eg. com is followed



④ If disc is moving on a curved surface then centre of curvature become an I-centre.



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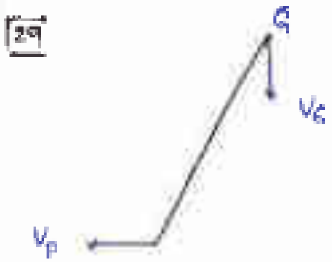


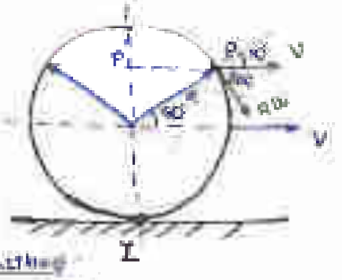
$\vec{V_G} = \vec{V_P} + \vec{V}_{G/P}$

$V_A = OA\,\omega$

$\therefore \vec{V}_{G/P} = (PG \cdot \omega_{PG})\,\hat{e}_t$

($V_{G/P}$ has one component perpendicular to PG)

30



$\overline{V_P}$ Rolls without slip

$V = R\omega$

$V_{rel} = \sqrt{V^2 + (R\omega)^2 + 2(V)(R\omega)\cos 60}$

$= \sqrt{V^2 + V^2 + 2V^2 \cdot \frac{1}{2}}$

$\boxed{V_{rel} = \sqrt{3}\,V}$

I - method

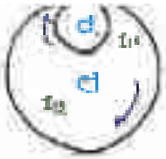
$\frac{V_P}{IP} = \frac{V}{IR}$

$\frac{V_P}{\frac{3}{2}R} = \frac{V}{R}$

$IP' = \sin 30 = \frac{IP'}{R}$

$IP = \frac{R}{2} + R = \frac{3}{2}R$

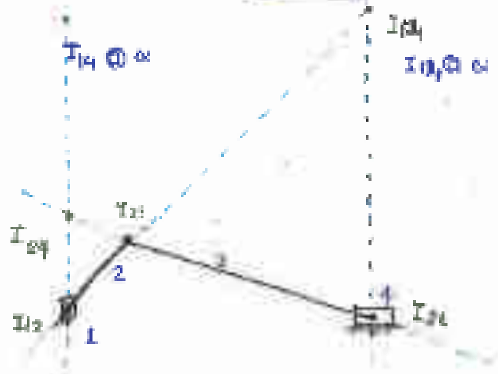
Ex



→ If a simple four-bar mechanism input link is parallel to output link angular speed of coupler will always be zero.

I_{13} I_{13} @ ∞ ⇒ $\boxed{\omega_3 = 0}$

Ex



→ ω_2 (given)

V_4 (slider velocity)

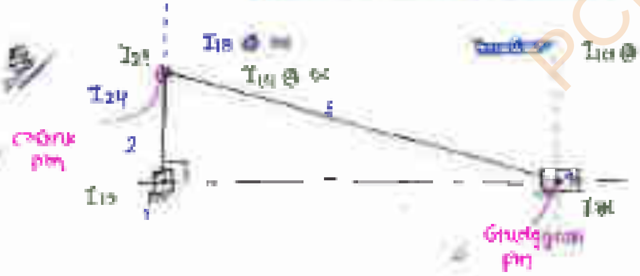
$P_{24} = P_{42}$

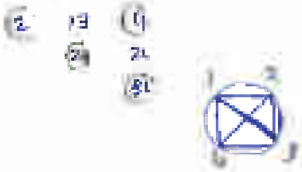
$V_{P_{24}} = V_{P_{42}}$

(link 4) (link 2)

$\boxed{V_{slider} = (I_{12} - I_{24})\,\omega_2}$

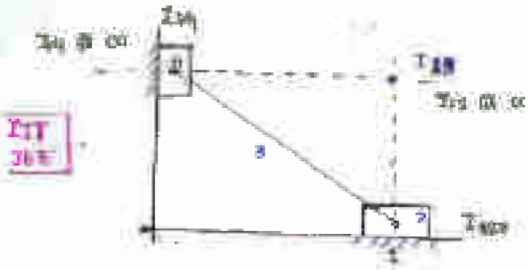
Ex





– In a single slider mechanism at an instant when crank is perpendicular to line of action I_{13} @ ∞ which leads $\boxed{\omega_3 = 0}$

(c)



12 13 (14)
(23) 24 =
(34)

I_{13} exists
I_{24} @ ∞

⇒ Rubbing velocity of pin

$$\boxed{V_{pin} = r_{pin}(\omega_1 \pm \omega_2)}$$

→ If links are rotating in opposite direction (+ve)
same direction (−ve)

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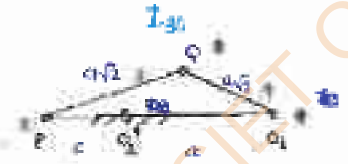
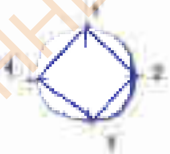




$\omega_2(I_{12} I_{23}) = \omega_3(I_{13} I_{23})$

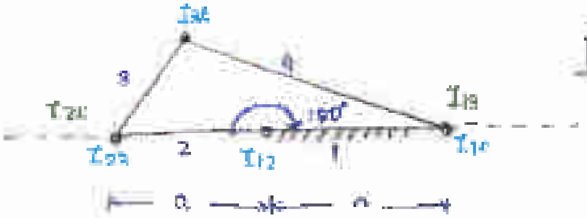
$\omega_3 = \frac{(2)(a)}{(2a)}$

$$\boxed{\omega_3 = 1 \text{ rad/s}}$$

Derived / write

$\omega_4(I_{14} I_{24}) = \omega_2(I_{12} I_{24}) \Rightarrow \omega_4 = \frac{(2)(a)}{2a}$

$$\boxed{\omega_4 = 1 \text{ rad/s}}$$



(i) $V \cdot R = \frac{\omega_{i/p}}{\omega_{o/p}} = \frac{2}{1}$

$\boxed{VR = \frac{2}{1}}$

(ii) Deltoid linkage (or) kite linkage
Galloway linkage

(iii) $\gamma = 90°$ (transmission angle) [couples ⟂ o/p link]

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$\vec{V_C} = \vec{V_O} + \vec{V}_{C/O}$

$|\vec{V_C}| = OC \cdot \omega_2 \Rightarrow \boxed{\omega_2 = \frac{V_C}{OC}}$

$\omega_2 =$ (known)

$V_4 = (?)$

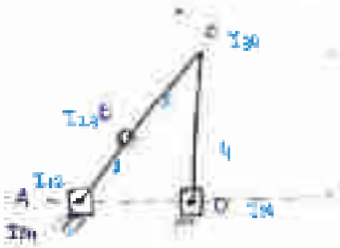
$\omega_2 (I_{12} I_{24}) = V_4$

$\frac{OC}{\sin(90-\beta)} = \frac{OH(I_{12} \cdot I_{14})}{\sin(\alpha+\beta)}$

$I_{12} \cdot I_{14} = OC \cdot \sin(\alpha+\beta) \sec\beta$

$\boxed{V_4 = V_C \sin(\alpha+\beta) \sec\beta}$

16



$\omega_{OT} = (?) = \omega_4$

$\omega_4 (I_{14} I_{24}) = \omega_2 (I_{12} \cdot I_{24})$

$= 0$

→ In given problem pressure angle (μ) corresponding to rocker will be at the extreme position (toggle position) so $\underline{\omega_{rocker} = 0}$ [M.A = ∞]

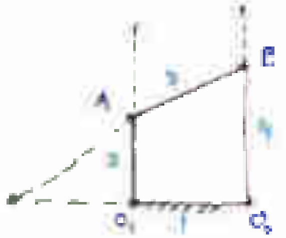
$\boxed{\omega_2 = 2 \text{ rad/s}}$

$V_{\text{rubbing velocity}} = r_{cs}(\omega_3 + \omega_4)$

$V_1 = r_{g}(\omega_1 + \omega_2) = 10(0 + 2) = 20$
$V_2 = r_{cs}(\omega_2 + \omega_3) = 10(2 + 0) = 20$
$V_3 = r_{cs}(\omega_3 + \omega_4) = 10(0 + 1) = 10$
$V_4 = r_{cs}(\omega_1 + \omega_4) = 10(0 + 1) = 10$

$\omega_2 (I_{12} I_{24}) = \omega_4 (I_{14} I_{24})$
$\omega_2 (\ell) = \omega_4 (2\ell)$
$\boxed{\omega_4 = 1 \text{ rad/s}}$

Q 17 Alternative approach



$\vec{V}_A = \vec{V}_{O_1} + \vec{V}_{A/O_1}$
$\boxed{|\vec{V}_A| = O_1A \; \omega_2}$

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$
$= \vec{V}_A + (\omega_3 \times AB)$
$\boxed{\vec{V}_B = \vec{V}_A}$ $\quad \omega_3 = 0$

$\vec{V}_B = \cancel{\vec{V}_{O_2}} + \vec{V}_{B/O_2}$
$\boxed{\vec{V}_B = O_2B \cdot \omega_4}$

$O_1A \; \omega_2 = O_2B \; \omega_4$
$\boxed{\ell_{in} \, \omega_{in} = \ell_{out} \, \omega_{out}}$ ← for parallel links

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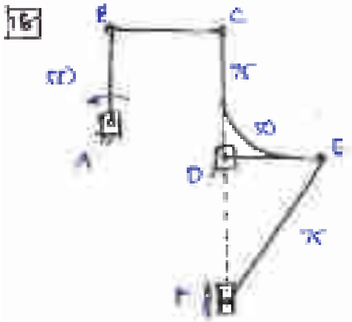

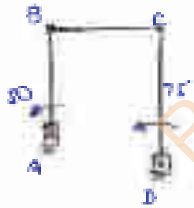

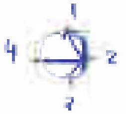

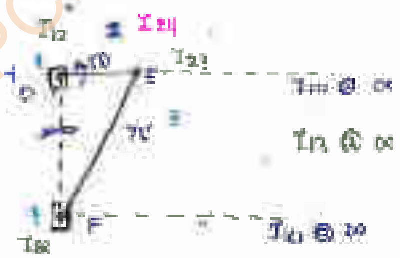

AB // CD

$\ell_{in} \, \omega_{in} = \ell_{out} \, \omega_{out}$
$50 \times 3 = \omega \times 75$
$\boxed{\omega_{out} = 2 \text{ rad/s}}$

→ $\omega_{CD} = 2$ rad/s (ccw)
→ $\omega_{DE} = 2$ rad/s (cw)

$\omega_2 (I_{12} I_{24}) = \omega_4 (I_{14} - I_{24})$
$(2)(50) = V_F$
$\boxed{V_F = 100 \text{ cm/s}}$

$\vec{V}_E = \vec{V}_D + \vec{V}_{E/D}$

$\boxed{\vec{V}_E = DE \cdot \omega_3}$

$\vec{V}_F = \vec{V}_E + \vec{V}_{F/E}$

$= \vec{V}_E + (\vec{\omega}_3 \times \vec{EF})$

$\boxed{\vec{V}_F = \vec{V}_E}$

Hence $|\vec{V}_F| = DE \cdot \omega_3$

$\boxed{V_{slider} = 5\,\omega_2}$ angular velocity of crank

20

$\boxed{\omega_3 = 0}$ → $\boxed{I_{13} @ \infty}$

If $I_{13} @ \infty$
Crank is ⊥ to line of stroke.

$V_{slider} = \ell\,\omega_2$
(or) $\omega_2 = 1(\ell)$ [check]

$V_{slider} = r\,\omega_2$

$2 = 2(1)\,\omega_2$

$\boxed{\omega_2 = 2 \text{ rad/s}}$

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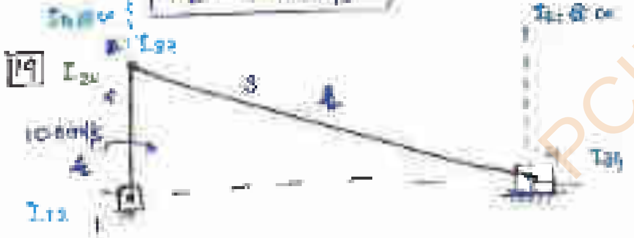
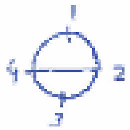


$\frac{\ell}{r} = 4$



$V_{slider} = 1$ m/s

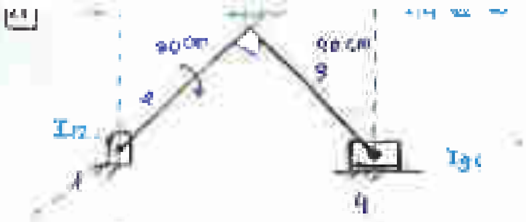
$\omega_4 (I_{14} I_{24}) = \omega_2 (I_{12} I_{24})$

$1 = 10\,(I_{12} I_{24})$

$\boxed{r = 0.1}$

$\frac{\ell}{0.1} = 4$

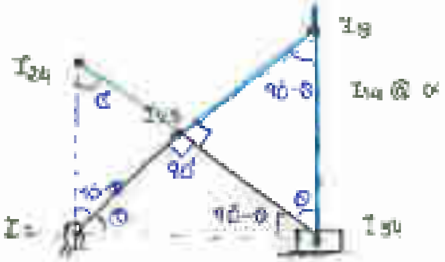
$\boxed{\ell = 0.4}$ C.R length



$V_{supp} = 8(\omega_2 + \omega_3)$

$\omega_2 (I_{12} \cdot I_{23}) = \omega_3 (I_{13} \cdot I_{23})$

$(30)(80) = \omega_3 (58.3)$



$$\frac{I_{13} \cdot I_{34}}{I_{14} \, I_{23}} = \frac{I_{12} \, I_{23}}{I_{24} \, I_{34}} = \frac{I_{12} \, I_{34}}{I_{14} \, I_{24}}$$

$$\frac{90}{I_{13} I_{23}} = \frac{80}{40}$$

$I_{13} I_{23} = 58.5$ cm

$\boxed{\omega_3 = 5.62 \text{ rad/s}}$

→ $V_{pin} = 4.5(\omega_3 + \omega_2)$

$= 4.5(5.625 + 10)$

$\boxed{V_{pin} = 39 \text{ m/s}}$

→ for slider velocity

$\omega_4 (I_{14} \cdot I_{24}) = \omega_2 (I_{12} \, I_{24})$

$V_{slider} = \omega_2 (I_{12} \, I_{24})$

$\boxed{V_{slider} = 376 \text{ cm/s}}$

$\tan\theta = \frac{90}{80}$

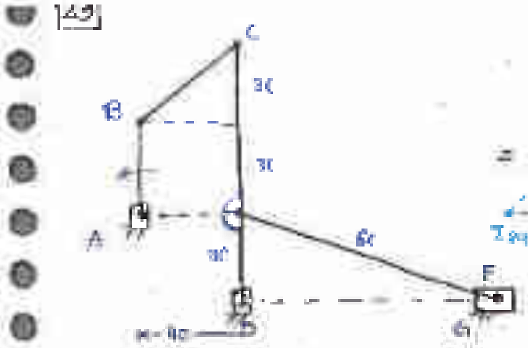
$\boxed{\theta = 83.12}$

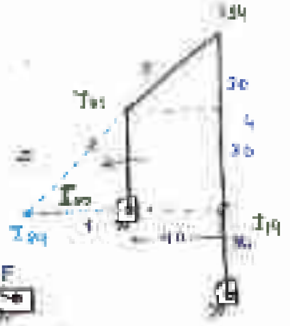
$\sin\theta = \frac{30}{I_{12} I_{24}}$

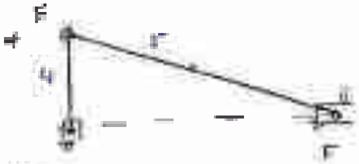
$I_{12} I_{24} = \frac{30}{\sin 83.12}$

$I_{12} I_{24} = 33.2$ cm

49







→ $\omega_2 = 6$ rad/s (given)

$\omega_4 = (?)$

$\omega_2 (I_{12}-I_{24}) = \omega_4 (I_{14}-I_{24})$

$\omega_2 (40) = \omega_4 (40)$

$\omega_4 = \frac{(6)(40)}{(40)}$

$\frac{80}{x+40} = \frac{3x}{x}$

$3x = x + 40$

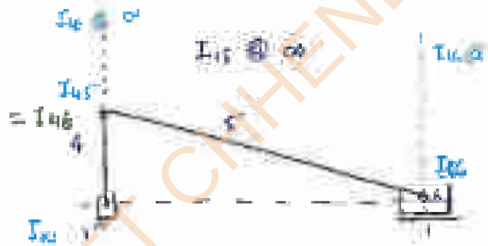
$\boxed{x = 40}$

1) Here two links are parallel so ;

$\boxed{\ell_{14}\, \omega_{14} = \ell_{64}\, \omega_{64}}$

$\therefore (80)(6) = (90)\,\omega$

$\boxed{\omega_4 = 2 \text{ rad/s}}$



2) $\omega_5 = (?)$

$\omega_4 (I_{14}-I_{45}) = \omega_5 (I_{15}-I_{45})$

I_{15} @ ∞

$\boxed{\omega_5 = 0}$

3) $V_{slider} = (?)$

$\omega_4 (I_{14} I_{46}) = \omega_6 (I_{16} \cdot I_{46})$

$(2)(30) = V_{slider}$

$\boxed{V_{slider} = 60 \text{ cm/s}}$

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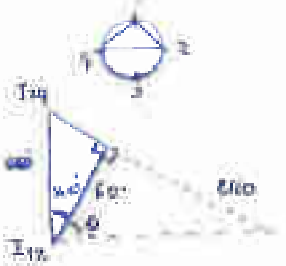
$\omega_2 (I_{12}-I_{24}) = \omega_4 (I_{14} \cdot I_{24})$

$10 (61.84) = V_{slider}$

$\boxed{V_{slider} = 618.4 \text{ cm/s}}$

$= 6.184$ m/s

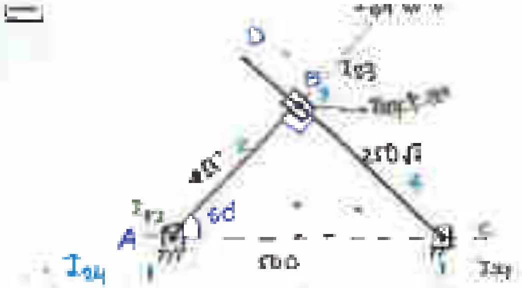


$\tan \theta = \frac{240}{60} = 4$

$\boxed{\theta = 75.96}$

$\cos(75.96) = \frac{60}{I_{12} I_{24}}$

$\boxed{I_{12} I_{24} = 61.84}$



Observation

(i) Crank & slotted bar mechanism

(ii) Crank length is half of fixed link length

$QRR = \frac{2}{1}$

(iii) $\angle ABC = 90^\circ$ so crank ⊥ to slotted bar

(iv) $\omega_{CD} = 0$ since it is at its extreme position (toggle)

→ $\omega_{AB} = 10$ rad/s (CCW)

$V_B = (?)$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} \rightarrow (0)$$

$\boxed{\omega_4 = 0}$

→ $V_{slider} = \omega_2 (I_{12} I_{23}) = \omega_3 (I_{13} I_{35})$

$= 10 (250)$

$\boxed{V_{slider} = 2.5 \text{ m/s}}$

Alternative

→ $\vec{V}_{B_2} = \cancel{\vec{V}_A} + \vec{V}_{B_2/A}$

$= \vec{\omega}_2 \times \vec{AB}_2$

$\boxed{|\vec{V}_{B_2}| = AB_2 \cdot \omega_2}$

→ $\vec{V}_{B_3} = \vec{V}_{B_2} + \cancel{\vec{V}_{B_3/B_2}}$

$\boxed{|\vec{V}_{B_3}| = AB_2 \omega_2}$

→ $\vec{V}_{B_3} = \vec{V}_{B_4} + \vec{V}_{B_3/B_4}$

$\vec{V}_{B_4} = \vec{V}_C + \vec{V}_{B_4/C}$

$= \cancel{\vec{V}_C} + \vec{\omega}_4 \times \vec{B/C}$ $(\omega_4 = 0$

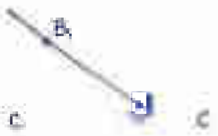
$0 = \vec{V}_{B_4}$

$\vec{V}_{B_3} = \vec{V}_{B_3/B_4}$

$|\vec{V}_{B_3/B_4}| = AB \cdot \omega_2 = |\vec{V}_{B_3}|$

$= 250 \times 10$

$\boxed{|\vec{V}_{B_3/B_4}| = 2.5 \text{ m/s}}$



$QRR = \frac{1}{2} = \frac{\beta}{\alpha}$

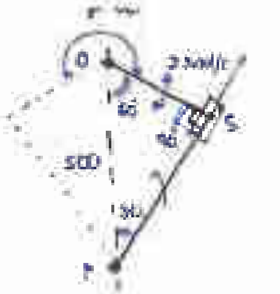
$\frac{\beta}{\alpha} = \frac{1}{2}$

$2\beta = \alpha$

$\alpha + \beta = 360$

$2\beta + \beta = 120$

$\boxed{\beta = 120}$ & $\boxed{\alpha = 240}$



→ $\cos 60° = \frac{OS}{500}$

$\boxed{OS = 250 \text{ mm}}$

7 max speed (ω_4)

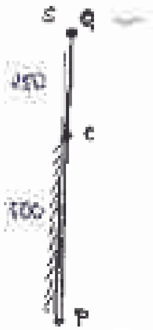
$\vec{V_c} = \cancel{\vec{V_O}} + \vec{V_{S/O}}$

$|\vec{V_c}| = |\vec{V_{S/O}}|$

$= OS \cdot \omega_2$

$= 250 \times 2$

$= 500 \text{ mm/s}$



$V_c \longrightarrow C$

$V_Q \longrightarrow P$

→ $\vec{V_Q} = \cancel{\vec{V_P}} + \vec{V_{Q/P}}$

$= PQ \cdot \omega_4$

$= 750\, \omega_4$

S & Q on same pt

$500 = 750\, \omega_4 \Rightarrow \boxed{\omega_4 = 2/3}$ rad/s

8 velocity approach

$\boxed{\theta = 70.53}$

$\hat{OA} = \hat{i}\cos\theta + \hat{j}\sin\theta$

$\omega = 10$ rad/s (CCW)

$\boxed{\vec{\omega_2} = 10\hat{k}}$

$\vec{V_A} = \cancel{\vec{V_O}} + \vec{V_{A/O}}$

$= \vec{\omega_2} \times \vec{AO}$

$= 10\hat{k} \times 80(\hat{i}\cos\theta + \hat{j}\sin\theta)$

$\boxed{\vec{V_A} = 800\cos\theta\,\hat{j} - 800\sin\theta\,\hat{i}}$

$\hat{OA} = \hat{i}\cos\theta + \hat{j}\sin\theta$

$\hat{AB} = \hat{i}\sin\theta - \hat{j}\,cc$

→ $\vec{V_B} = \vec{V_A} + \vec{V_{BA}}$

$= \vec{V_A} + (\vec{\omega_3} \times \vec{AB})$

$= \vec{V_A} + \omega_3\hat{k} \times 240(\hat{i}\sin\theta - \hat{j}\cos\theta)$

$\boxed{\vec{V_B} = \vec{V_A} + 240\,\omega_3\sin\theta\,\hat{j} + 240\,\omega_3\cos\theta\,\hat{i}}$

for motion

$\vec{\omega_3} = \omega_3\hat{k}$

⇒ $V_B \hat{i} = 600\cos\theta \hat{j} - 600\sin\theta \hat{i} + 240\,\omega_3 \sin\theta \hat{j} + 240\,\omega_3 \cos\theta \hat{i}$

$V_B = 240\,\omega_3 \sin\theta - 600\sin\theta$

$0 = 600\cos\theta + 240\,\omega_3 \sin\theta$

$\boxed{\omega_3 = -0.625 \text{ rad}}$ (CW)

$\boxed{V_B = -516.4 \text{ mm/s}}$ (− x direction)

34

$\omega_{CD} = 2$ rad/s (CCW) $= \omega_4$

$\omega_{AB} = ?$

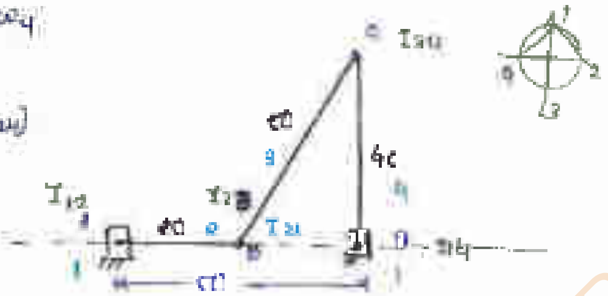
ω_4 $\omega_4 (I_{14}\ I_{24}) = \omega_2 (I_{12}\ I_{24})$

$(2)\ \omega_4 (30) = \omega_2 (20)$

~~$\omega_4\ \omega_4 (30) = \omega_2 (20)$~~

$(2)(30) = \omega_2 (20)$

$\boxed{\omega_2 = 3 \text{ rad/s}}$



35

$\omega_{AB} = 1$ rad (CW) $= \omega_2$

ω_{BC}, $\omega_{CD} = ?$

⇒ $\omega_2 (I_{12}\ I_{23}) = \omega_3 (I_{13}\ I_{23})$

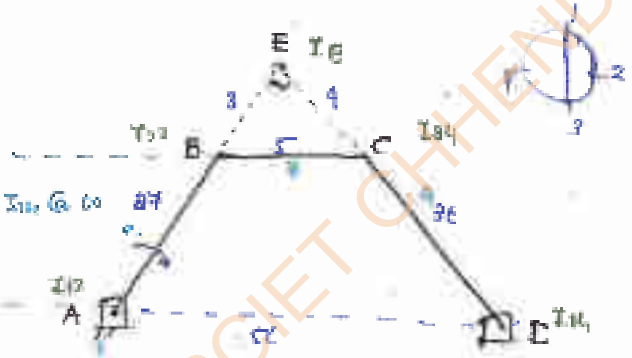
$\omega_2 (27) = \omega_3 (3)$

$\boxed{\omega_3 = 9 \text{ CCW}}$

⇒ $\omega_3 (I_{13}\ I_{34}) = \omega_4 (I_{14}\ I_{34})$

$(9)(4) = \omega_4 (36)$

$\boxed{\omega_4 = 1 \text{ rad/s (CW)}}$

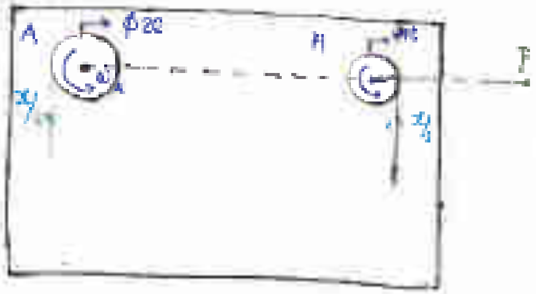


36 the displacement of A in 'x' the same as H

$V_A = V_H$

$10\,\omega_A = 5\,\omega_H$

$\boxed{\frac{\omega_A}{\omega_H} = \frac{1}{2}}$



$\frac{\omega_A}{\omega_H} = \frac{HP}{AP}$ → $\omega_A (AH) = \omega_H (PH)$

$\frac{1}{2} = \frac{HP}{AP}$

$AP = 2 HP$

$AH + HP = 2HP$

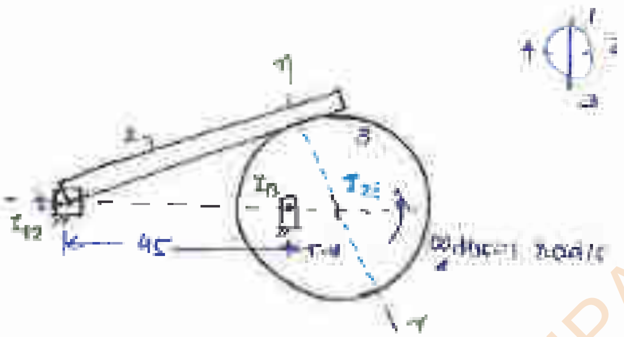
$\boxed{AH = HP}$

39

$\omega_2 (I_{12} I_{23}) = \omega_3 (I_{13} I_{23})$

(i) → $\omega_2 (50) = \omega_3 (5)$

$\boxed{\omega_3 = 0.1 \text{ rad/s}}$



40

AB = 1 m

$V_A = 2$ m/s

$V_P = ?$

→ $\Delta IAP \cong \Delta IPB$

IP is comm^n

AP = PB ∵ P is M.P

$\angle IAP = \angle IBP$

→ IP is $\perp^r$ to AB   Vp is along AB

$\tan 30° = \frac{IP}{BP}$

$\boxed{IP = 0.5 \tan 30} = 0.288$ m

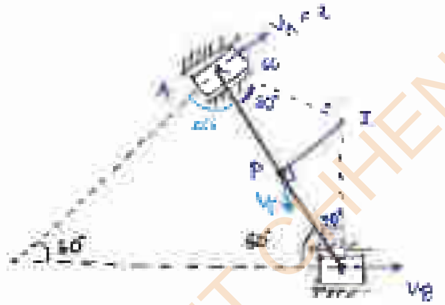
$IB^2 = \sqrt{(IP)^2 + (BP)^2}$

$\boxed{IB = 0.577 \text{ m} = IA}$

$\frac{V_A}{IA} = \frac{V_P}{IP}$

$V_P = \frac{(2)(0.288)}{0.577}$

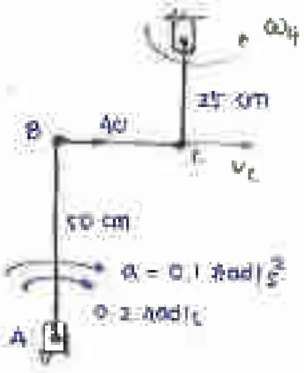
$\boxed{V_P = 1 \text{ m/s}}$



Since : AB ∥ CD so

$\boxed{\ell_{in}\,\omega_{in} = \ell_{out}\,\omega_{out}}$

so, ℓ_{AB} [illegible] 60 → $\boxed{\omega_3 = 0}$



→ $\ell_{in}\,\omega_{in} = \ell_{out}\,\omega_{out}$

$(50)(0.2) = (25)\,\omega_{out}$

$\boxed{\omega_{out} = 0.4\ \text{rad/s}}$

Acc. analⁿ

→ $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$= 0 + \vec{a}_{n\,B/A} + \vec{a}_{t\,B/A}$

$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{AB}) + (\vec{\alpha}_2 \times \vec{AB})$

$= -0.2\hat{k} \times (-0.2\hat{k} \times 50\hat{j}) + (-0.1\hat{k} \times 50\hat{j})$

$= -0.2\hat{k} \times (10\hat{i}) + 5\hat{i}$

$\boxed{\vec{a}_B = -2\hat{j} + 5\hat{i}}$

→ $\vec{a}_C = \vec{a}_B + \vec{a}_{C/B}$

$= \vec{a}_B + \vec{a}_{n\,C/B} + \vec{a}_{t\,C/B}$

$= \vec{a}_B + \underbrace{\vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{BC})}_{(0)} + \vec{\alpha}_3 \times \vec{BC}$

$= \vec{a}_B + \alpha_3\hat{k} \times 40\hat{i}$

$\boxed{\vec{a}_C = \vec{a}_B + 40\alpha_3\hat{j}}$

→ $\vec{a}_C = -2\hat{j} + 5\hat{i} + 40\alpha_3\hat{j}$

$\boxed{\vec{a}_C = (40\alpha_3 - 2)\hat{j} + 5\hat{i}}$ — ①

Assume,

$\vec{\alpha}_3 = \alpha_3\hat{k}$ (ccw)

$\vec{\alpha}_4 = \alpha_4\hat{k}$

→ $\vec{a}_C = \vec{a}_D + \vec{a}_{C/D}$

$= \vec{a}_{n\,C/D} + \vec{a}_{t\,C/D}$

$= \vec{\omega}_4 \times (\vec{\omega}_4 \times \vec{DC}) + \vec{\alpha}_4 \times \vec{DC}$

$= 0.4\hat{k} \times (0.4\hat{k} \times (-25)\hat{j}) + \alpha_4\hat{k} \times (-25\hat{j})$

$= 0.4\hat{k} \times 10\hat{i} + 25\alpha_4\hat{i}$

$\boxed{\vec{a}_C = 4\hat{j} + 25\alpha_4\hat{i}}$ — ②

eqⁿ ① + ② $5 = 25\alpha_4 \Rightarrow \boxed{\alpha_4 = 0.2\ \text{rad/s}^2}$ (ccw)

$40\alpha_3 - 2 = 4 \Rightarrow \boxed{\alpha_3 = 0.15\ \text{rad/s}^2}$ (ccw)

→ If i/p link ∥ o/p link

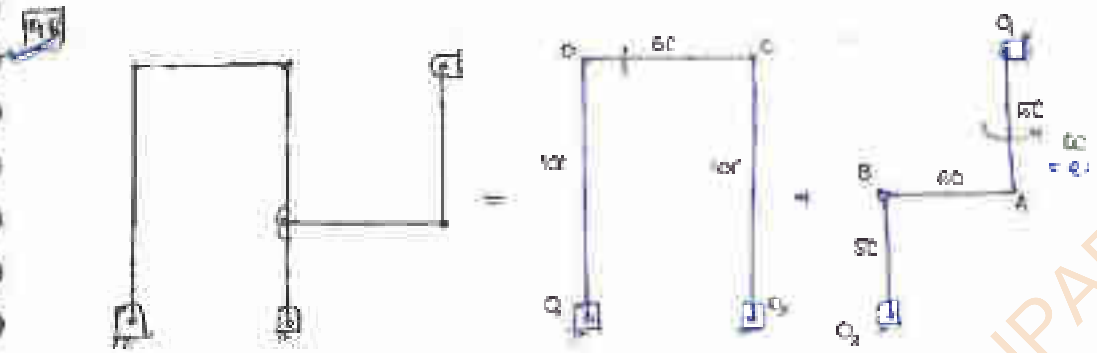
$\ell_{in}\omega_{in} = \ell_{out}\omega_{out}$

$\boxed{\ell_{in}\alpha_{in} = \ell_{out}\alpha_{out}}$

⇒ $50 \times \alpha_2 = 25 \times \alpha_4$

$50 \times 0.1 = 25 \times \alpha_4$

$\boxed{\alpha_4 = 0.2}$ rad/s²



In O_1ABO_2

→ $\ell_{in}\omega_{in} = \ell_{out}\omega_{out}$

$(50)(2) = (50)\,\omega_{out}$

$\boxed{\omega_{out} = \omega_{O_2B} = 2 \text{ rad/s}}$

→ $\alpha_{in} = 0$ (given)

$\ell_{in}\alpha_{in} = \ell_{out}\alpha_{out}$

$\boxed{\alpha_{out} = 0} = \alpha_{O_2B}$

In O_3CDO_4

$(O_3C)(\omega_{O_3C}) = (O_4D)(\omega_{O_4D})$

$(100)(2) = (100)(\omega_{O_4D})$

$\boxed{\omega_{O_4D} = 2 \text{ rad/s}}$

→ $O_3C\,(\alpha_{O_3C}) = O_4D\,(\alpha_{O_4D})$

$\boxed{\alpha_{O_4D} = 0}$

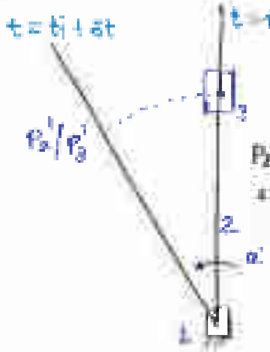
→ $\vec{V}_D = \cancel{\vec{V}_{O_4}} + \vec{V}_{D/O_4} \Rightarrow |\vec{V}_D| = O_4D \cdot \omega_{O_4D}$

$= 100 \times 2 \Rightarrow \boxed{|\vec{V}_D| = 200 \text{ mm/s}}$

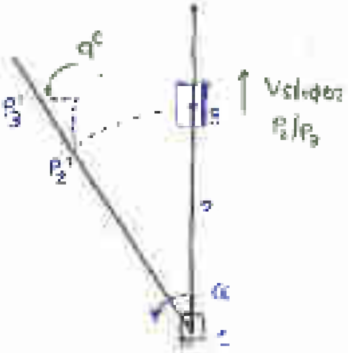
→ $\vec{a}_D = \cancel{\vec{a}_{O_4}} + \vec{a}_{D/O_4} \Rightarrow |\vec{a}_D| = O_4D\,(\omega_{O_4D})^2 + \cancel{(O_4D \cdot \alpha_{O_4D})}$

$= 100 \times 2^2$

$\boxed{|\vec{a}_D| = 400 \text{ mm/s}^2}$



P2/P3 are instantaneously coincident pts



→ $\boxed{V_{P_3/P_1} = \text{zero}}$

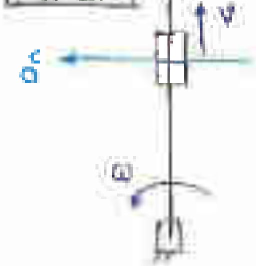
$$\boxed{\vec{a}_c = 2[\vec{\omega}_3 \times \vec{V}_{P_2/P_3}]}$$

⇒ Direction of coriolis acc

① Rotate the velocity vector by 90°

② The sense of rotation should be same as ω

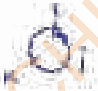
1st method

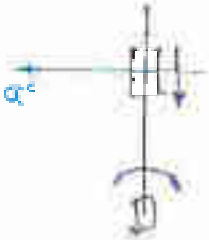


$\hat{a}^c$ = ω's direction × velocity's direction
(follow right hand thumb)

$= \hat{k} \times \hat{j}$

$= -\hat{i}$

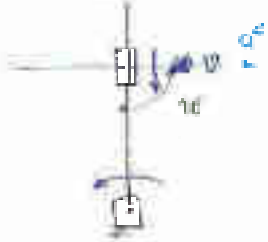




$\hat{a}_c = -\hat{k} \times -\hat{j}$

$= -\hat{i}$

2nd method



$\hat{a}_c = \hat{k} \times -\hat{j}$

$= \hat{i}$

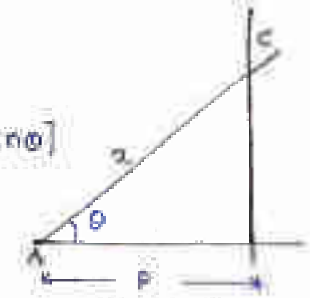
Rotate velocity vector by 90° as
the same direction of 'ω' as that of

$$\vec{AC} = |\vec{AC}| \cdot \hat{AC}$$

$$= x \cdot \hat{AC}$$

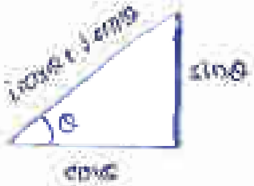
$$= \frac{p}{\cos\theta}\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

$$\boxed{\vec{AC} = p\left[\hat{i} + \hat{j}\tan\theta\right]}$$



$x \cos\theta = p$

$$\boxed{x = \frac{p}{\cos\theta}}$$



$$\boxed{\frac{d}{dt}(\text{unit vector}) = 0}$$

→ $\vec{v} = \frac{d\vec{AC}}{dt}$

$$= \frac{d}{dt}\left[p\{\hat{i} + \hat{j}\tan\theta\}\right]$$

$$= p\left[0 + \hat{j}\sec^2\theta\frac{d\theta}{dt}\right]$$

$$\boxed{\vec{v} = \frac{p\omega}{\cos^2\theta}\hat{j}}$$

→ $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\frac{p\omega}{\cos^2\theta}\right]$

$$= p\omega\frac{d}{dt}\left[(\cos\theta)^{-2}\right]$$

$$= p\omega\left[(-2)(\cos\theta)^{-3}\cdot(-\sin\theta)\frac{d\theta}{dt}\right]$$

$$\boxed{\vec{a} = \frac{2p\omega^2\sin\theta}{\cos^3\theta}}$$

$\boxed{55}$ $\ell_{min} = 60$

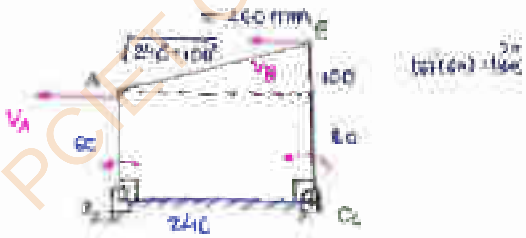
$\ell_{max} = 280$

$p = 240$

$q = 160$

$(60 + 280) \boxed{<} [240 + 160]$

→ crank rocker



$\boxed{56}$ for parallel links

$\ell_{in}\omega_{in} = \ell_{out}\omega_{out}$

$(60)(8) = (160)\omega_{out}$

$$\boxed{\omega_{out} = 3 \text{ rad/s}}$$

$\boxed{57}$ for parallel $\boxed{\alpha_3 = 0}$

$\boxed{57}$

{ Here α = 0

ε = 0

If α ≠ ε ≠ 0

Then R is

~~dir~~ known

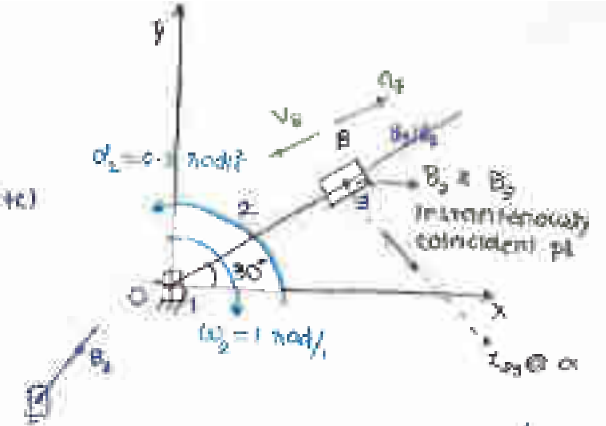
$|\vec{OB}| = 40$ cm

$\vec{OB} = \hat{i}\cos 30 + \hat{j}\sin 30$

$V_B = 0.2$ m/s

$a_B = 0.5$ m/s² (deceleration $\propto$ opposite)

$\vec{a}_{B_3/O} = (?)$



→ $\vec{a}_{B_3} = \vec{a}_{B_2} + \vec{a}_{B_3/B_2} + \vec{a}_c$

→ $\vec{a}_{B_3} = \vec{a}_c + \vec{a}_{B_3/O}$

⇒ $\vec{a}_{B_3} = \vec{a}_{B_2/O} + \vec{a}_{B_2/O} + \vec{a}_{B_3/B_2} + \vec{a}_c$

$= \vec{a}^n_{B_2/O} + \vec{a}^t_{B_2/O} + \vec{a}^n_{B_3/B_2} + \vec{a}^t_{B_3/B_2} + \vec{a}_c$

$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{OB}_2) + (\vec{\alpha}_2 \times \vec{OB}_2) + 0 + \vec{a}_B + 2[\vec{\omega}_2 \times \vec{V}_{B_3/B_2}]$

$\{\vec{a}^n_{B_3/B_2} = 0$ as $\omega_3 = 0$

∵ $\left\{\vec{a}^n_{B_3/B_2} = \omega_3^2 \ldots = \frac{V^2}{\rho}\right.$

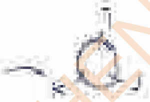
$\boxed{\omega_3 = 0}$ because I_{23} @ ∞

<u>Now</u>:

$\vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{OB}_2) = -1\hat{k} \times (-1\hat{k} \times 40\,(\hat{i}\cos 30 + \hat{j}\sin 30)$

$= -1\hat{k} \times (-40\cos 30\,\hat{j} + 40\sin 30\,\hat{i})$

$= -40\cos 30\,\hat{i} - 40\sin 30\,\hat{j}$



$\vec{\alpha}_2 \times \vec{OB}_2 = 0.5\hat{k} \times 40(\hat{i}\cos 30 + \hat{j}\sin 30)$

$= 20\cos 30\,\hat{j} - 20\sin 30\,\hat{i}$

$\vec{a}_B = 50\,(\hat{i}\cos 30 + \hat{j}\sin 30)$

$\vec{a}_{B_3} = \vec{a}_{B_2/O} + \vec{a}^t_{B_2/O} + \vec{a}_B + \vec{a}_c$

$2[\vec{\omega}_2 \times \vec{V}_{B_3/B_2}] = 2[-1\hat{k} \times 20(-\hat{i}\cos 30 - \hat{j}\sin 30)]$

$= 40\cos 30\,\hat{j} - 40\sin 30\,\hat{i}$

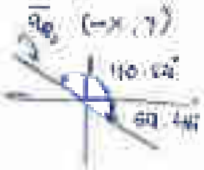
⇒ $\vec{a}_{B_3} = -40\cos 30\,\hat{i} - 40\sin 30\,\hat{j} + 20\cos 30\,\hat{j} - 20\sin 30\,\hat{i}$
$+ 50\cos 30\,\hat{i} + 50\sin 30\,\hat{j} + 40\cos 30\,\hat{j} - 40\sin 30\,\hat{i}$

$= (-40\cos 30 - 20\sin 30 + 50\cos 30 - 40\sin 30)\,\hat{i}$
$+ (-40\sin 30 + 20\cos 30 + 50\sin 30 + 40\cos 30)\,\hat{j}$

$\boxed{\vec{a}_{B_3} = -21.339\,\hat{i} + 56.961\,\hat{j}}$

→ $|\vec{a}_{B_3}| = 60.82 = \underline{0.60\text{ m/s}^2}$ ans.

$\theta = \tan^{-1}\left(\frac{56.96}{21.33}\right)$

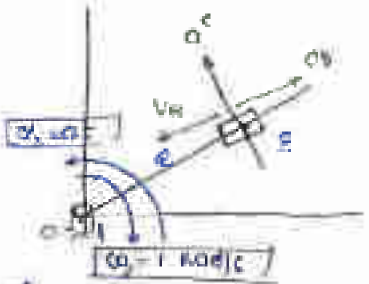


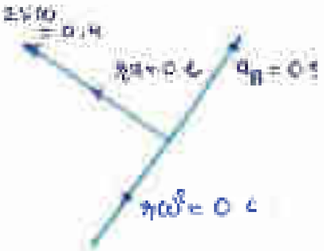
$\boxed{\theta_1 = -69.46°}$ & $\theta_2 = 180 - 69.46°$

$\boxed{\theta_2 = 110.54°}$

$\overline{a}_{B_3} = \overline{a}^n_{B/O} + \overline{a}^t_{B/O} + \overline{a}_B + \overline{a}_c$

$r\omega^2$ $r\alpha$ 0.5 $2V\omega$





=

$a_R = \sqrt{0.8^2 + 0.1^2}$

$\boxed{a_R = 0.806}$ m/s²

$\tan\theta = \frac{0.8}{0.1}$

$\theta = 80.53$

$\theta_{from\ x} = 80.53 + 30$

$\boxed{\theta_x = 110.53}$

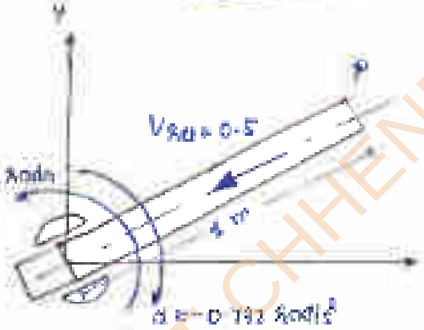
→ angle is taken from (+ve) x axis to resultant accn

24

$\overline{a}_{B_3} = \overline{a}^n_{B/O} + \overline{a}^t_{B/O} + \overline{a}_B + \overline{a}^c_B$

(Radial accn of arm w.r.t is base zero $\overline{a}_B = 0$

$= r\omega^2 + r\alpha + 2\omega V$



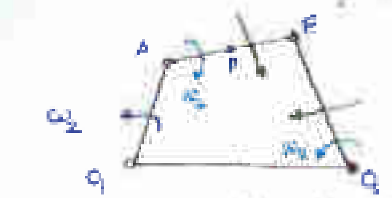
⇐

$\tan\theta = \frac{\sqrt{3}}{1}$

$\boxed{\theta = 60}$

$\boxed{\theta = 270 \text{ from +x axis}}$ (40 Resultant)

Q, simple four bar mechanism :-



Velocity Analysis

→ $\vec{V}_A = \vec{V}_{O_1} + \vec{V}_{A/O_1}$

$|\vec{V}_A| = O_1A \cdot \omega_2$

m d ⊥ to O_1A

✓ ✓

→ $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$= \vec{V}_A + \vec{\omega}_3 \times \vec{AB}$

m,d m d ⊥ to AB

✓ ✓ × ✓

→ $\vec{V}_B = \vec{V}_{O_2} + \vec{V}_{B/O_2}$

$= \vec{\omega}_4 \times \vec{O_2B}$

m d ⊥ to O_2B

× ✓

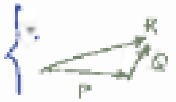
∴ $o_1a = \vec{V}_{A/O_1} = \vec{V}_A$

$ab = \vec{V}_{B/A}$

$|\vec{V}_{B/A}| = AB \cdot \omega_3$

$o_2b = \vec{V}_{B/O_2}$

$|\vec{V}_{B/O_2}| = (O_2B)\,\omega_4$



$\vec{R} = \vec{P} + \vec{Q}$

→ ✓ $\dfrac{AP}{AB} = \dfrac{ap}{ab}$ ✓

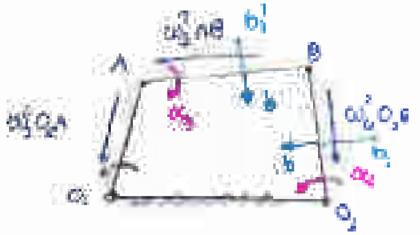
$ap =$ ___ (Bound)

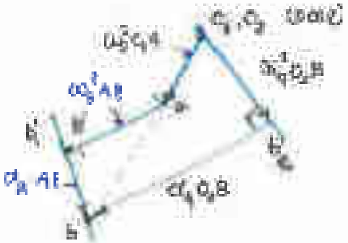
$\vec{V}_{P/A} = ap$

$\vec{V}_{P/B} = bp$

$\vec{V}_P = o_1p$

Acceleration Analysis





— $a_A = a_{O_1} + a_{A/O_1}$

$= \vec{a}^n_{A/O_1} + \vec{a}^t_{A/O_1}$

$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{O_1A}) + (\vec{\alpha}_2 \times \vec{O_1A})$

m d ⌞ ∥^{el} to link O_1A

— $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$= \vec{a}_A + \vec{a}^n_{B/A} + \vec{a}^t_{B/A}$

$= \vec{a}_A + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{AB}) + (\vec{\alpha}_3 \times \vec{AB})$

m d m d m d

✓ ✓ ✓ ✓ × ✓

⌞ ∥^{el} to AB ⌞ ⊥^{el} to $\omega_3^2 AB$

→ $\vec{a}_B = \vec{a}_{O_2} + \vec{a}_{B/O_2}$

$= \vec{a}^n_{B/O_2} + \vec{a}^t_{B/O_2}$

$= \vec{\omega}_4 \times (\vec{\omega}_4 \times \vec{O_2B}) + (\vec{\alpha}_4 \times \vec{O_2B})$

m d m d

✓ ✓ × ✓

⌞ ∥^{el} to O_2B ⌞ ⊥^{r} to $\omega_4^2 O_2B$

⇒ $\vec{a}_{B/A} = ab$

$\vec{a}_B = Ob$

★ Single Slider Mechanism :

Velocity :-

(Velocity dia.)

— $\vec{V}_A = \vec{V}_O + \vec{V}_{A/O}$

$= \vec{OA} \times \vec{\omega}_2$

m d ⌞ ⊥^{r} to OA

← $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$= \vec{V}_A + \vec{\omega}_3 \times \vec{AB}$

m d ⟶ ⊥^{r} to AB

× ✓

Accn.- $\vec{a}_A = \cancel{\vec{a}_O} + \vec{a}_{A/O} = \vec{a}^n_{A/O} + \vec{a}^t_{A/O}$

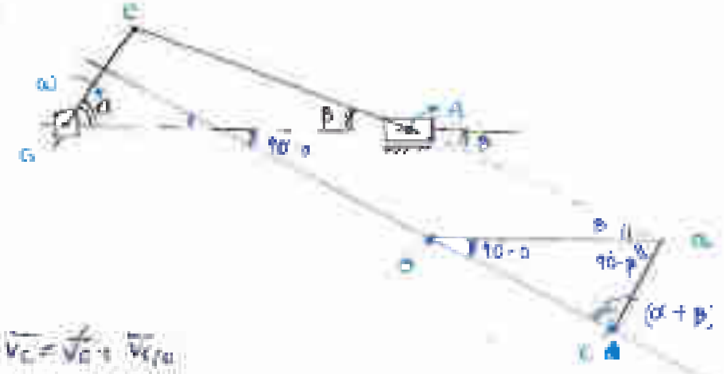
$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{OA}) + (\vec{\alpha}_2 \times \vec{OA})$

m d ⟵ ∥^{el} to OA

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \vec{a}^n_{B/A} + \vec{a}^t_{B/A}$

$= \vec{a}_A + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{AB}) + (\vec{\alpha}_3 \times \vec{AB})$

⊥^{r} to $AB\,\omega_3^2$



$\vec{V_C} = \vec{V_O} + \vec{V_{C/O}}$

$= OC \cdot \omega_3$

& $V_A = \vec{V_C} + \vec{V_{A/C}}$

$= V_C + AC \cdot \omega_4$

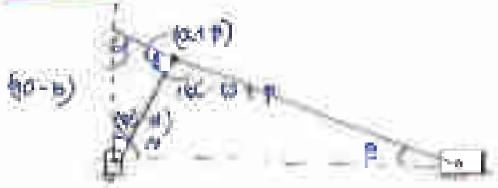
m d

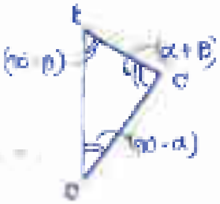
Apply sine Rule

$$\frac{OA}{\sin(\alpha+\beta)} = \frac{OC}{\sin(90-\beta)}$$

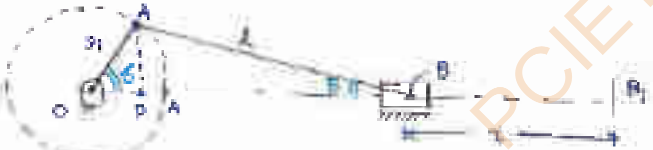
$$\boxed{V_A = V_C \sin(\alpha+\beta)\sec\beta}$$

Shortcut :-





⇒ Velocity and Acceleration analysis of slider crank mechanism (Analytic method)



— $n = l/r$

Displacement of slider $= x = B_1B$

$= B_1O - BO$

$= (r+l) - (OP + PB)$

$= (r+l) - (r\cos\theta + l\cos\beta)$

$= (r + nr) - (r\cos\theta + nr\cos\beta)$

$= r[(n+1) - (n\cos\beta + \cos\theta)]$

But $n = \frac{l}{r}$

$r\sin\theta = l\sin\beta$ (AP)

$\sin\theta = n\sin\beta$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{\sin^2\theta}{n^2}}$$

$$= \sqrt{\frac{n^2-\sin^2\theta}{n^2}} = \frac{\sqrt{n^2-\sin^2\theta}}{n}$$

so Displacement $= x = r[(n+1) - (n\cdot\frac{\sqrt{n^2-\sin^2\theta}}{n} + \cos\theta)]$

$$\boxed{\text{Displacement of slider/piston} \quad x = r[(1-\cos\theta) + (n-\sqrt{n^2-\sin^2\theta})]}$$

@ $\theta = 0°$ $\boxed{x = 0}$

$\theta = 180$ $\boxed{x = 2r}$

Hence $\boxed{\text{Stroke length} = 2 \times \text{Crank Radius}}$

Special case: $n = 1$

i.e $r = l$

$\boxed{r = l}$

$\boxed{\text{Stroke length} = 4r}$ (practically not possible)

→ Velocity of Slider :

$$v = \frac{dx}{dt} = \frac{d}{dt}\{r[(1-\cos\theta) + (n-\sqrt{n^2-\sin^2\theta})]\}$$

$$= r\left[\frac{d}{dt}(1-\cos\theta) + \frac{d}{dt}(n-\sqrt{n^2-\sin^2\theta})\right]$$

$$= r\left[0-(-\sin\theta)\frac{d\theta}{dt} + \{0-\frac{1}{2}(n^2-\sin^2\theta)^{-1/2}(0-2\sin\theta\cos\theta\frac{d\theta}{dt})\right]$$

$$= r\left[\sin\theta\cdot\omega + \frac{\sin2\theta}{2\sqrt{n^2-\sin^2\theta}}\cdot\omega\right]$$

$$= r\omega\left[\sin\theta + \frac{\sin2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

neglecting $\sin^2\theta$ ...

$$\boxed{V_{approx} = r\omega\left[\sin\theta + \frac{\sin2\theta}{2n}\right]}$$

@ $\theta = 90°$ ⇒ $\boxed{V_{slider} = r\omega}$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[r\omega\left\{\sin\theta + \frac{\sin 2\theta}{2n}\right\}\right]$$

$$= r\omega\left[\cos\theta\,\frac{d\theta}{dt} + \frac{2\cos 2\theta}{2n}\cdot\frac{d\theta}{dt}\right]$$

$$= r\omega\left[\cos\theta\,\omega + \frac{\cos 2\theta}{n}\,\omega\right]$$ 'ω' const

$$\boxed{a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]}$$ ← where ω' const

→ Angular velocity & Angular acceleration of connecting rod

GATE - 0

$$\sin\beta = \frac{\sin\theta}{n}$$

$$\frac{d\beta}{dt} = \omega_{CR}, \quad \frac{d\theta}{dt} = \omega_{crank}$$

$$\cos\beta\,\frac{d\beta}{dt} = \frac{1}{n}\cos\theta\cdot\frac{d\theta}{dt}$$

$$\Rightarrow \omega_{CR} = \frac{\omega\cos\theta}{n\cos\beta}$$

$$\left\{\begin{array}{l}\cos\beta = \sqrt{1-\frac{\sin^2\theta}{n^2}}\\ n\cos\beta = \sqrt{n^2-\sin^2\theta}\end{array}\right.$$

$$\boxed{\omega_{CR} = \frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}}$$

→ angular acceleration of connecting rod :

$$\alpha_{CR} = \frac{d\omega_{CR}}{dt}$$

$$= \frac{d}{dt}\left[\frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}\right]$$

$$= \frac{d}{dt}\left[\omega\cos\theta\,(n^2-\sin^2\theta)^{-1/2}\right]$$

$$= \frac{d}{dt}\,\omega\left[\cos\theta\left(-\frac{1}{2}\right)(n^2-\sin^2\theta)^{-3/2}\underset{(0-2\sin\theta\cos\theta)\frac{d\theta}{dt}}{} + (-\sin\theta)(n^2-\sin^2\theta)^{-1/2}\frac{d\theta}{dt}\right]$$

$$= \omega^2\left[\frac{\sin 2\theta\cdot\cos\theta}{2(n^2-\sin^2\theta)^{3/2}} - \frac{\sin\theta}{(n^2-\sin^2\theta)^{1/2}}\right]$$

$$\boxed{\alpha_{CR} = \omega^2\left[\frac{\sin 2\theta\cdot\cos\theta}{2(n^2-\sin^2\theta)^{3/2}} - \frac{\sin\theta}{(n^2-\sin^2\theta)^{1/2}}\right]}$$

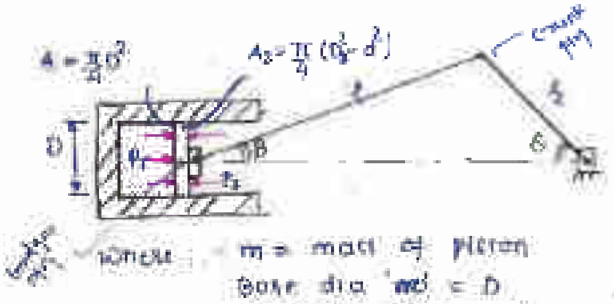
$\theta = 45^\circ$

$\omega = \text{const}$

$$\alpha_{CR} = \frac{d}{dt}\left[\frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}\right]$$

$\boxed{\alpha_{CR} = C}$ $\therefore$ ω const

→ Dynamic Force Analysis (in single slider mechanism)



where : m = mass of piston

bore dia 'and' = D

whist pin dia = d

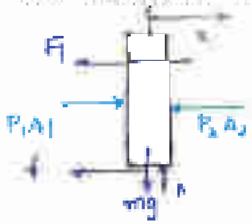
(or) c.R. dia

P_1 = pressure exerted by working substance

P_2 = pressure exerted by surrounding

(i) piston effort - net force acting on piston

in Horizontal engine



$$F_{P_{net}} = (P_1A_1 - P_2A_2) - f - F_I$$
$$= (P_1A_1 - P_2A_2) - f - m a_{piston}$$

$$\boxed{F_{P_{net}} = (P_1A_1 - P_2A_2) - f - m r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]}$$

force to slider

lubricating greater force than

In vertical Engine

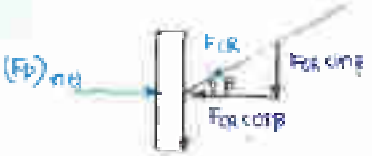
$$\boxed{F_{P_{net}} = (P_1A_1 - P_2A_2) - f - m r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right] \pm mg}$$

where : from TDC to BDC '+mg'
BDC to TDC '−mg'

NOTE:- ~~Force in connecting Rod~~ / ~~thrust in~~ gudgeon pin

— stress = [illegible]

— $(F_P)_{net} \geq$ slider force



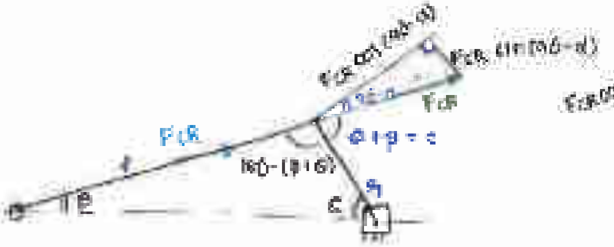
$F_{CR}\cos\beta = (F_P)_{net}$

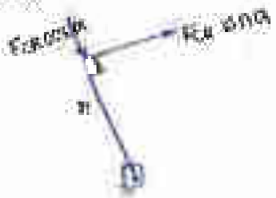
$$\boxed{F_{CR} = \frac{(F_P)_{net}}{\cos\beta}}$$

(iii) Normal thrust b/w cylinder wall & piston :-

$$\boxed{N = F_{CR}\sin\beta}$$

(iv) Turning moment in the crankshaft $\boxed{\vec{T} = \vec{R} \times \vec{F}}$





$\vec{T} = \vec{R_1} \times \vec{F}$

$T = F_{CR}\sin\alpha \cdot R_1$

$$\boxed{T = \frac{F_P}{\cos\beta}\,\sin(\theta+\beta)\cdot R_1}$$

$\left\{ F_{CR} = \frac{F_{P\,net}}{\cos\beta} \right.$

(v) Thrust force in crank pin

$= F_{CR}\cos\alpha$

$$\boxed{F_{T_{crank\,pin}} = F_{CR}\cos(\theta+\beta)}$$

✓

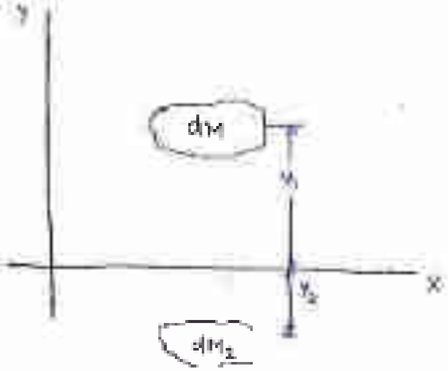
Note $T = f(\theta, \beta)$ but $\sin\beta = \frac{\sin\theta}{n}$

so $\boxed{T = f(\theta)}$

- Since the turning moment is function of θ (crank angle) & in order to run the engine we require uniform torque. Hence we require a device to make torque const. (Name of device is flywheel)
- If it is ask to calculate θ at which force in gudgeon pin changes its direction then solve it by taking $\boxed{F_{P\,effect} = 0}$ (for calculating gudgeon pin force)

piston effect = 0

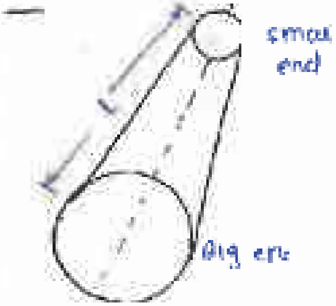
$$\boxed{x_{cm} = \frac{m_1x_1 + m_2x_2 + \dots}{m_1 + m_2 + \dots}}$$



— second moment of mass (dm_1) $= y_1(y_1 dm_1)$

$$\boxed{dI = y_1^2 dm_1}$$

—

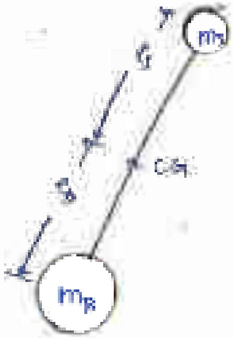


m = total mass of C.R

m_s = mass @ small end

m_b " @ big end

$\boxed{m = m_s + m_b}$ ← ①



$m_s l_s = m_b l_b$

$$\boxed{m_s = m_b \frac{l_b}{l_s}}$$

— $m = m_s + m_b = m_b + m_b \frac{l_b}{l_s}$

$$\boxed{m_b = \frac{m\, l_s}{l_b + l_s}}$$

$$\boxed{m_s = \frac{m\, l_b}{l_b + l_s}}$$

— $I_{act} = mk^2$

— $I_{eq\ system} = m_b l_b^2 + m_s l_s^2$

$I_{actual} = I_{act} = I_{eq\ sys}$

$\boxed{Ex}$ $l = 100$ cm , $l_b = 40$ cm

$m = 100$ kg $k = 80$ cm

$l_b + l_s = l \rightarrow \boxed{l_s = 60 \text{ cm}}$

— $m_b = \frac{m\, l_s}{l_s + l_b} = \frac{(100)(60)}{(60)+(40)} \Rightarrow \boxed{m_b = 60 \text{ kg}}$

— $m_s = \frac{m\, l_b}{l_s + l_b} = \frac{(100)(40)}{(60)+(40)} \Rightarrow \boxed{m_s = 40 \text{ kg}}$

— $I = m_b l_b^2 + m_s l_s^2 = (40)(0.4)^2 + (60)(0.6)^2$

$= 24$ kg-m²

$r = 20$ cm

$l = 80$ cm

→ $F_{t/CR} = \frac{F\cos(\theta+\phi)}{\sin\beta}$

$F_{t(CR)} = \frac{(F_p)_{net}}{\cos\beta}$

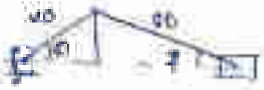
at mid of stroke → $\boxed{\theta = 90^\circ}$

→ $F_{t(CR)} = \frac{2\ kN}{\cos(14.50)}$

$\boxed{F_{t(CR)} = 2.065\ kN}$

$n = \frac{l}{r} = 4$

$\sin\beta = \frac{\sin\theta}{n} = \frac{1}{4} = \boxed{14.77 = \beta}$



→ Twisting moment

$T = \frac{(F_p)_{net}}{\cos\beta} \sin(\theta + \beta) \times R$

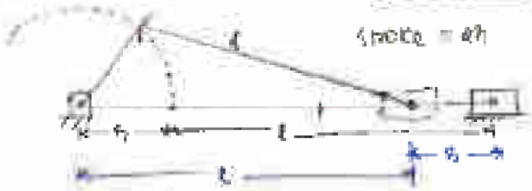
$= \frac{2\ kN}{\cos(14.50)} \sin(90 + 14.77) \times 0.2$

$\boxed{T = 0.4\ kN \cdot m}$

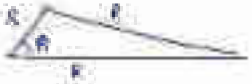
Analysis : when piston is at middle of stroke length

$\boxed{\theta = 82.82^\circ}$

Stroke = 2r



θ = 90° is not position → $l^2 = r^2 + x^2$ ≯



$l^2 = l^2 + r^2 - 2lr\cos\theta$

$r^2 = 2lr\cos\theta$

$\cos\theta = \frac{r}{2l}$

$\theta = \cos^{-1}\left[\frac{1}{8}\right]$

$\boxed{\theta = 82.81^\circ}$

[58] $T = I\alpha$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

at $t = 0$ $\omega = 0$ $\theta = \frac{1}{2}\alpha t^2$

θ ↑ average increasing → 0, π, 2π, 3π → ⓑ ×

← has operation not linear

B.11

$r = 900$ mm

$N = 600$

$\theta = 90^\circ$

$F_t = 5$ kN $= (F_P)$

$\rightarrow T = \dfrac{(F_P)_{\max} \sin(\theta + \phi)}{\cos\phi} \cdot r$

$= \dfrac{5 \cdot \sin(90 + \phi)}{\cos\phi} (0.2)$

$\boxed{T = 1 \text{ kN-m}}$

$\sin\phi = \dfrac{\sin\theta}{n}$ $n = 4$

$= \dfrac{1}{4} \Rightarrow \boxed{\phi = 14.47}$

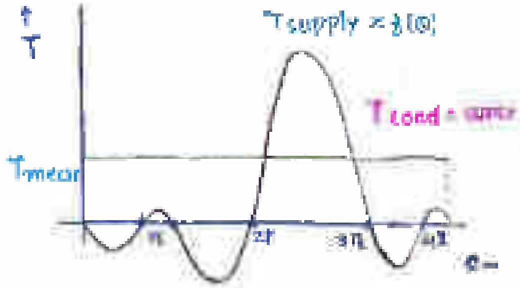
Energy Reservoirs

- Thermal Reservoirs [TER]
  - Source
  - Sink
- Mechanical Energy Reservoirs [MER]
  - Ex – flywheel

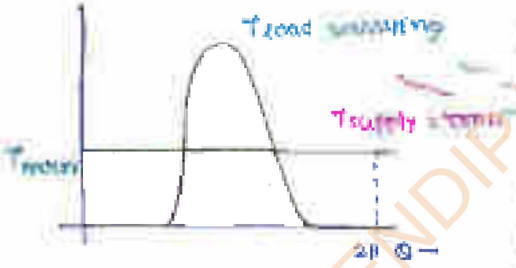
Rot. K.E $= \frac{1}{2} I \omega^2$

→ Uses of flywheel :

Uses
- In Engine
- In punching press



4 - stroke single cylinder I.C engine



→ Flywheel in engines

(i) $T_{act} > T_m$

$T_f = T_{act} - T_m$

flywheel will store energy

$\left(\frac{1}{2} I \omega^2\right) \uparrow$

$\omega \uparrow$

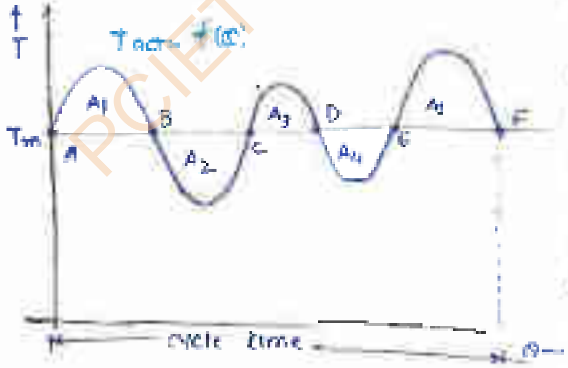
flywheel accelerating

(ii) $T_{act} < T_m$

flywheel will supply energy

$\omega \downarrow$

flywheel decelerating



⇒ B = $E_B = E_A$ + area of T vs θ dia b/w A & B

$$\Rightarrow E_B = E_A + \int_{\theta_A}^{\theta_B} (T_{act} - T_m)\, d\theta$$

$$\boxed{E_B = E_A + A_1}$$

$$\Rightarrow \boxed{E_C = E_B - A_2}$$

$$E_C = E_A + A_1 - A_2$$

$$\Rightarrow \boxed{E_D = E_C + A_3}$$

$$= E_A + A_1 - A_2 + A_3$$

$$\Rightarrow \boxed{E_E = E_D - A_4}$$

$$= E_A + A_1 - A_2 + A_3 - A_4$$

$$\Rightarrow \boxed{E_F = E_E + A_5}$$

$$= E_A + A_1 - A_2 + A_3 - A_4 + A_5$$

→ $\boxed{E_F = E_A}$

$$A_1 - A_2 + A_3 - A_4 + A_5 = 0$$

$$\boxed{A_1 + A_3 + A_5 = A_2 + A_4}$$

— let E_B is max

E_C is min

$E_{max} - E_{min}$ = max^n fluctuation of $(\Delta K.E)_{max}$

$$= \frac{1}{2} I \omega_{max}^2 - \frac{1}{2} I \omega_{min}^2$$

$$\boxed{(\Delta K.E)_{max} = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)}$$

→ $(\Delta K.E)_{max} = \frac{1}{2} I (\omega_{max} - \omega_{min})(\omega_{max} + \omega_{min})$

$$\boxed{(\Delta K.E)_{max} = I(\omega_{max} - \omega_{min})\, \omega_{mean}}$$

Here ω_{max} & ω_{min} close to each other so take arithmetic mean → ω_{mean}

→ $(\Delta K.E)_{max} = I(\omega_{mean})^2 \left(\frac{\omega_{max} - \omega_{min}}{\omega_{mean}}\right)$

$$\boxed{(\Delta K.E)_{max} = I \omega_m^2 \cdot C_s} \quad \Rightarrow \quad \boxed{\Delta E = 2 E C_s}$$

where, $C_s = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}}$

C_s = co-efficent of fluction of speed

$C_s = \frac{\text{max}^n \text{ fluctuation of speed}}{\text{mean speed}}$

$\omega_{max} = 205$

$\omega_{mean} = 200$

$\omega_{min} = 195$

$\omega_{max} - \omega_{min} = C_s\, \omega_{mean}$

$$\boxed{\frac{N_{max} - N_{min}}{N_{mean}} = C_s}$$

— If C_s value is small ⇒ fluctuation is small

Note :-
The prime job of flywheel is to reduce the speed fluctuation in a cycle.

	C_s
Pumps	1/10 - 1/20
Reciprocating I.C. eng:	1/100 - 1/20
Aircraft	1/1000

→ fluctuation → vibration → Dynamic loading → strength (↓) → failure (↑)

'C_s' should be small

$(\Delta K.E)_{max} = I \omega_m^2 C_s$

$I C_s$ = const

$$\boxed{C_s \propto \frac{1}{I}}$$

$\boxed{I = mR^2}$ (Ring flywheel)

$\boxed{I = \frac{mR^2}{2}}$ (Disc flywheel)

→ If nothing is mentioned in problem take Ring flywheel $\boxed{I = mR^2}$ — because it having less fluctuation of speed compare to disc.

[*] Co-efficient of fluctuation of energy (C_E)

→ $C_E = \frac{\text{max}^n \text{ fluctuation of energy}}{\text{work done / cycle}} = \frac{(\Delta KE)_{max}}{WD/cycle}$

$\boxed{\text{Work done / cycle = net area of T vs } \theta \text{ diag}}$

— W.D/cycle = net energy Reqⁿ per cycle

$\boxed{\text{W.D/cycle} = T_{mean} \times \text{cycle time}}$

Two stroke engine → $\boxed{\text{cycle time} = 2\pi}$

Resisting torque is const T_m = const N = 1000 rpm
P = ?

$P = \int T d\theta = \int 10000$

$$WD = \int T d\theta = \int_0^{2\pi} (10000 + 1000 \sin 2\theta - 1200 \cos 2\theta)\, d\theta$$

$$= 10000(2\pi) + 1000\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} - 1200\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= 20000\pi - 500[\cos 4\pi - \cos 0] - 600[\sin 4\pi - \sin 0]$$

$$\boxed{WD/\text{cycle} = 62800 \text{ J}}$$

W/D cycle $= 10000(2\pi) = T_{mean} \times 2\pi$

$$\boxed{T_{mean} = 10000 \text{ N·m}}$$

$\Rightarrow P = \frac{2\pi \times 10000}{60} \Rightarrow \boxed{P = 104.7 \text{ kW}}$

→ Any eqn of T the Block Region [ ] indicates the $\boxed{T_{mean}}$

→ Two stroke Engine → cycle time = 2π
  Four stroke Engine → cycle time = 4π

[1c]

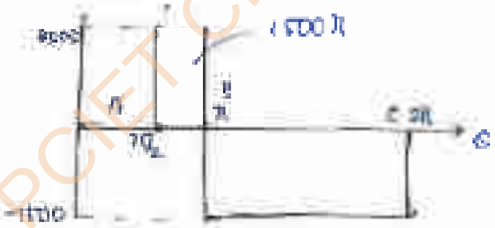
W/D/cycle = net area of +ve ⊙ dia

$= 3000 \times \frac{\pi}{2} - 1500 \times \pi$

$$\boxed{WD/\text{cycle} = 0}$$

W.D/cycle $= T_m \times 2\pi = 0$

$$\boxed{T_m = 0}$$

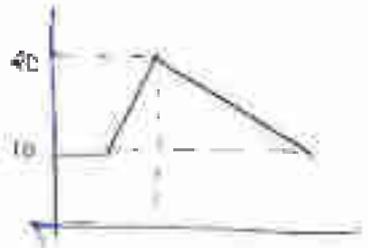


→ E_A

→ $E_B = E_A + 1500\pi$ ← max

$E_C = E_A + 1500\pi - 1500\pi$

$= E_A$ ← min



$E_{max} - E_{min} = E_A + 1500\pi - E_A$

$\Rightarrow \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2) = 1500\pi$

$\Rightarrow \frac{1}{2} I (40^2 - 10^2) = 1500\pi \Rightarrow \boxed{I = 6.28 \text{ kg·m}^2}$

14

$N_{avg} = 400$ RPM

$C_s = \pm 0.5\% = 1\%$

→ $(\Delta K.E) = I \omega_{mean}^2 C_s$

$2600 = I \left(\frac{2\pi \times 200}{60}\right)^2 \times \left(\frac{1}{100}\right)$

$2600 = I (4.38)$

$$\boxed{I = 593.6 \text{ kg}\cdot\text{m}^2}$$

16

M → Rim, $I = mR^2$; disc, $I = \frac{mR^2}{2}$ → $R/2$

→ $\frac{M}{2}$, R ; $\frac{M}{2}$, $R/2$

$I = I_R + I_m$

$= mR^2 + \frac{mR^2}{2} = \frac{m}{2}R^2 + \frac{m}{2(2)}\left(\frac{R}{2}\right)^2$

$= \frac{mR^2}{2} + \frac{mR^2}{16}$

$$\boxed{I = \frac{9}{16} mR^2}$$

~~$I = I_R + I_M$~~

~~$= mR^2 + \frac{mR^2}{2}$~~

~~$= \frac{mR^2}{2} + \frac{mR^2}{4}$~~

~~$\boxed{I = \frac{3}{4} mR^2}$~~

15

$T = 400$ N·m

$N = 120$ RPM

$C_s = \pm 2\% = 0.04$

→ $(\Delta K.E) = I \omega_{mean}^2 C_s$

$400 = I \left(\frac{2\pi \times 120}{60}\right)^2 (0.04)$

$$\boxed{I = 65 \text{ kg m}^2}$$

12

$C_s = \pm 2\% = 4\%$

$N_{mean} = 600$ rpm → $\omega_{mean} = 62.8$

$(K.E)_{max} = 5000$ J

$5000 = I (62.8)^2 \left(\frac{4}{100}\right)$ $\qquad I\omega^2 C_s = E_m$

$I = 31.69$ kg m²

→ another flywheel attached same size $I_2 = 2I$

$C_s \propto \frac{1}{I} \Rightarrow \frac{C_{s2}}{C_{s1}} = \frac{I_1}{I_2} \Rightarrow \boxed{C_{s2} = 0.02}$

$R_1 = R_1 \rightarrow R_2 = 2R$

$M_1 = M_1 \rightarrow M_2 = M$

$\rightarrow (K \cdot E) = I\omega_m^2 C_s$

$\boxed{C_s \propto \frac{1}{I}} \rightarrow \frac{C_{s_2}}{C_{s_1}} = \frac{I_1}{I_2} = \frac{\frac{1}{2} m R_1^2}{\frac{1}{2} m_2 R_2^2} = \frac{R_1^2 \cdot 4}{(R_1)^2 \cdot 4}$

$\boxed{C_{s_2} = 0.01} \rightarrow 1\%$

[10] E_A

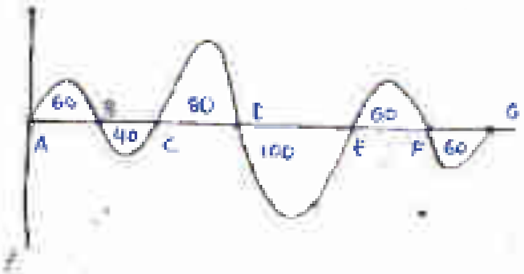
$E_B = E_A + 60$

$E_C = E_A + 60 - 40 = E_A + 20$

$E_D = E_A + 20 + 80 = E_A + 100$

$E_D > E_B > E_C > E_F$

$\boxed{R > P > Q > S}$

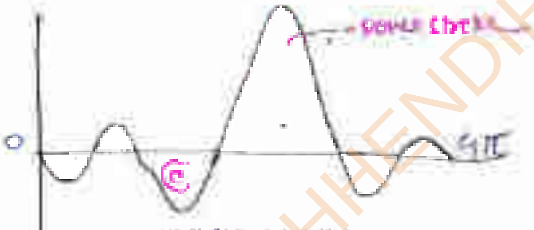


[9]

— If there is more than one power stroke then multi cylinder

— Power stroke – 720 – 4π

— comp stroke – 180 – 360



1 cm² = 100 N·m

$T_m \times 4\pi = [-0.5 + 1 - 2 + 2.5 - 0.5 + 0.5] 100$

$T_m = \frac{100}{\pi}$ N-m

[8]

$E_A = E_O + 100$

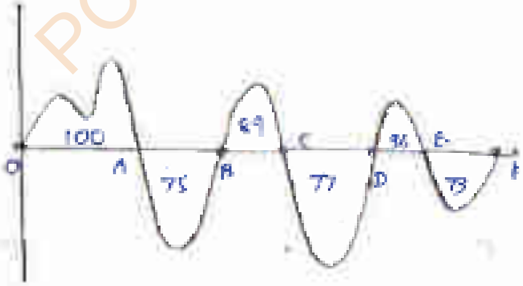
$E_B = E_O + 25$

$E_C = E_O + 119$

$E_D = E_O + 37$

$E_F = E_O + 73$

$\boxed{E_F = E_O}$ true



[7] N = 1200 rpm , I = 1 , C_s = 4%

$N_{avg} = \frac{N}{2}$, C_s = 4%

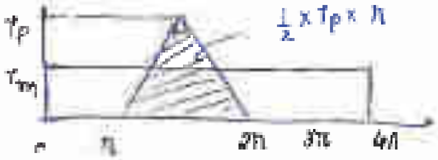
$I_1 \omega_{mean_1}^2 C_{s_1} = I_2 \omega_{mean_2}^2 C_{s_2}$

$I_1 (1200)^2 (4\%) = I_2 (600)^2 (4\%)$

$T_m \times 4\pi = \frac{1}{2} \times T_p \times \frac{\pi}{4}$

$10 \times 4\pi = T_p \times \frac{\pi}{4}$

$\boxed{T_p = 320 \text{ N m}}$



5

$\eta = 0.5$

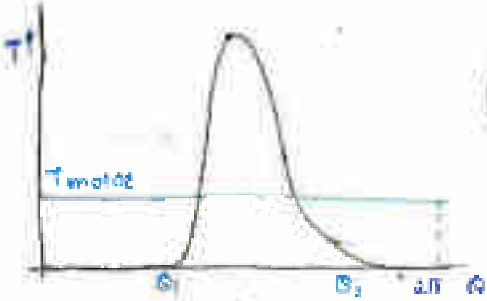
$N = 280$ rpm

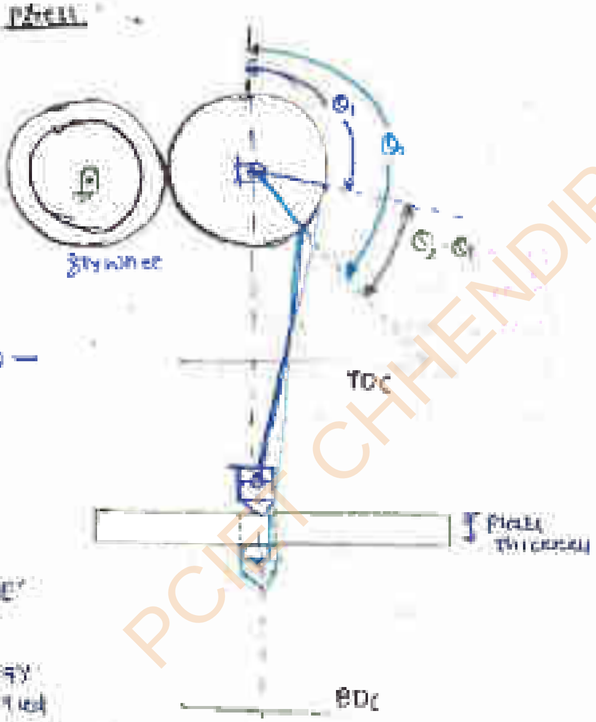
600 holes → per hr

$600/_{60\times60} = 0.166$ Hole/s

$t = 0.45$

→ Flywheel in punching press





Energy supply by motor / cycle

$\boxed{E_{motor} = T_m \times 2\pi}$

→ let energy req^d per cycle 'E'

Energy req^d for punching = Energy consumed by motor during punching + Energy supplied by flywheel

$E_{flywheel}$ = Energy req^d during punching − Energy req^d supplied by motor during punching

$2\pi \longrightarrow E$

$1 \longrightarrow E/_{2\pi}$

$\theta_2 - \theta_1 \longrightarrow \frac{E}{2\pi}(\theta_2 - \theta_1)$

$E_{flywheel} = E - \frac{E}{2\pi}(\theta_2 - \theta_1)$

$$\boxed{E_{flywheel} = E\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)}$$ → This valid for when idle stroke is not consume any energy

where $E_{flywheel} = \frac{1}{2} I(\omega_{max}^2 - \omega_{min}^2)$

$= I\omega_{mean}^2 C_f$

$$\boxed{\frac{\theta_2 - \theta_1}{2\pi} = \frac{\text{thickness of plate}}{2 \times \text{stroke}} = \frac{\text{punching time}}{\text{cycle time}}}$$

5. $K = 0.5$ m (RGm)

$N_1 = 260$ rpm ← $N_2 = 230$ r.pm (speed should not drop so
{ N_1 & N_2 is very close

600 holes per Hr

punching time = 1.5 sec

Energy req^n = 10,000 J

P_{motor} = 2 kW

m = (?)

→ $E_{flywheel} = E_{req^n} - E_{supply \text{ during punching}}$

{ P_{motor} = 2000 J/s
1 sec → 2000 J
1.5 sec → (?) =

$= 10000 - 3000$

$\boxed{E_f = 7000}$

→ $\frac{1}{2} I(\omega_{max}^2 - \omega_{min}^2) = 7000$

$I \left(\frac{2\pi}{60}\right)^2 (260^2 - 230^2) = 14000$

17 56
26 77

$\boxed{I = 86.93 \text{ kg m}^2}$

→ $I = mK^2$

$86.93 = m(0.5)^2$

$\boxed{m = 347.7 \text{ kg}}$

4. d = 40 mm, t = 30 mm ; $\tau_u = 7$ Nm/mm^2 (Shear area)

Stroke = 100 mm , t = 10 sec → cycle time

$V_{mean} = 25$ m/s , C_f = 8% = 0.08

P_{motor} = (?) F_4

← Sheared area $A = \pi d t = 3765$ mm^2

1 mm^2 → 7 N-m

3765 m^2 → (?)

$\boxed{E_{punch} = 26370 \text{ J}}$

✓ by the motor in one cycle

(supplying energy it may assume)

In 10 sec energy supplied by motor $= 26576.0\ J$

1 sec " " " $= 2657.6\ J/s$

$P_m = 2.65\ kW$

— $E_f = E_{req^n} - E_{supply}$ by motor during punching

$= 26376.6 - 2957.6$

~~= 25358.3~~

$= 23198.4$

— plate thickness

— $E_F = E\left[1 - \frac{\theta_2 - \theta_1}{2\pi}\right]$ check

$= E\left[1 - \frac{50}{2\times100}\right]$

$I\omega_m^2 C_s = 22419.6$ Joule

$I = mk^2$

$k\,\theta = v$

$\boxed{V_m = k\omega}$

R.o.g $= k = r_m$

$\underbrace{\omega_m^2\ m(k^2)}_{V^2}\ C_s = 22419.6$

$m = \frac{22419.6}{(60)^2 \times (0.02)}$

$\boxed{m = 1195.71\ kg}$

[1] Four stroke cycle 4π = 4π

$P = 60\ kW$

$N = 300$ rpm

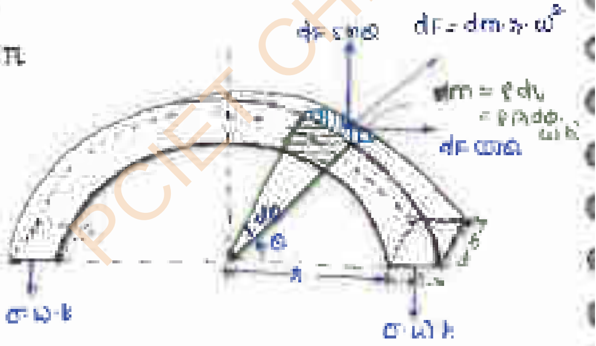
$C_e = 0.7$

$C_s = 0.02$

$\sigma_{max} = 6\ MN/m^2$

$r_m = (?)$

$\rho = 7200\ kg/m^3$



$2\sigma wb = \int dF \sin\theta$

$2\sigma wb = \int dm\, r\, \omega^2 \sin\theta$

$= \int (r\,d\theta\cdot w\cdot b)\,\rho\cdot r\,\omega^2\cdot \sin\theta$

$2\sigma = \int_0^{\pi} (\rho r\,d\theta)\, r\,\omega^2 \sin\theta\, d\theta$

$= \rho r^2\omega^2 \int_0^{\pi} \sin\theta\, d\theta$

$2\sigma = \rho v_m^2 [-\cos\theta]_0^{\pi} = \rho v_m^2 [-\cos\pi + \cos 0]$

$\boxed{\sigma = \rho v_m^2}$

$\rho = 7800\ kg/m^3$

$\sigma = \rho V_m^2$

$V_m = \sqrt{\frac{5000000}{7800}} \Rightarrow \boxed{V_m = 25.25\ m/s}$

→ $V_m = \frac{\pi D_m N}{60} \Rightarrow D_m = \frac{25.25 \times 60}{\pi \times 300}$

$\boxed{D_m = 1.6\ m}$

→ $(K.E)_{max} = I \omega_m^2 C_s$

$C_E = \frac{(\Delta K.E)_{max}}{W.D/cycle}$

$W.D/cycle = T_m \times cycle\ time$

$P = \frac{2\pi N T_m}{60}$

$T_m = \frac{60 \times 80 \times 10^3}{2\pi \times 300}$

$\boxed{T_m = 2546.47\ N\text{-}m}$

→ $W.D/cycle = 2546.47 \times 4\pi$

$\boxed{W.D/cycle = 32000\ J}$

$C_E = \frac{(\Delta KE)_{max}}{W.D/cycle}$

$(\Delta KE)_{max} = 32000 \times 0.9$

$\boxed{(\Delta KE)_{max} = 28800\ J} = I \omega_{mean}^2 C_s$

→ $28800 = I \left(\frac{2\pi \times 300}{60}\right)^2 \times 0.02$

$\boxed{I = 1459.02\ kg\text{-}m^2}$

2 Four stroke engine :

cycle time = 4π

$C_s = 0.01$

W.D/cycle = net area of T vs θ dia.

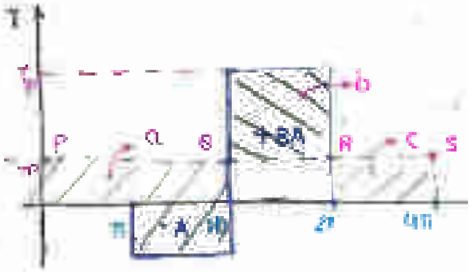
$= -A + 3A = 2A$

power = 80 kW

$N_m = 240$ rpm

$80 \times 10^3 = \frac{2\pi \times 240 \times T_m}{60}$

⇒ $\boxed{T_m = 3185.7\ Nm}$



$T_m \times 4\pi = 2A$

$795.7 \times 4\pi = 2A$

$\boxed{A = 5000}$

→ $T_p \times \pi = 15000 = 3A$

$T_p = \frac{15000}{\pi}$

$\boxed{T_p = 4774.64 \text{ N·m}}$

~ E_p

$E_Q = E_p - a \Rightarrow E_{min}$

$E_R = E_p - a + b \Rightarrow E_{max}$

$E_S = E_p$

$\Delta E = E_R - E_Q$
$= b$

$b = (T_p - T_m)\pi$

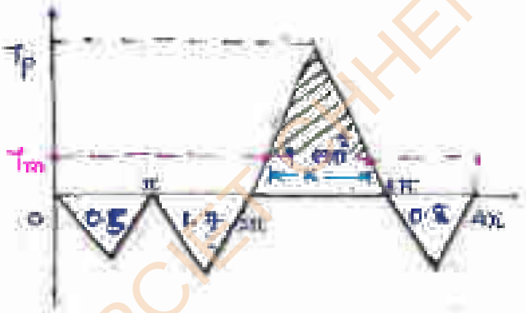
$I\omega_m^2 C_s = (T_p - T_m)\pi$

$\boxed{I = 1976.5 \text{ kg m}^2}$

$A = -0.5 - 1.7 + 9 - 0.8$
$= -3 + 9$
$= 6$

$\Rightarrow 1 \text{ cm}^2 = 1400 \text{ J}$

$6 \text{ cm}^2 = 8400 \text{ J}$



WD/cycle = net area bet^n T vs θ dia.

$T_m \times 4\pi = 8400$

$\boxed{T_m = 668.45 \text{ N-m}}$

→ Expansion stroke ⇒ $\frac{1}{2} \times T_p \times \pi = 9 \times 1400$

$\boxed{T_p = 8021.47 \text{ N·m}}$

→ for energy fluctuation at C it is min & hatched portion is max fluctuation area $(T_p - T_m)$ triangle

$\frac{x}{\pi} = \frac{T_p - T_m}{T_p} \Rightarrow \boxed{x = 2.879 \text{ rad}}$

$$\frac{1}{2}I(\omega_1^2-\omega_2^2) = \frac{1}{2}(2\cdot 780)(6021.00 - 565.65)$$

$$I(\omega_1^2-\omega_2^2) = 21147.11$$

$$I\left(\frac{2\pi}{60}\right)^2(102^2-98^2) = 21147.11$$

$$\boxed{I = 2412.9\ kg\ m^2}$$

13. Power of punching m/c = 2 kW (motor)

cycle time is = T

punching stroke | idle stroke

cycle time: $\frac{T}{4}$ | $\frac{3T}{4}$

Energy: $\frac{3}{4}\times(2T) = \frac{3}{2}T$ | $\frac{1}{4}\times(2T) = \frac{T}{2}$

1 sec → 2 kJ

T sec → 2T

total Energy = 2T

$$\rightarrow C_E = \frac{\text{max}^n \text{ fluctuation of energy}}{\text{W.D / cycle}} \rightarrow (\text{flywheel energy})$$

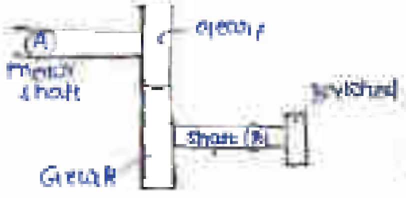
E flywheel = total energy Req^n for punching − energy supplied by motor during punching

$$= \frac{3T}{2} - (2T)\left(\frac{T}{4}\right)$$

$$= T$$

$$C_E = \frac{T}{2T} = 0.5 \Rightarrow \boxed{C_E = 0.5}$$

14.



reduction ratio = 4

$$\frac{\omega_A}{\omega_B} = \frac{4}{1} = \frac{n_B}{n_A}$$

→ let by ratio :

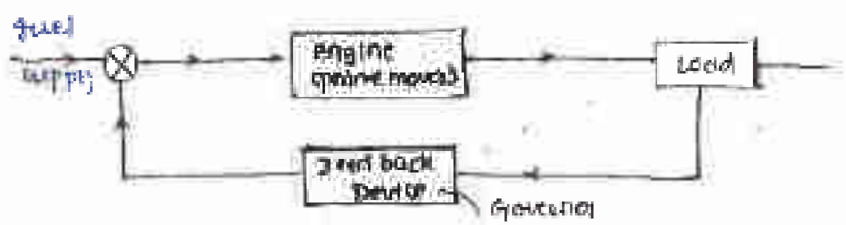
$$(\Delta K.E)_{max,B} = (\Delta K.E)_{max,A}$$

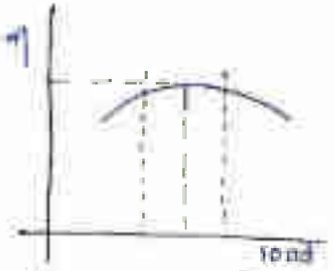
$$I(\omega_{m_B}^2)\cdot C_{s_1} = I(\omega_{m_A}^2)\cdot C_{s_2}$$

$$I(\omega_B)^2(0.004) = I(\omega_A)^2\cdot C_{s_2}$$

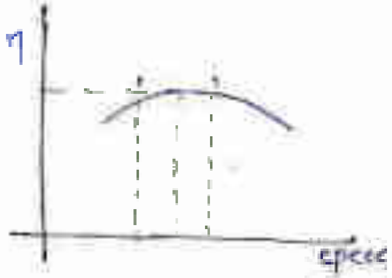
$$\boxed{C_{s_2} = 0.004} = \pm 0.2$$

$$= \pm 0.2\% \text{ (answer)}$$





(Load vs η)

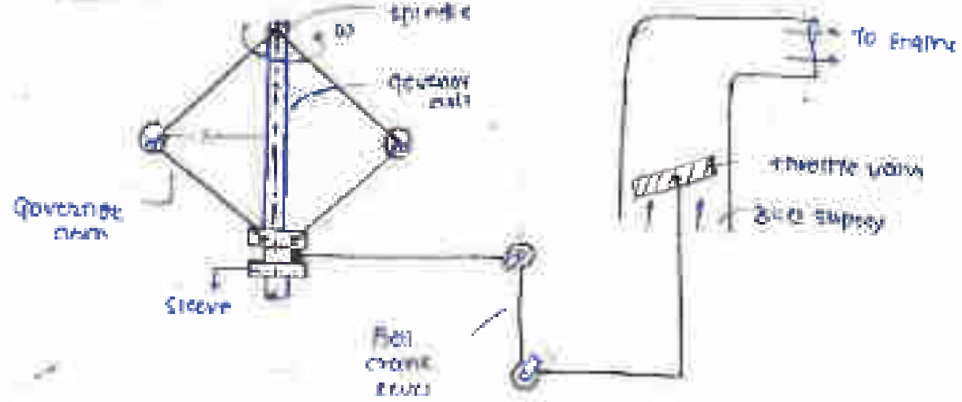


(η vs speed)

- Governors are feedback devices which regulates the fuel supply whenever there is variation in load at output of primemover.

Flywheel	Governor
- Flywheel works continuously (each stroke)	- Governor works intermediately
- Flywheel regulates the speed fluctuation within a cycle.	- Governor regulates the speed fluctuation in between two cycle.
- Flywheel regulates each cycle (within) fluctuation	- Governors regulates entire cycle (between) fluctuation

⇒ Working of Governor

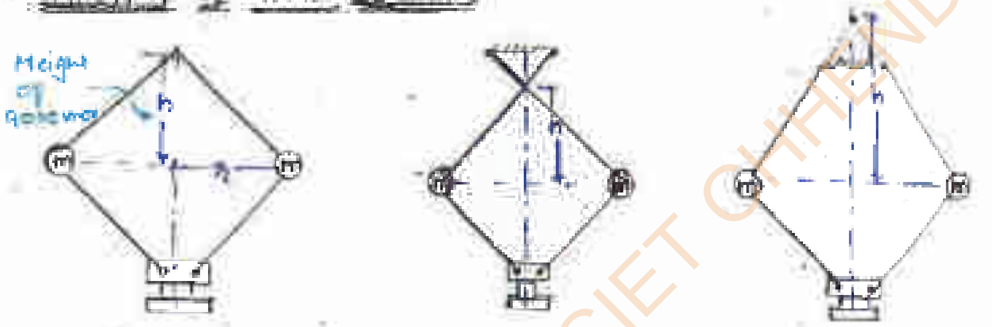


or ↓
Sleeve comes down
throttle valve open
fuel supply ↑

→ Classification of Governors:

- Governors
  - Centrifugal Governors
    - Pendulum type (PSU)
      - Watt Governor (Q)
    - Gravity controlled
      - Porter (Q)
      - Proell (Q)
    - Spring controlled
      - Hartnell ✓
      - Wilson Hartnell
      - Hartung
  - Inertial Governors
    - Q: Inertia

→ Analysis of Watt Governor:



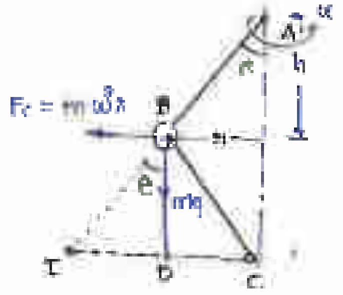
Simple watt governor

crossed arm watt

open arm type watt governor

Height of Governor — Distance between plane carrying governor balls to the point where governor arms intersect with governor axis

→ Analysis of watt governor



- Sleeve mass negligible
- friction of governor arms negligible

$\Sigma M_I = 0$

$\Rightarrow m x \omega^2 BD = mg\, ID$

$\Rightarrow \omega^2 r = g\left(\frac{ID}{BD}\right)$

$\Rightarrow \omega^2 r = g \tan\theta$

$\Rightarrow \boxed{\omega^2 = \frac{g}{h}}$

$\tan\theta = \frac{r}{h}$

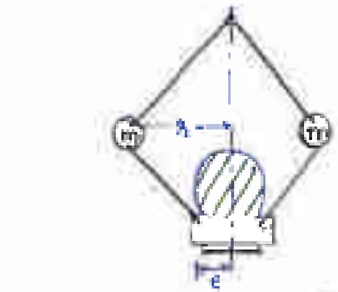
h

→ $\omega^2 \propto \frac{1}{h}$ or $\omega \propto \frac{1}{\sqrt{h}}$ ⇒ $\boxed{\frac{\omega_1}{\omega_2} = \sqrt{\frac{h_2}{h_1}}}$

→ $\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}$

$\boxed{h = \frac{895}{N^2}}$ → $N^2 = \frac{895}{h}$

Q) Porter Governor



m = mass of balls
M = sleeve mass

→ inertia of governor arms are negligible

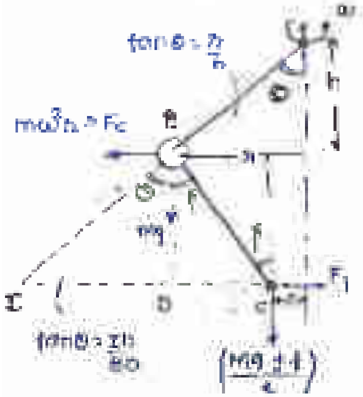
$\Sigma M_I = 0$ (ACW +ve)

→ $m r \omega^2 (BD) - mg(DI) - \left(\frac{Mg \pm f}{2}\right) IC = 0$

⇒ $m r \omega^2 (BD) = mg(DI) + \left(\frac{Mg \pm f}{2}\right) IC$

$= mg\left(\frac{DI}{BD}\right) + \left(\frac{Mg \pm f}{2}\right)\left(\frac{IC}{BD}\right)$

$= mg\left(\frac{DI}{BD}\right) + \left(\frac{Mg \pm f}{2}\right)\left(\frac{CD}{BD} + \frac{DI}{BD}\right)$



$m r \omega^2 = mg \tan\theta + \left(\frac{Mg \pm f}{2}\right)(\tan\beta + \tan\theta)$

$m\omega^2 r = \tan\theta\left[mg + \left(\frac{Mg \pm f}{2}\right)\left(1 + \frac{\tan\beta}{\tan\theta}\right)\right]$

$= \frac{r}{h}\left[mg + \left(\frac{Mg \pm f}{2}\right)(1 + K)\right]$

$\left\{ K = \frac{\tan\beta}{\tan\theta} \right.$

$$\boxed{\omega^2 = \frac{2mg + (Mg \pm f)(1+K)}{2mh}}$$

where, $K = \frac{\tan\beta}{\tan\theta}$.

Special case

if $K = 1$ i.e. $\theta = \beta$

i.e. Governor arms are equal length and both arms are pivoted on governor axis

2mh

$$\boxed{\omega^2 = \frac{mg + (Mg \pm f)}{mh}}$$

→ 1) friction is neglected

$$\boxed{\omega^2 = \left(\frac{m+M}{m}\right)\cdot\frac{g}{h}}$$

when (friction neglected)

→ 2) sleeve mass neglected

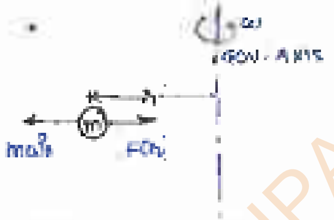
$$\boxed{\omega^2 = \frac{g}{h}}$$

(when sleeve mass neglected) → Watt Governor

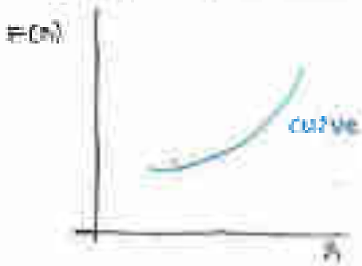
Governor terminology

1) Controlling force | Resisting force

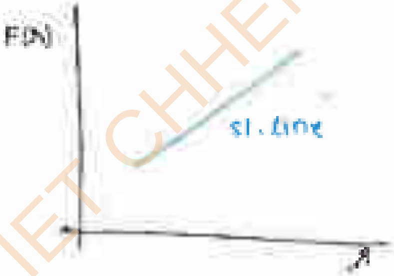
- The resultant of all frictional force that forces (in centrifugal governor) & spring forces is known as controlling forces



for gravity controlled governor



for spring controlled gov



Slope of controlling force curve = $\frac{dF(r)}{dr}$

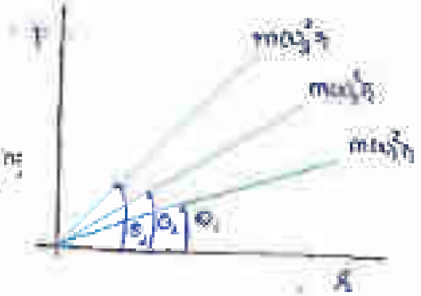
→ Centrifugal force $\boxed{F = mr\omega^2}$

Slope of centrifugal force curve $= m\omega^2 = \frac{F}{r}$

$$\boxed{\tan\theta_3 > \tan\theta_2 > \tan\theta}$$

→ by Increasing speed

$$\boxed{\omega_3 > \omega_2 > \omega_1}$$

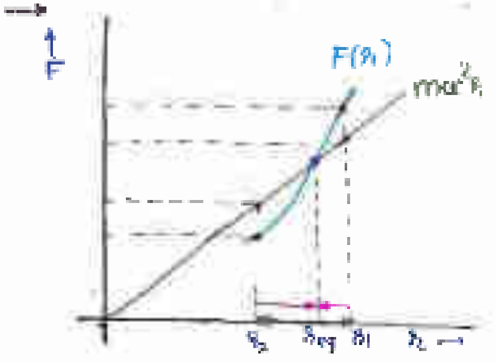


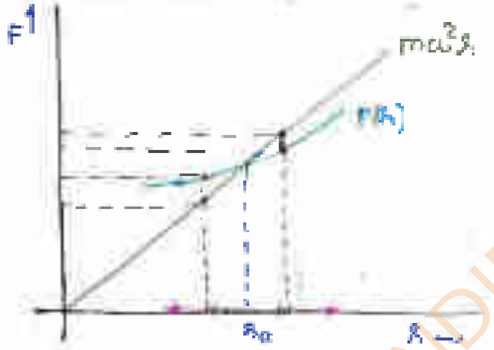
(e) Types of equilibrium

a] stable eq^n →

b] unstable eq^n →

c] Neutral eq^n →





Stable governor

slope of restoring force curve > slope of disturbing force

→ $\frac{dF(r)}{dr} > \frac{F}{r}$

⇒ $\boxed{\frac{dF(r)}{dr} > m\omega^2}$

unstable governor

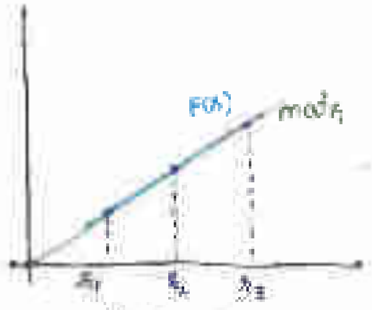
slope of restoring force < slope of disturbing force

$\boxed{\frac{dF(r)}{dr} < \frac{F}{r}}$

$\boxed{\frac{dF(r)}{dr} < m\omega^2}$

→ restoring force → decrease speed (↓)
disturbing force → increase

\# Neutral equation ⇒ [Isochronous governor]



slope of restoring = slope of
force curve   disturbing force

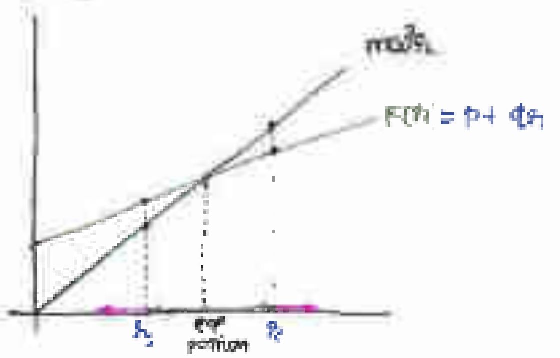
$\frac{dF(r)}{dr} = \frac{F}{r}$

$\boxed{\frac{dF(r)}{dr} = m\omega^2}$

→ practically it is not possible

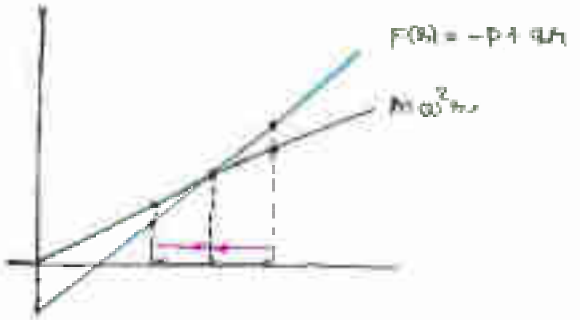
$r_1 < r_2 < r_3$

$\boxed{\omega_1 = \omega_2 = \omega_3 = \text{const}}$



unstable eqⁿ:
p & q are "+ve"
$p > 0$
$q > 0$

$\boxed{\text{unstable}}$



stable eqⁿ:
$p < 0$; −ve
$q > 0$; +ve

→ Range:

$\boxed{\text{Range of governor} = \omega_{max} - \omega_{min}}$

Ex. Range of isochronous governor = 0

→ Sensitiveness:

- A ability of governor to sense the change in output load & have a sleeve displacement accordingly.
- If for a same change in load sleeve displacement of governor A is more than that of governor B then governor is said to be more sensitive than B.

→ Co-efficient of sensitiveness or sensitiveness of governor.

$$\boxed{\text{co-efficient of sensitiveness (or) sensitiveness} = \frac{\omega_{mean}}{\omega_1 - \omega_2}}$$ ← GATE, ISRO IES, PSC

Ex. sensitiveness of isochronous governor is ∞

$$\text{co-efficient of sensitiveness (or) sensitiveness} = \frac{\omega_1 - \omega_2}{\omega_{mean}}$$ ← Check from

- When a governor is connected to some prime mover at that it will take the fluctuation of the prime mover & its sensitivity will be $\frac{N_1 - N_2}{N_{mean}}$

- If sensitiveness of governor is large then rather than working at some equilibrium position the sleeve will tend to oscillate that is vibrate about mean position, so this [illegible] governor is said to be hunting

Effort of Governor

- Mean force exerted on the sleeve is known as effort

Porter governor

$$h = \frac{2mg + (Mg \pm f)(1+K)}{2m\omega^2} \quad \text{—} \; (i)$$

let $\boxed{\omega_1 = (1+c)\,\omega}$ fraction

$$\omega_1^2 = \frac{2mg + (Mg \pm f)(1+K)}{2mh_1}$$

if $\omega_1 > \omega$
sleeve moves up.

$$\omega_1^2 = \frac{2mg + (Mg \pm f + E)(1+K)}{2mh}$$

$$h = \frac{2mg + (Mg \pm f + E)(1+K)}{2m\omega_1^2} \quad \text{—} \; (ii)$$

From eq^n (i) & (ii)

$$\Rightarrow \frac{2mg + (Mg \pm f)(1+K)}{2m\omega^2} = \frac{2mg + (Mg \pm f + E)(1+K)}{2m\omega_1^2}$$

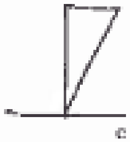
$$\Rightarrow \frac{2mg + (Mg \pm f)(1+K)}{2m\omega^2} = \frac{2mg + (Mg \pm f + E)(1+K)}{2m(1+c)^2\omega^2}$$

$$\Rightarrow [2mg + (Mg \pm f)(1+K)](1 + c^2 + 2c) = 2mg + (Mg \pm f + E)(1+K)$$

$$\Rightarrow 2mg + (Mg \pm f)(1+K) + 2c\,[2mg + (Mg \pm f)(1+K)]$$
$$= 2mg + (Mg \pm f)(1+K) + E(1+K)$$

$$\boxed{\frac{E}{2} = \frac{c}{(1+K)}\,[2mg + (Mg \pm f)(1+K)]}$$

effort of governor



It is a mean force which varies from 0 to maximum [illegible] by c.

→ the workdone by effort is known as power.

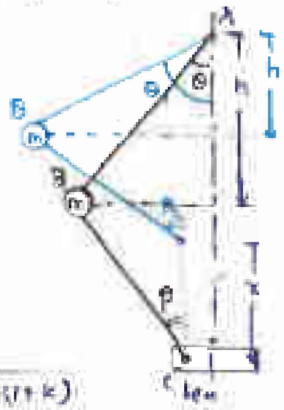
$\boxed{\text{power} = \text{effort} \times \text{sleeve displacement}}$

$$\therefore h = \frac{2mg + (Mg \pm f)(1+K)}{2m\omega^2}$$

$$\Rightarrow h_1 = \frac{2mg + (Mg \pm f)(1+K)}{2m\omega_1^2}$$

$$\Rightarrow h_1 = \frac{2mg + (Mg \pm f)(1+K)}{2m(1+c)^2\omega^2}$$

$$\therefore \frac{h_1}{h} = \frac{2mg + (Mg \pm f)(1+K)}{2m(1+c)^2\omega^2} \times \frac{2m\omega^2}{2mg + (Mg \pm f)(1+K)}$$



$$\Rightarrow \boxed{\frac{h_1}{h} = \frac{1}{(1+c)^2}}$$

→ x = Displacement sleeve

$$x = (AB\cos\theta + BC\cos\beta) - (AB_1\cos\theta_1 + B_1C_1\cos\beta_1)$$

$$= (h + BC\cos\beta) - (h_1 + B_1C_1\cos\beta_1)$$

$$x = (h - h_1) + (BC\cos\beta - B_1C_1\cos\beta_1)$$

⇒ If θ = β (sleeve longer same)

$$x = 2AB\cos\theta - 2AB_1\cos\theta_1$$

$\boxed{x = 2(h - h_1)}$ ← when θ = β (K = 1), sleeve displacement

→ $\text{power} = \frac{E}{2} \times x$

$$\frac{E}{2} = \frac{c}{(1+K)}[2mg + (Mg \pm f)(1+K)]$$

If K = 1, θ = β

$$\Rightarrow \frac{E}{2} = \frac{c}{2}[2mg + (Mg \pm f)(2)]$$

$$\boxed{\frac{E}{2} = c[mg + (Mg \pm f)]}$$

$$\Rightarrow \boxed{\text{power} = \frac{E}{2} \times x}$$

$$\text{power} = \frac{E}{2} \times 2(h - h_1) = \frac{E}{2} \times 2h\left[1 - \frac{1}{(1+c)^2}\right]$$

$$\left\{\frac{h_1}{h} = \frac{1}{(1+c)^2}\right.$$

$$= \frac{E}{2} \times 2h\left[\frac{1 + 2c - 1}{1 + c^2}\right]$$

$$= c[mg + (Mg \pm f)] \times 2h\left(\frac{2c}{1+2c}\right)$$

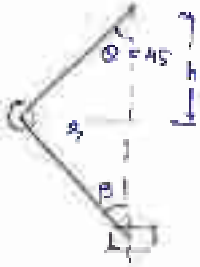
$$\boxed{\text{power} = \frac{4c}{1+2c}\left[mg + (mg \pm f)r\right]}$$

If friction neglected f = 0

$$\text{Power} = \frac{4c^2}{1+2c}\left[mg + Mg\right] h$$

$$\boxed{\text{power} = \frac{4c^2}{1+2c}(m + M) g r}$$ by neglecting friction

③ K = 1
m = 1
M = 20
x = h
N = 15



$\cos 45° = \frac{h}{50}$

$$\omega^2 = \frac{2mg + (Mg \pm f)(1+K)}{2mh}$$ h in m

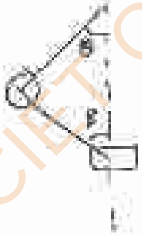
$$= \frac{2mg + 2Mg}{2mh} = \left(\frac{m+M}{m}\right)\frac{g}{h}$$

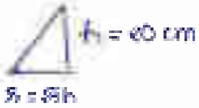
$$= ~~[illegible]~~ \left(\frac{1+20}{1}\right)\left(\frac{9.81}{0.35}\right)$$

$$\boxed{\omega = 24 \text{ rad/s}}$$

④ $r = \sqrt{3}h$

h = 20 cm





$$\omega^2 = \frac{2mg \pm (Mg \pm f)(1+K)}{2mh}$$

$$= \left(\frac{m+M}{m}\right)\frac{g}{h}$$

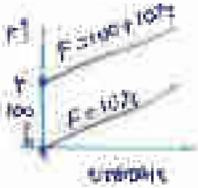
$$= \left(\frac{2+10}{2}\right) \times \frac{9.81}{0.2}$$

$$\boxed{\omega = 17.15 \text{ rad/s}}$$

⑩ 800 = P + q(70)
700 = P + q(60)

$\boxed{q = 10}$ N/m 800 − 700 = P ⇒ $\boxed{P = 100}$ N

→ F = 100 + 10x



{ P < 0 the (Hve) Governor unstable.
to become isochronous P = 0, q > 0

$CF = 50r_1 - 1000 \rightarrow 90V-2$

$CF = 100r - 1000 \rightarrow 80V-3$

$(\text{slope})_1 > (\text{slope})_2$

$r_3 > r_2 > r_1$

$\Rightarrow \omega_1 > \omega_2 > \omega_3$

$F(r) = pr + q$

(ii) $r_1 = 40$ cm

$CF_1 = 0$

$CF_1 > 0 \Rightarrow r_1 > 40$ cm

$\Rightarrow CF_2 > 0 \Rightarrow r_2 > 50$ cm

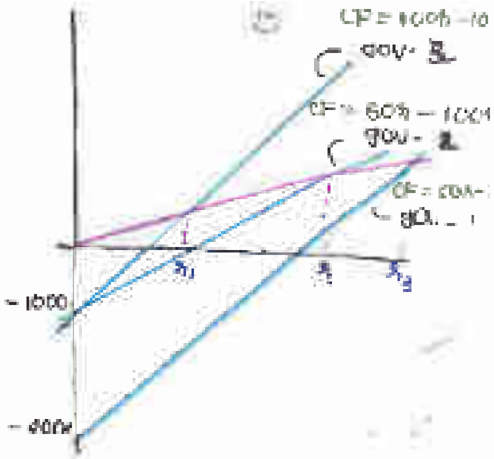
(iii) $50 < r < 60$

(e) $r_1 = 55 \Rightarrow CF_2 = 1700$

$CF_3 = 1700$

$(CF)_2 > (CF)_3 \rightarrow$ change in ... test
Radius of governor
Sleeve displacement test
Sensitive ... test
} Governor ②

(iv) $\omega_1 < \omega_2 < \omega_3$



[4] CF ↓ — spring force ↓

$\downarrow F_s = kx$

$F_s \propto x \Rightarrow K \downarrow$

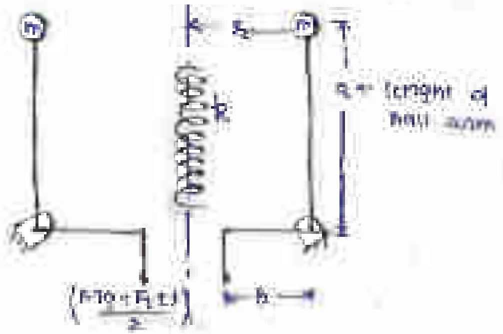
[7] $14 = p + 2q$

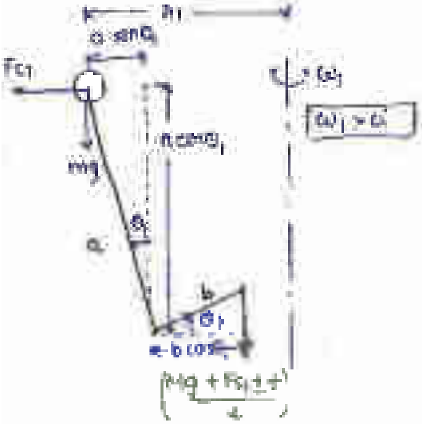
$30 = p + 6q$

$16 = 4q \Rightarrow \boxed{q = 4}$ N/cm

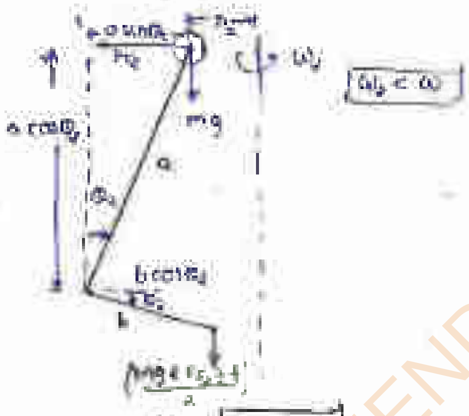
$\boxed{p = 2}$ N

$p > 0$ & $q > 0 \Rightarrow$ unstable





case-(i) $\boxed{\omega_1 > \omega}$



case-(ii) $\boxed{\omega_2 < \omega}$

$$F_{C_1} \cdot a\cos\theta_1 + mg\, a\sin\theta_1 = \frac{Mg + F_{S_1} \pm f}{2} \cdot b\cos\theta_1$$

$$\Rightarrow m r_1 \omega_1^2 \, a\cos\theta_1 + mg\, a\sin\theta = \frac{Mg + F_{S} \pm f}{2} \cdot b\cos\theta_1$$

$$F_{C_2}\, c\cos\theta_2 \pm mg\, a\sin\theta_2 = \frac{Mg \pm F_{S_2} \pm f}{2} \cdot b\cos\theta_2$$

$$\Rightarrow m r_2 \omega_2^2 \, a\cos\theta_2 = mg\, a\sin\theta_2 + \left(\frac{Mg + F_{S} \pm f}{2}\right) b\cos\theta_2$$

→ **Special case**

If θ_1 & θ_2 are very small then

$$\boxed{m r_1 \omega_1^2 \cdot a = \frac{Mg + F_{S_1} \pm f}{2} \cdot b}$$
$$\boxed{m r_2 \omega_2^2 \cdot a = \frac{Mg + F_{S_2} \pm f}{2} \cdot b}$$

$$\Rightarrow \frac{r_1 \omega_1^2}{r_2 \omega_2^2} = \frac{Mg + F_{S_1} \pm f}{Mg + F_{S_2} \pm f}$$

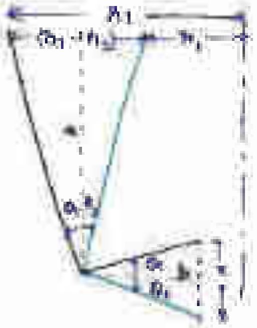
If friction neglected

$$\frac{r_1 \omega_1^2}{r_2 \omega_2^2} = \frac{Mg + F_{S_1}}{Mg + F_{S_2}}$$

condition for isochronous
$$\boxed{\frac{r_1}{r_2} = \frac{Mg + F_{s1}}{Mg + F_{s2}}}$$

Note : only spring controlled governors could be isochronous

→ sleeve displacement



$$\frac{x}{b} = \frac{(r_1 - r_2)}{a}$$

$$\boxed{x = \frac{b}{a}(r_1 - r_2)}$$

→ Stiffness of spring :

$$\left(\frac{Mg + F_{s1} \pm f}{2}\right)b - \left(\frac{Mg + F_{s2} \pm f}{2}\right)b = m r_1 \omega_1^2 a - m r_2 \omega_2^2 a$$

$$\Rightarrow \frac{b}{2}\left[\cancel{Mg} + F_{s1} \pm \cancel{f} - \cancel{Mg} - F_{s2} \pm \cancel{f}\right] = ma(r_1 \omega_1^2 - r_2 \omega_2^2)$$

$$\Rightarrow \frac{b}{2}\left[F_{s1} - F_{s2}\right] = (m r_1 \omega_1^2 - m r_2 \omega_2^2)a$$

$$\boxed{F_{s1} - F_{s2} = (F_{c1} - F_{c2})\frac{2a}{b}}$$ where $F_c = mr\omega^2$

$$\Rightarrow (F_s)_{net} = (F_{c1} - F_{c2})\frac{2a}{b}$$

$$k \cdot x = F_{c1} - F_{c2}\left(\frac{2a}{b}\right)$$

$$k = \frac{F_{c1} - F_{c2}}{\frac{b}{a}(r_1 - r_2)}\left(\frac{2a}{b}\right)$$ $\left\{ x = \frac{b}{a}(r_1 - r_2) \right.$

$$\boxed{k = 2\left(\frac{a}{b}\right)^2\left(\frac{F_{c1} - F_{c2}}{r_1 - r_2}\right)}$$

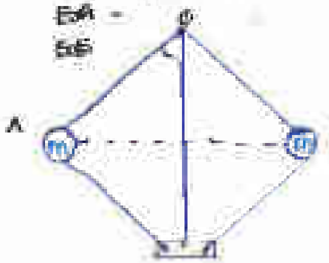
B) equilibrium speed & also calculate % change in speed for 50 mm rise in the level of balls.

OA = 640 mm

θ = 30°

EA =

EB



— simple watt governor

①

$\boxed{h = 0.554\ m}$

$\omega_1^2 = \frac{g}{h_1} = \frac{9.81}{0.554}$

$\boxed{\omega_1 = 4.20\ rad/s}$

②

$h_2 = 0.554 - 0.05$

$\boxed{h_2 = 0.504\ m}$

$\omega_2^2 = \frac{g}{h} = \frac{9.81}{0.504}$

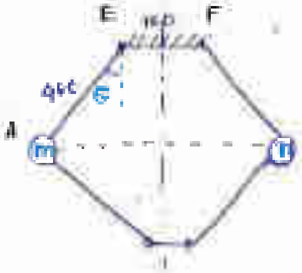
$\boxed{\omega_2 = 4.4107\ rad/s}$

% change speed $= \frac{N_2 - N_1}{N_1} \times 100$

$= \underline{4.64\%}$

EA = 460.

EF = 160, θ = 30



①

$\boxed{h_1 = 0.4156\ m}$

$\tan 30 = \frac{80}{h'}$

$\boxed{h' = 0.134\ m}$

so, $\boxed{h_1 = 0.5541\ m}$

$\omega_1^2 = \frac{g}{h} = \frac{9.81}{0.5541} \Rightarrow$

$\boxed{\omega_1 = 4.20\ rad/s}$

②

$h = y + 0.3856$

$\cos\theta_1 = \frac{0.3856}{0.460} \Rightarrow \boxed{\theta = 40.38°}$

$\tan 40.38° = \frac{80}{y} \Rightarrow \boxed{y = 0.09406}$

total displa = 0.3856 + 0.09406

$\boxed{h_1 = 0.4446\ m}$

$\omega_2 = \frac{g}{h_1} = \frac{9.81}{0.4446} \Rightarrow \boxed{\omega_2 = 4.619\ rad/s}$

% $\frac{\omega_2 - \omega_1}{\omega_1} = \underline{9.74}$

The governor ball radius of rotation at min & max speeds are 105 mm & 140 mm respectively M = 20 kg , m = 5 kg determine range of speed

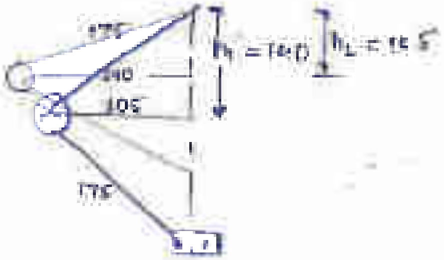
(i) when friction is absent

(ii) friction = 15 N.

→ arm length = 175 mm

$r_1 = 105$

$r_2 = 140$



Case-(i) same length arm $\boxed{k = 1}$

→ $\omega_1^2 = \frac{2mg + (Mg \pm f)(1 + k)}{2mh_1}$

$= \frac{(2)(5)(9.81) + \{(20)(9.81) + 0\}(1 + 1)}{(2)(5)(0.140)}$

$\boxed{\omega_1 = 15.71 \text{ rad/s}}$

Case-(ii) $\omega_2^2 = \frac{2mg \pm (Mg \pm f)(1 + k)}{2mh_2}$

$= \frac{(2)(5)(9.81) + \{(20)(9.81) + 0\}(2)}{(2)(5)(0.105)}$

$\boxed{\omega_2 = 21.63 \text{ rad/s}}$

Range $= \omega_2 - \omega_1$

$\boxed{\text{Range} = 2.595 \text{ rad/s}}$

Case-(ii) : Friction considered $k = 1$

take $h = r$ in $h_1 = 105$ & 140
$h_1 = 140$ to 105 $r \downarrow$ f … f
… f (−ve)

→ $\omega^2 = \frac{mg + (Mg \pm f)}{mh}$

If smaller $f(-ve)$
If big $f(+ve)$

$\omega_1^2 = \frac{(5)(9.81) + (20)(9.81) - 15)}{(5)(0.14)}$

$\boxed{\omega_1 = 18.36 \text{ rad/s}}$

→ $\omega_2^2 = \frac{(5)(9.81) + (20)(9.81) + 15)}{(5)(0.105)}$

$\boxed{\omega_2 = 22.36 \text{ rad/s}}$

Range $= \omega_2 - \omega_1$

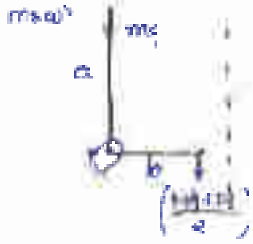
$\boxed{\text{Range} = 4.126 \text{ rad/s}}$

Given $r_1 = 25$ cm

$\omega = 20$ rad/s

$K = 200$ N/cm

$a = b$



$$(m r_1 \omega^2)(a) = \left(\frac{F_s}{2}\right)(b)$$

$$(1)(0.25)(20)^2 = \frac{K x}{2} = \frac{(200)x}{2}$$

$$\boxed{x = 1 \text{ cm}}$$

[6] $a = b = \frac{8}{2}$

$\omega = 20$

$r_2 = 40$ cm

$m = 1$ kg

$$(m r_2 \omega^2)\, a = \left(\frac{F_s}{2}\right)(b)$$

$$1 \times \frac{40}{100} \times (20)^2 \times a = \frac{F_s}{2}$$

$$\boxed{F_s = 320 \text{ N}}$$

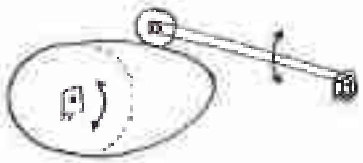
- It is higher pair mechanism
- cam is main rotating or oscillating element & follower follows the motion

→ Classification of cam & follower

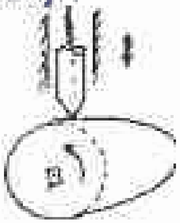
i) On the basis of type of motion :

a) oscillatory cam & follower mechanism

b) translating cam & follower mechanism



oscillatory cam & follower

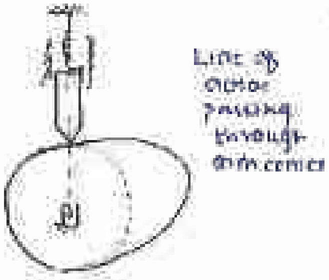


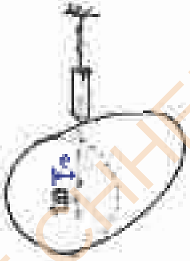
translating

2) On the basis of offset being provided

a) Radial cam

b) offset / eccentric cam





3) On the basis of type of shape of follower

a) Rise - Dwell - Return - Dwell [R-D-R-D]

b) Dwell - Rise - Return - Dwell [D-R-R-D]

4) On the basis of the shape of follower

a) knife edge follower 

b) Roller follower 

c) flat face follower 

d) Mushroom follower 

5) On the basis of type of motion of follower

a) uniform velocity

b) uniform Accⁿ & Retardation [Parabolic]

c) simple harmonic [SHM]

d) cycloidal motion

[illegible]

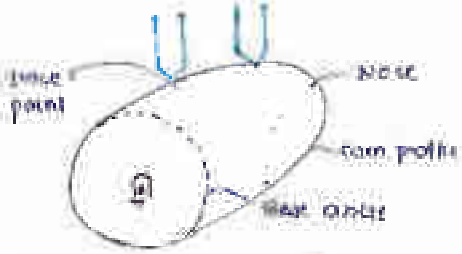
- Since, cam is main transmission rotating element but in order to discuss the terminology, we consider after inversion of cam & follower mechanism that is we consider cam as fixed member & follower is moving on it

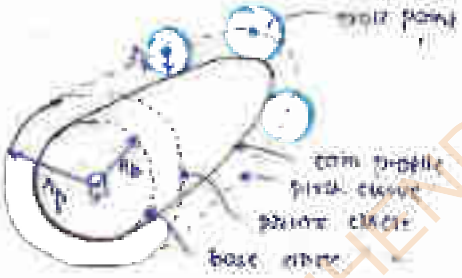
① Base circle

- It is the smallest circle tangential to cam profile
- Radius of base circle determine the size of cam
- Base circle will never intersect with cam profile

② Trace point :

- The reference point on follower whose locus is pitch curve is known as trace point
- In case of knife edge follower tip of the knife is trace point.
- In case of roller follower centre of roller is trace point





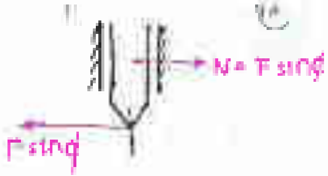
③ Prime circle

- The smallest circle tangential to pitch curve is known prime circle.

$$\boxed{R_{prime} = R_{base} + R_{roller}}$$

→ Pressure angle :

- It is the angle between direction of velocity of the follower (or line of movement of follower) and common normal at the point of contact
- Pressure angle of for a cam profile is not constant
- In a cam & follower mechanism the force transmission always takes place along the common normal that's why it is also known as line of action



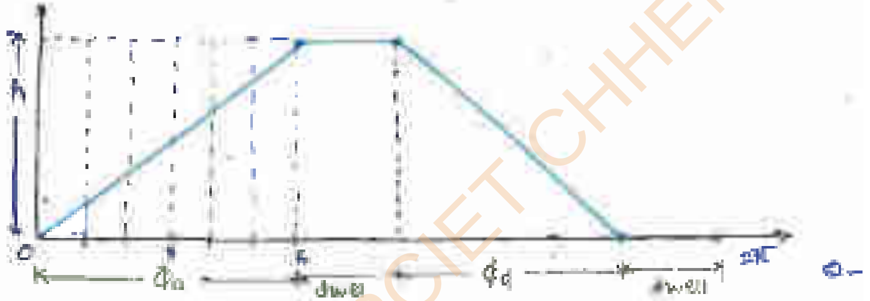
- A large value of pressure angle is avoided as it leads to Jamming.
- F sinϕ components forms a couple from normal forces exerted by the guide and it tends to bend the stem of follower.
- The point on cam profile where pressure angle is maximum is known as pitch point.
- All the pitch point for cam profile actually lie in a circle concentric to the base circle which is known as pitch circle.
- $\phi_{max} \leq 30°$

⇒ Types of motion of follower:

Note ● Displacement of follower is always measured from base c

Note ● In order to calculate pressure angle we used pitch curve. (common normal should to pitch curve.)

① Uniform velocity [R - D - R - D]



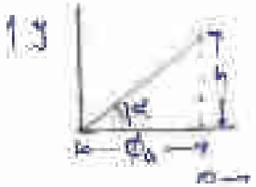
(Displacement diagram)

⇒ ϕ_a = angle of ascent / rise

ϕ_d = angle of descent / return

h = max distⁿ travelling by follower

→ Displacement eqⁿ



$y = \text{const} \cdot \theta$
slope of line

$$\boxed{\tan\alpha = \frac{h}{\phi_a}}$$

$$\boxed{y = \left(\frac{h}{\phi_a}\right)\cdot\theta}$$

[illegible]

$v = \frac{dy}{dt} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d}{d\theta}\left[\frac{h\theta}{\phi_a}\right]$

$\boxed{v = \frac{h\omega}{\phi_o}}$ = const

→ Accⁿ eqⁿ

$a = \frac{dv}{dt}$ here v = const

$\boxed{a = 0}$

→ Jerk equation

$j = \frac{da}{dt}$

$\boxed{j = 0}$

[2] uniform acceleration or Retardation (parabolic motion)

$y = a\theta^2 + b\theta + c$

@ $\theta = 0$, $y = 0$

@ $\theta = \frac{\phi_a}{2}$, $y = \frac{h}{2}$

@ $\theta = 0$, $\frac{dy}{d\theta} = 0$

1st condⁿ ⇒ $0 = a(0)^2 + b(0) + c$ ⇒ $\boxed{c = 0}$

2nd " ⇒ $\frac{dy}{d\theta} = 2a\theta + b$ ⇒ $\boxed{b = 0}$

3rd condⁿ ⇒ $y = a\theta^2$ ⇒ $\frac{h}{2} = a\left(\frac{\phi_a}{2}\right)^2$ ⇒ $\boxed{a = \frac{2h}{\phi_a^2}}$

→ Displacement eqⁿ

$y = a\theta^2$ ⇒ $\boxed{y = 2h\left(\frac{\theta}{\phi_a}\right)^2}$

→ velocity eqⁿ

$v = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \omega\frac{dy}{d\theta} = \omega\frac{d}{d\theta}\left[2h\left(\frac{\theta}{\phi_o}\right)^2\right] = \frac{2h\omega}{\phi_o^2}\, 2\theta$

$\boxed{v = \frac{4h\omega}{\phi_o^2}\theta}$

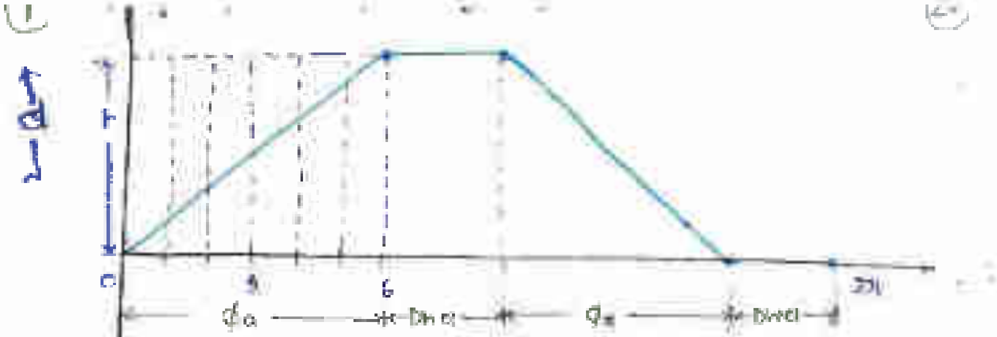
→ Accⁿ eqⁿ

$a = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \omega\frac{d}{d\theta}\left[\frac{4h\omega}{\phi_o^2}\theta\right]$

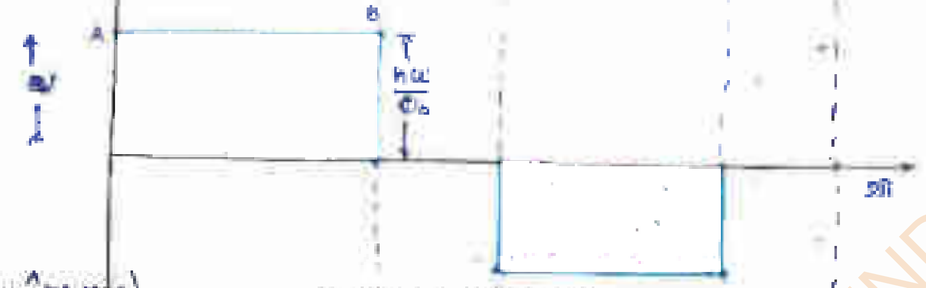
$\boxed{a = \frac{4h\omega^2}{\phi_o^2}}$ = const

→ Jerk

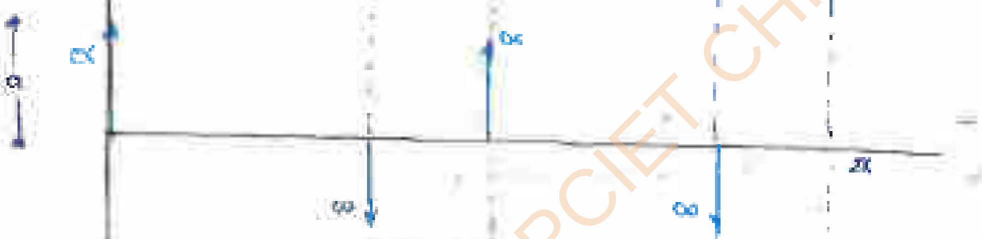
$j = \frac{da}{dt}$ ⇒ $\boxed{j = 0}$



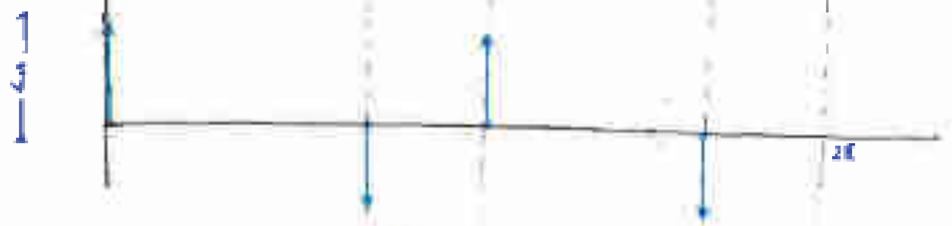
(i) Displacement diagram



(ii) velocity diagram

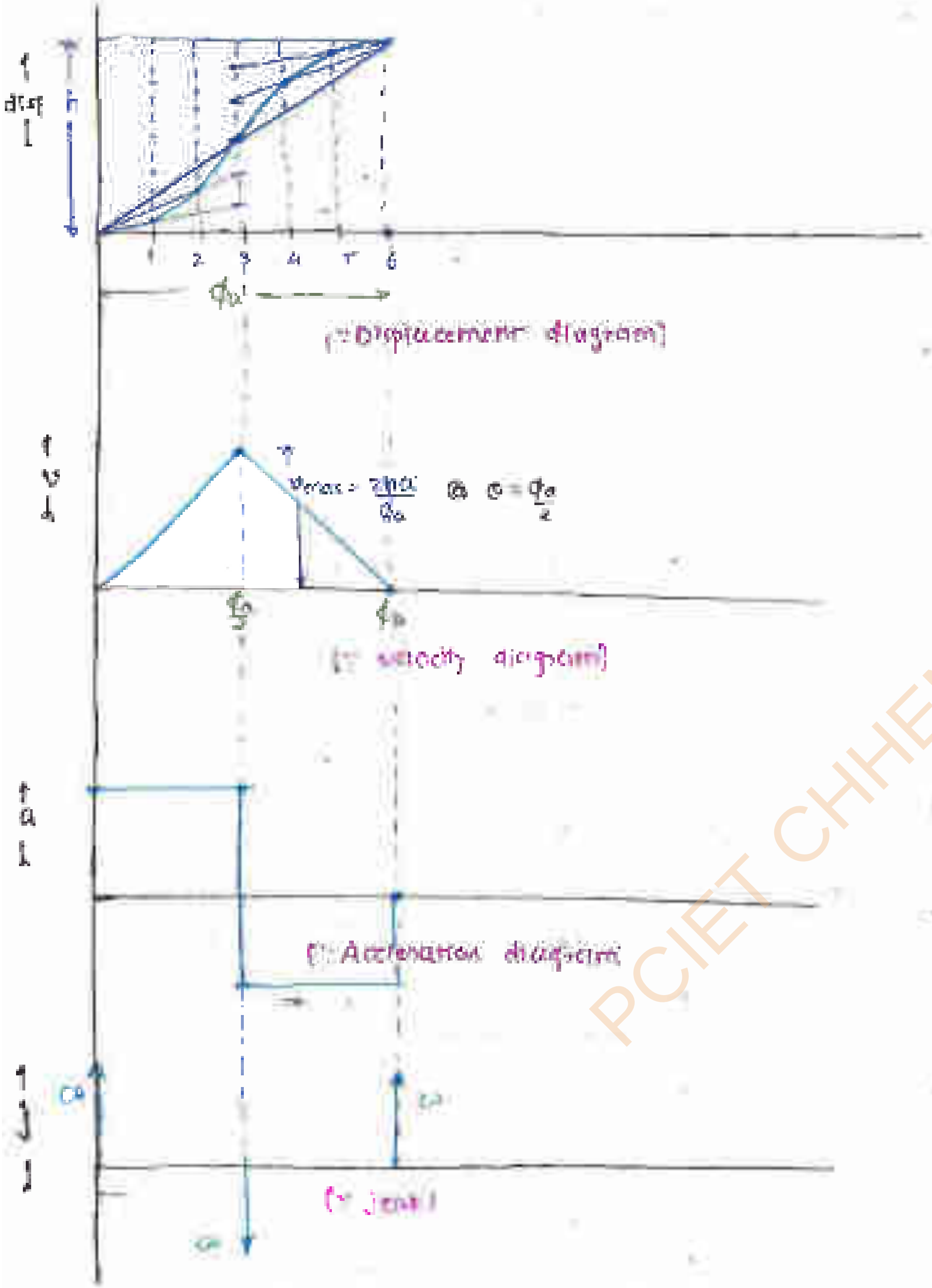


(iii) acceleration diagram



(iv) Jerk

[∵ uniform velocity R-D-R-D]



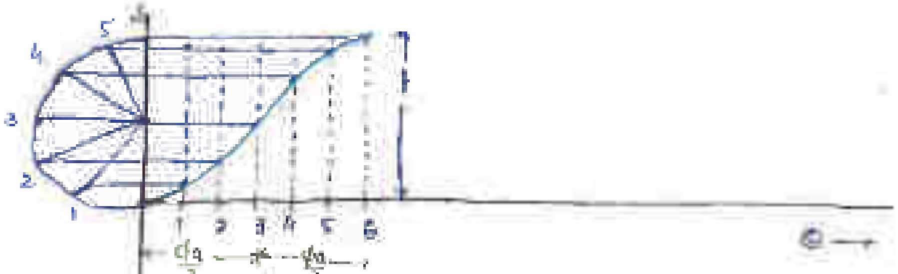
(∴ Displacement diagram)

(∴ velocity diagram)

(∴ Acceleration diagram)

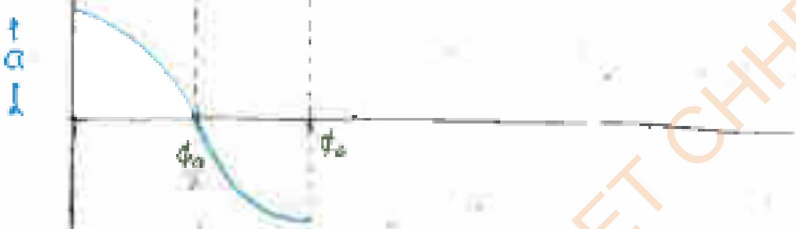
(∴ jerk)

[∵ uniform acc^n or retardation]



Displacement dia.

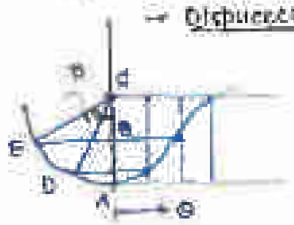
(velocity diagram -[sinosoidal]



(Acceleration dia.) [sinosoidal]

(Jerk diagram)

→ Displacement eqⁿ



$$AB = AC - BC = \frac{h}{2} - EC\cos\beta = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$y = \frac{h}{2}\left[1 - \cos\beta\right]$$

$$\boxed{y = \frac{h}{2}\left[1 - \cos\left(\frac{\pi\theta}{\phi_a}\right)\right]}$$

$$\text{at } \left.\begin{array}{l} \beta \to \pi \\ y \to h \\ \theta \to \phi_a \end{array}\right\} \frac{\beta}{\pi} = \frac{\theta}{\phi_a} = \frac{y}{h}$$

→ velocity eqⁿ

$$v = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \omega\frac{dy}{d\theta} = \omega\frac{d}{d\theta}\left[\frac{h}{2}\left(1 - \cos\left(\frac{\pi\theta}{\phi_a}\right)\right)\right]$$

$$= \frac{h\omega}{2}\left[0 - \left(-\sin\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}\right)\right]$$

$$\boxed{v = \frac{\pi h\omega}{2\phi_a}\sin\left(\frac{\pi\theta}{\phi_a}\right)}$$

θ	0	$\phi_a/2$	ϕ_a
v	0	$\frac{\pi h\omega}{2\phi_a}$	0

hence $\boxed{v_{max} = \frac{\pi h\omega}{2\phi_a}}$ @ $\theta = \frac{\phi_a}{2}$

→ Accⁿ eqⁿ

$$a = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \omega\frac{d}{d\theta}\left[\frac{\pi h\omega}{2\phi_a}\cdot\sin\left(\frac{\pi\theta}{\phi_a}\right)\right] = \frac{\pi h\omega^2}{2\phi_a}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}$$

$$\boxed{a = \frac{\pi^2 h\omega^2}{2\phi_a^2}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)}$$

θ	0	$\phi_a/2$	ϕ_a
a	$\frac{\pi^2 h\omega^2}{2\phi_a^2}$	0	$-\frac{\pi^2 h\omega^2}{2\phi_a^2}$

$$j = \frac{da}{dt} = \frac{d\theta}{dt}\,\frac{da}{d\theta} = \omega\,\frac{da}{d\theta}$$

$$= \omega\,\frac{d}{d\theta}\left[\frac{\pi^2\omega^2 h}{2\phi_a^2}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)\right]$$

$$= \frac{\pi^2 h\omega^3}{2\phi_a^2}\left[-\sin\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}\right]$$

$$\boxed{j = -\frac{\pi^3 h\omega^3}{2\phi_a^3}\cdot\sin\left(\frac{\pi\theta}{\phi_a}\right)}$$

θ	0	$\frac{\phi_a}{2}$	ϕ_a
j	0	$-\frac{\pi^3 h\omega^3}{2\phi_a^3}$	0

[4] Cycloidal Motion

— Displacement eq^n

$$\boxed{y = \frac{h}{\pi}\left[\frac{\pi\theta}{\phi_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right]}$$

— Velocity eq^n

$$v = \frac{dy}{d\theta}\cdot\frac{d\theta}{dt} = \omega\,\frac{d}{d\theta}\left[\frac{h}{\pi}\left\{\frac{\pi\theta}{\phi_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right\}\right]$$

$$= \frac{h\omega}{\pi}\left[\frac{\pi}{\phi_a} - \frac{1}{2}\cos\left(\frac{2\pi\theta}{\phi_a}\right)\cdot\frac{2\pi}{\phi_a}\right]$$

$$\boxed{v = \frac{h\omega}{\phi_a}\left[1 - \cos\left(\frac{2\pi\theta}{\phi_a}\right)\right]}$$

θ	0	$\phi_a/4$	$\phi_a/2$	$3\phi_a/4$	ϕ_a
v	0	$\frac{h\omega}{\phi_a}$	$\frac{2h\omega}{\phi_a}$	$\frac{h\omega}{\phi_a}$	0

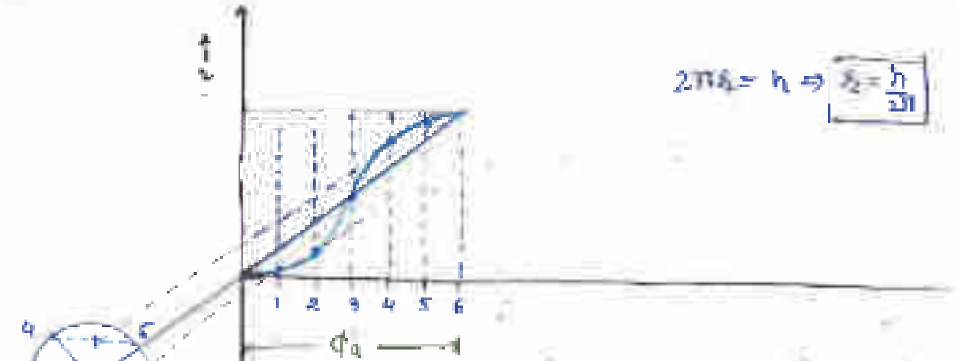
— Acc^n eq^n

$$a = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt} = \omega\,\frac{d}{d\theta}\left[\frac{h\omega}{\phi_a}\left\{1 - \cos\left(\frac{2\pi\theta}{\phi_a}\right)\right\}\right]$$

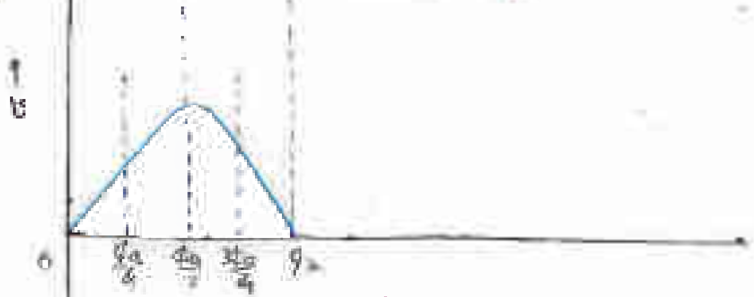
$$= \frac{h\omega^2}{\phi_a}\left[0 - \left(-\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right)\frac{2\pi}{\phi_a}\right]$$

$$\boxed{a = \frac{2\pi h\omega^2}{\phi_a^2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)}$$

$2\pi r_2 = h \Rightarrow \boxed{r_2 = \frac{h}{2\pi}}$

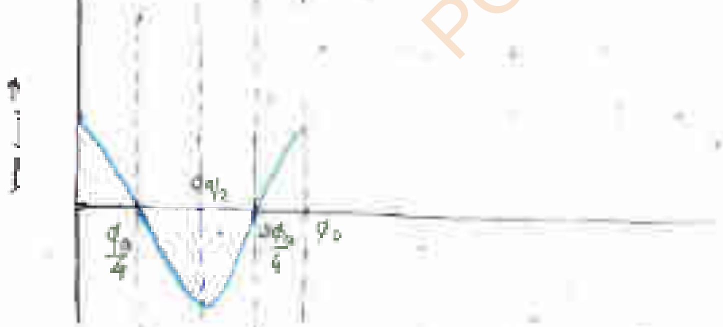


(∵ Displacement diagram)



(Velocity diagram)

(Accⁿ diagram)



(Jerk dia.)

a	0	$\frac{2\pi h\omega^2}{\phi_o^2}$	0	$-\frac{2\pi h\omega^2}{\phi_o^2}$	0

$\frac{2\pi h\omega^2}{\phi_o^2}$ is a_{max}

→ jerk eq^n

$$j = \frac{da}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d}{d\theta}\left[\frac{2\pi h\omega^2}{\phi_o^2} \sin\left(\frac{2\pi\theta}{\phi_o}\right)\right]$$

$$j = \frac{2\pi h\omega^3}{\phi_o^2} \cos\left(\frac{2\pi\theta}{\phi_o}\right)\frac{2\pi}{\phi_o}$$

$$\boxed{j = \frac{4\pi^2 h\omega^3}{\phi_o^3} \cos\left(\frac{2\pi\theta}{\phi_o}\right)}$$

θ	0	$\phi_o/4$	$\phi_o/2$	$3\phi_o/4$	ϕ_o
j	$\frac{4\pi^2 h\omega^3}{\phi_o^3}$	0	$-\frac{4\pi^2 h\omega^3}{\phi_o^3}$		$\frac{4\pi^2 h\omega^3}{\phi_o^3}$

NOTE → cycloidal motion is best possible motion among all four. Hence it is used for high speed device

→ for moderate speed we can used SHM and for low speed uniform velocity

	V_{max}	a_{max}
uniform velocity	$\frac{h\omega}{\phi_o}$	—
SHM	$\frac{\pi h\omega}{2\phi_o}$	$\frac{\pi^2 h\omega^2}{2\phi_o^2}$
cyclodial	$\frac{2h\omega}{\phi_o}$	$\frac{2\pi h\omega^2}{\phi_o^2}$

✓ $\boxed{(V_{max})_{cycloidal} > (V_{max})_{SHM} > (V_{max})_{UCE}}$

$\phi_a = 90°$

$\omega = 2$ rad/s

$\theta = \frac{2}{3}\phi_a$

$\rightarrow\ y = \frac{h}{2}\left[1 - \cos\frac{\pi\theta}{\phi_a}\right] = \frac{4}{2}\left[1 - \cos\pi\left(\frac{2}{3}\frac{\phi_a}{\phi_a}\right)\right]$

$\boxed{y = 3\text{ cm}}$

$\rightarrow\ v = \frac{h}{2}\sin\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}\cdot\omega = \frac{\pi h\omega}{2\phi_a}\sin\left(\frac{\pi\theta}{\phi_a}\right)$

$= \frac{\pi\times4\times2}{2\times\pi/2}\sin(120)$

$\boxed{v = 7\text{ cm/s}}$

$\rightarrow\ a = \frac{\pi h\omega^2}{2\phi_a}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a} = \frac{\pi^2 h\omega^2}{2\phi_a^2}\cos\left(\frac{\pi\theta}{\phi_a}\right)$

$= \frac{\pi^2\times4\times2^2}{2\times\pi^2/4}\cos(120)$

$\boxed{a = -16\text{ cm/s}^2}$

6

$\phi_0 = \pi$

$h = 10$ cm

$v_{max} = 25$ cm/s

$\omega = (?)$

$a_{max} = (?)$

$v_{max} = \frac{\pi h\omega}{2\phi_a} = \frac{\pi\times10\times\omega}{2\times\pi} = 25$

$\boxed{\omega = 5}$ rad/s

$a_{max} = \frac{\pi^2 h\omega^2}{2\phi_a^2} = \frac{\pi^2\times10\times5^2}{2\times\pi^2} \Rightarrow \boxed{a_{max} = 125\text{ cm/s}^2}$

7

$\phi_a = \pi/2$

$h = 6$ mm

$\omega = 1$ rad/s

at $y = 3$ mm $\rightarrow \boxed{\theta = \phi_a/2}$ (coz at h 3 mm as half φ)

$v = \frac{\pi h\omega}{2\phi_a}\sin\left(\frac{\pi\theta}{\phi_a}\right) = \frac{\pi\times6\times1}{2\times\pi/2} \rightarrow \boxed{v = 6\text{ mm/s}}$

$a = \frac{\pi^2 h\omega^2}{2\phi_a^2}\cos\left(\frac{\pi\theta}{\phi_a}\right) = \ldots\cos(\pi/2) \rightarrow \boxed{a = 0}\text{ mm/s}^2$

15)

$y = 10 + 5\sin\theta$

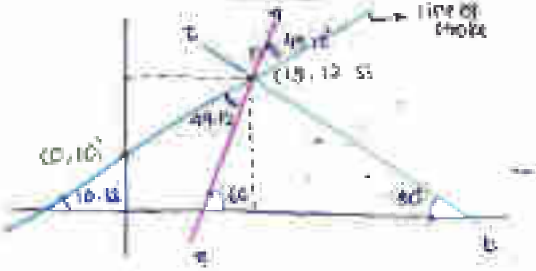
@ $\theta = 30$, $\boxed{P = ?}$

$\frac{x}{15} = \cos\theta$ & $\frac{y-10}{5} = \sin\theta$

$\Rightarrow \left(\frac{x}{15}\right)^2 + \left(\frac{y-10}{5}\right)^2 = 1$ $\Rightarrow$ $G = (0, 10)$

at $\theta = 30$; $x_p = 15\cos 30$ $y_p = 10 + 5\sin 30$

$\boxed{x_p = 13}$ $\boxed{y_p = 12.5}$



$\tan\alpha = \frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{12.5 - 10}{13 - 0}\right)$

$\boxed{\theta_1 = 10.88}$

→ slope of tangent $= \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{0 + 5\cos\theta}{-15\sin\theta}$

$= -\frac{1}{3\tan\theta}$

@ $\theta = 30°$ ⇒ $\frac{dy}{dx} = -\frac{1}{3\tan 30}$ ⇒ $-\frac{1}{\sqrt{3}} = 1$

$\boxed{\alpha' = -30}$ from x 40

→ $m_1 m_2 = -1$

$-\frac{1}{\sqrt{3}}$ $m_2 = -1$

$m_2 = \sqrt{3}$

$\tan\beta = \sqrt{3}$

$\boxed{\beta = 60}$ from x to ccw)

$\boxed{Ans = 49.18}$

✓ Angle betn normal & line of stroke

19) $y = 2x^2 - 7x - 2$ $c = 2(16) - 7(4) + c$
$= 32 - 28$

at $x = 4$, $y = 2$

~~@ x = 4, y~~

→ Radial com line of action passing through centre

$\boxed{x_p = 4}$ & $\boxed{y_p = 2}$

$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{4 - 0}$

$\boxed{\theta = 26.56}$

→ slope of tangent $\frac{dy}{dx} = \Big|_{(x_p, y_p) = (4, 2)}$

$= 4x - 7$

$= 16 - 7 = 9$

$\tan\beta = 9$

$\boxed{\beta = 83.65°}$

→ common normal

$m_1 m_2 = -1$

$m_2 = -1/9$

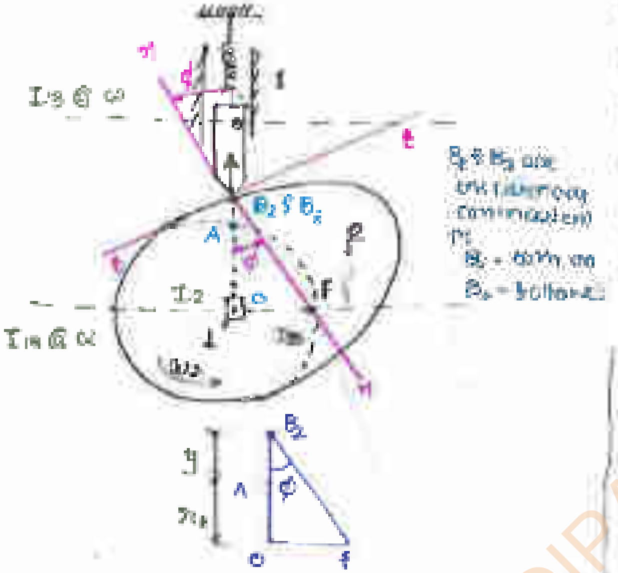
$\boxed{\alpha = -6.34°}$

$\left(\text{slope of tangent}\right)\left(\text{slope of normal}\right) = -1$

$\boxed{Ans = 32.4°}$

- It is angle betⁿ comⁿ normal to the pitch curve (or trace) point and line of stroke

- B_2 & B_3 are instantaneously coincident pt



$V_{B_2} = OB\ \omega_2$

$\rightarrow V_{B_3} = (I_{13} \cdot B_3)\ \omega_3$

$V_{B_3} = (I_{23} \cdot C)\ (\omega_2 ? \omega_3)$

$\boxed{V_{B_3} = (I_{23} \cdot C)\ \omega_2}$

$V_{B_3} = (OP)\ \omega_2$

$\rightarrow \tan\phi = \frac{OP}{OB_2} = \frac{V_{B_3}/\omega_2}{OB_2}$

$\tan\phi = \frac{V_{B_3}}{\omega_2\ OB_2}$

$\rightarrow \tan\phi = \frac{dy/dt}{(r_b + y)\ d\theta/dt}$

$$\boxed{\tan\phi = \frac{dy/d\theta}{r_b + y}}$$

NOTE :- In case of roller follower. $\tan\phi = \frac{dy/d\theta}{(r_b + r_R) + y}$

$$\boxed{\tan\phi = \frac{dy/d\theta}{(r_b + r_R) + y}}$$

where $r_b + r_R = r_{prime}$

NOTE - In case offset provided

$$\boxed{\tan\phi = \frac{dy/d\theta - e}{\sqrt{r_b^2 - e^2} + y}}$$

when e = eccentricity being provided

NOTE

- During rise stroke $\frac{dy}{d\theta}$ (+ve) therefore offset provided reduces pressure angle during rise.
- During return stroke $\frac{dy}{d\theta}$ (-ve) hence, offset are increases pressure angle during return

cam center, the cam should rotate A-C-W

If it provided to left side of cam center, cam should rotate C-W

A large value of φ should be avoided as it causes more friction, more wear, jamming, bending of the follower.

N = F cosφ (in vg dirn)

f = μN

= μ F cosφ

→ f = f^n(cosφ)

As φ ↑ → friction ↑

Larger value of ϕ_{max} are possible with parabolic & cycloidal motion

& moderate value with SHM

& smaller value for uniform velocity

− for order to above comparision

lift of follower,
Angle of action (rise action)
ω of cam
} Same cond^r

The point of max velocity usually coincide with inflection point (the point where curvature is changing) as it correspond to max. steepness of displacement diagram

parabolic & cycloidal motion required largest size of cam
SHM → moderate size
uniform velocity → smallest size

Important points for selection of cam profile.

1) There should be smooth jerkless motion
2) size of cam should be smaller
3) Inertia of cam should not be much large
4)

Important points for follower.

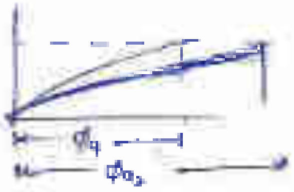
1) knife edge follower
↳ simplest follower
↳ contact stresses are large
↳ result in more wear & tear
↳ knife-edge follower can be called as roller follower have zero radius

[4]
↳ The sliding friction of knife-edge follower is converted into rolling friction
↳ If the cam profile is steep, roller follower gets jam
↳ It involves high inertia

[5] Flat face follower
↳ It can be used for relatively steep cam
↳ In order to minimize the contact stress we provide a spherical end to the flat face follower known as Mushroom follower.

[1]

$$\tan\phi = \frac{dy/d\theta}{R_b + y}$$



$$v = \frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$$

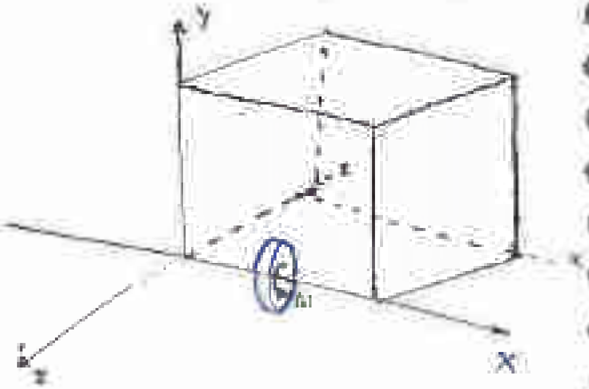
Motion	Displacement eqⁿ	velocity	accⁿ	Jerk
1) [illegible] uniform velocity	$y = \left(\frac{h}{\phi_a}\right)\cdot\theta$	✓	0	0
2) uniform accⁿ	$y = 2h\left(\frac{\theta}{\phi_a}\right)^2$	✓ Vmax @ $\theta = \frac{\phi_a}{2}$	✓	0
3) SHM	$y = \frac{h}{2}\left[1 - \cos\left(\frac{\pi\theta}{\phi_a}\right)\right]$	✓ Vmax @ $\theta = \frac{\phi_a}{2}$	✓ amax @ $\theta = 0$ & $= \phi_a$	✓
4) cycloidal	$y = \frac{h}{\pi}\left[\frac{\pi\theta}{\phi_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right]$	✓ Vmax @ $\theta = \frac{\phi_a}{2}$	✓ amax @ $\theta = \frac{\phi_a}{4}$ & $\frac{3\phi_a}{4}$	✓ Jerk max @ $\theta = 0$ & ϕ_a, $\frac{\phi_a}{2}$

→ cylindrical cam have reciprocating motion of follower.

→ Roller follower used in engine.

- Spin motion

- The rotation of rotor (or engine parts) is known as spin motion
- The plane on which it can be observed or parallel to it is called a plane of spin.



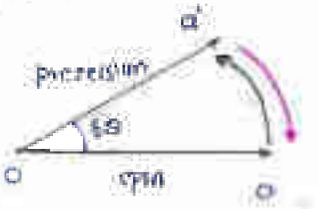
- Precession

- The rotation or oscillating of axis of spin (that is main rotating element) is called precession

→ Whenever the main rotating object starts to precess about some axis as result a change in its angular momentum which gives rise to couple known as gyroscopic couple

→ This gyroscopic couple tends to change the position of the object.

→ There will be an reactive gyroscopic couple exerted by the bearing whose magnitude equals to active gyroscopic couple but direction will be opposite & it will always give the same effect



- Observer at 'o'
- Rotor is rotating 'cw'
- Shaft is moving towards left

	Axis	plane
spin	+x	yz
precession	+y	xz
Reac' cple	+z	xy

$aa' = oa(\delta\theta)$

$\frac{d}{dt}(aa') = \frac{d}{dt}(oa\cdot\delta\theta)$

$\frac{d}{dt}(aa') = oa\frac{d}{dt}(\delta\theta) \Rightarrow \boxed{\frac{d}{dt}(aa') = (oa)\omega_p}$

$C = \frac{d}{dt}(mv)$

$C = \frac{d}{dt}(I\omega)$

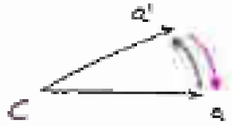
$\boxed{C = I\omega_s\omega_p}$ N-m

(span of RH in spin plane)

→ Keep the fingers of RH in spin of direction & then curl the fingures in dirn of moving then thumb indicate the gyroscopic axis

① On aeroplane

- Observer is on tail side
- Rotor rotating CW.
- Plane turning



plane will moves towards righ

	Axis	plane
spin	+x	yz
precession	+z	xy
Rec'd anc	−y	xz

⟸ Obs. plane is taking right turn

Ex Observer tail side.
Rotating rotor C.W.
Plane taking right turn

	Axis	plane
spin	+x	yz
precession	−y	xz
Red anc	−z	xy

Nose will come down.

② Naval ships

Steering

- If ship is taking right or left turn on curved path

Pitching

- Oscillation of ship about transverse axis

Rolling

- Oscillation of ship about longitudinal axis

- The axis which passing through c/s is longitudinal / parlel and then other two transverse axis.

- observer is standing in stern side
- ship is moving towards port side
- Rotor is rotating c/w

	Axis	plane
spin	x	yz
precession	y	xz
Reactive gyro.	z	xy

→ bow will raise

2. Effect of gyroscopic couple in pitching

observer is standing stern side
Bow is coming down
Rotor is rotating c/w

	Axis	plane
spin	x	yz
precession	−z	xy
Rea. gyro	y	xz

→ towards port

Effect of gyroscopic couple in Rolling

observer is standing stern side
Rotor is rotating g.c/w
ship is Rolling

	Axis	plane
spin	+x	yz
precession	+x/−x	yz
Rea. gyro	−	−

In rolling there is No Gyroscopic Couple.

[illegible] gyro scope small effect of [illegible] of is [illegible] in [illegible]

- In steering :-

$\boxed{C = I \omega_s \omega_p}$

where ω_s = angular speed of spin
ω_p = " " of precession (where vehicle is moving turn)
I = mass MOI of rotor

$\boxed{\omega_p = \frac{v}{R}}$

- In pitching :
- Assume it is SHM.

$\boxed{\theta = \theta_0 \sin \omega t}$

where $\theta_{max} = \theta_0$ → precession
= max. angular displacement from the mean position

Displacement eq^n $\boxed{\theta = \theta_0 \sin \omega t}$ → $\theta_{max} = \theta_0$

velocity eq^n (precession) $\boxed{\dot{\theta} = \theta_0 \omega \cos \omega t}$ → $(\omega_p)_{max} = \dot{\theta}_{max} = \theta_0 \omega_1$

Accl^n eq^n $\boxed{\ddot{\theta} = -\theta_0 \omega^2 \sin \omega t}$ → $(\ddot{\theta})_{max} = \theta_0 \omega^2$

$\ddot{\theta} = -\omega^2 \theta$

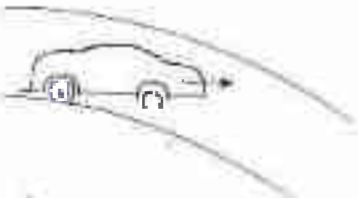
accl^n ∝ – Displacement

→ Acceleration ∝ (– Displacement) ⇒ show SHM motion that displacement is a force or accl^n acting is opposite dir^n

→ Effect of Gyroscope in Automobile :

→ Vehicle is moving in forward direction & going to turn a right turn engine rotating similar to wheel

	Axis	Plane
~~spin~~	~~+x~~	~~yz~~
~~Precession~~	~~–y~~	~~xz~~
~~Reactive axis~~	~~+z~~	~~xy~~



	Axis	Plane
spin	–z	xy
Precession	–y	xz
Reac. gy.	–x	yz

iii) $r = [illegible]$

$v = 20$ m/s

$\omega_s = 100$ rad/s

$I = 10$ kg m²

$C = I\,\omega_s\,\omega_p$

$= I\,\omega_s\left(\frac{v}{R}\right)$

$= 10 \times 100 \times \left(\frac{20}{100}\right)$

$\boxed{C = 200 \text{ N m}}$

[9] $m = 6000$ kg

$N = 2400$ rpm → $\omega_s = 251.2$ rad/s

dirⁿ of rotation of rotor is C.W. viewed from stern

$k = r_g = 450$ mm

$I = mk^2 = 6000\,(0.45)^2$

$= 1215$ kg m²

(i) Steering (steers to left) (port side)

$R = 60$ m

$v = 16$ knot $= 1860 \times 16$ m/hr $= 9.5$ m/s

$\omega_p = \frac{9.5}{60} = 0.155$ rad/s

The $C = I\,\omega\,\omega_p$

	Axis	Plane
Spin	+X	YZ
precession	+Y	XZ
Rctⁿ of	Z	XY

Bow is up (rise)

$C = (1215)(0.155)(251.2)$

$\boxed{C = 47.3 \text{ kN m}}$

(ii) pitching

Bow is descending

$\boxed{\theta = \theta_0 \sin\omega t}$ (pitching is SHM)

$\theta_{max} = \theta_0 = 7.5 \times \frac{\pi}{180} = 0.13089$

$\dot\theta = \theta_0\,\omega \cos\omega t$

$\dot\theta_{max} = \theta_0\,\omega$ so, $\omega = \frac{2\pi}{T} = \frac{2\pi}{18} = 0.3490$

$= (0.13089)(0.3490)$

$= (1215)(251.2)(0.04586)$

$\boxed{C = 13.980 \text{ kN m}}$

	Axis	plane
spin	+x	yz
precession	+z	xy
Reac^n gyro	+y	xz

(descending)

ship is turning left turn → on port side ↑

(acc)

$\dot{\omega}_{max} = \phi_0 \omega_1^2 = \underline{0.0158}$ rad/s²

(iii) $\omega_p = 0.035$ rad/s

$\omega = 80\pi$ rad/s

$C = I\omega\omega_p = 1215\,(80\pi)\,(0.035)$

$\boxed{C = 10.68 \text{ kN m}}$

No effect of gyroscopic coupling ↗

$\boxed{6}$ $N = 3000$ rpm → $\omega_s = 314.0$ rad/s

$I = 47.25$ kg m²

$\omega_p = \frac{2\pi}{T}$; $T = 17$ sec

$\omega_p = \frac{2\pi}{17} = 0.3694$

$C = I\omega_s\omega_p = (47.25)(314)(0.3694)$

$\boxed{C = 5.48 \text{ kN-m}}$

	Axis	plane
spin	+x	yz
precession	−y	xz
Reac^n gyro	−z	xy

Bow will comes down

[1]

	Axis	plane
spin	+x	yz
prec	+y	xz
Red^n gm	+z	xy

Nose raise.

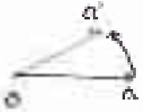
[2]

	Axis	plane
spin	+x	
prec	−y	
Red^n a	−z	

depress bow

[4]

+x
+z
−y

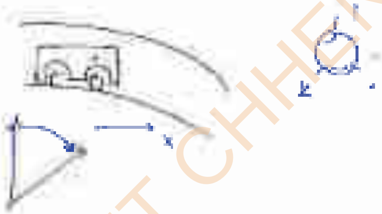


turn ship toward starboard

[5]

+z
−y
+x

~~outer wheel~~
inner wheel



[6]

+x	yz
+z/−z	xy
+y/−y	yz

in Horizontal plan.

Ex A uniform disc of 200 mm diameter has mass of 10 kg it is mounted of centrally on the horizontal shaft which being in bearing, which are 150 mm apart. Disc spin = 2000 rpm in c.c.w dir^n looking from right hand side bearing shaft precess uniform velocity of 60 rpm in Horizontal plane in A.C.W. when looked from top determine reaction at each bearing due to mass of gyroscopic effect

→ R = 100 mm
m = 10 kg

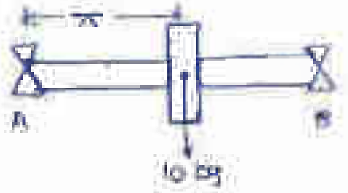
Ring I = mR²

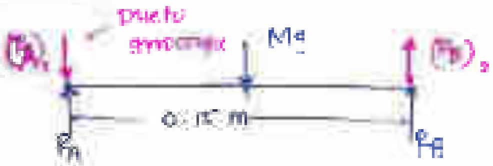
Disc $I = \frac{mR^2}{2}$

→ $\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi(2000)}{60} = 209.37$

→ $\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi(50)}{60} = 5.23$

$C = I\omega\omega_p \Rightarrow \boxed{C = 54.60\ N\text{-}m}$





→ $R_{A_1} = R_{B_1} = \frac{Mg}{2} = \frac{(10)(9.81)}{2} \Rightarrow \boxed{R_{A_1} = R_{B_1} = 49.05}$

→ R_{A_2} & R_{B_2} is reaction due to gyro couple

$(R_A)_2 \times l = 54.60 \Rightarrow (R_A)_2 = \frac{54.60}{0.15}$

$\boxed{(R_A)_2 = 350.2\ N} - (R_B)_2$

$(R_A)_{net} = \sqrt{}$

	Axis	Plane
Axes		
Spin	+x	yz
Precn	+y	xz
Reac couple	+z	xy

$(R_A)_{net} = 49.05 - 365.54 = -316.4$

$(R_B)_{net} = 49.05 + 365.54 = 414.57$

Q. It is given spin of 1000 rpm about its axis which horizontal flywheel is suspended at a point 15 cm from the plane of rotation of flywheel determine the motion of the flywheel

$(\omega_p) = (?)$

$m = 10$ kg

$k = 0.2$ m

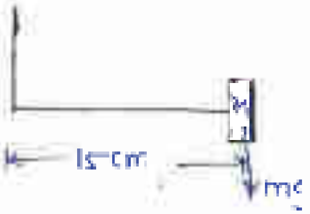
$I = \frac{mk^2}{2} \Rightarrow \boxed{I = 0.4}$ kg m^2

$N_{spin} = 1000$ rpm

$Mg \cdot \ell = I\omega_s\omega_p$

$10 \times 9.81 \times \frac{15}{100} = 0.4 \times \left(\frac{2\pi \times 1000}{60}\right)\omega_p$

$\boxed{\omega_p = 0.352 \text{ rad/s}}$



Q. A thin disc of mass m & radius R is mounted on light rod of length 2a which is freely hinged at O & other end of rod, & other end of rod is being supported by a light string. disc spin with an angular speed ω as shown in figure & whole assembly rotate about a vertical axis through O with an angular ω_p, determine the tension in spring

→ $C = I\omega\omega_p$

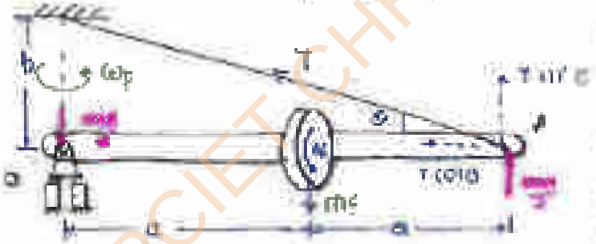
~~$mg\,a = ...b$~~

$\Sigma M_x = 0$

~~$\frac{mg}{2}(2a) + mg(a)$~~

Acw (+ve)

$T\sin\theta\,(2a) - mg(a) + I\omega\omega_p = C$



	Axis	plane
spin	+x	yz
prec	+y	xz
Reac. cy.	+z	xy

$T\sin\theta\,2a = mg\,a - \left(\frac{mR^2}{2}\right)\omega\omega_p$

$T = \left\{mga - \left(\frac{mR^2}{2}\right)\omega\omega_p\right\}\frac{1}{2a\sin\theta}$

but $\sin\theta = \frac{h}{\sqrt{4a^2+h^2}}$

$T = \left\{mga - \left(\frac{mR^2}{2}\right)\omega\omega_p\right\}\frac{\sqrt{h^2+4a^2}}{(2ah)}$

- process of removing unbalanced force or unbalanced couple either by adding some extra mass or by removing excess mass known as balancing

→ **Types of Balancing :**

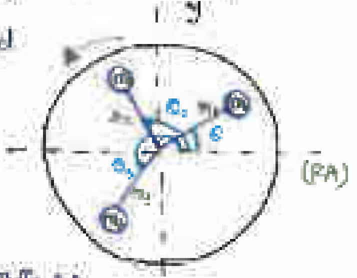
(i) **Static Balancing :-**

- when the masses are rotating in same plane

→ If system is statically balanced

$\Sigma F_x = 0$

$\Sigma F_y = 0$



$m_1 r_1 \omega^2 \cos\theta_1 + m_2 r_2 \omega^2 \cos\theta_2$
$+ m_3 r_3 \omega^2 \cos\theta_3 = 0$

$m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 = 0$

$\boxed{\Sigma m_i r_i \cos\theta_i = 0}$ & $\boxed{\Sigma m_i r_i \sin\theta_i = 0}$

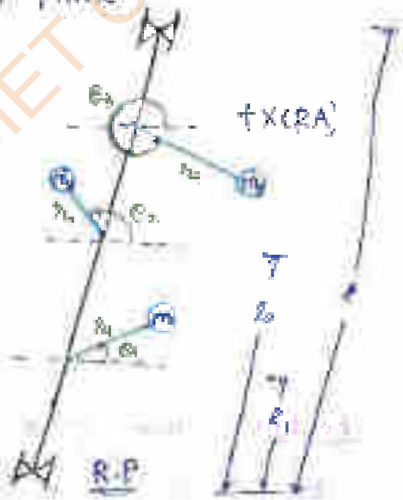
→ If a system is statically balanced then force **polygon** will be closed

— In statically balanced system the reaction will be equal in magnitude & same in direction

(ii) **Dynamic Balancing :-**

- when the masses lies on different plane

- If the system is dynamically balanced then force polygon as well as couple polygon will be closed



→ $\Sigma F_x = 0$

$$\boxed{\sum_{i=1}^{n} m_i r_i \cos\theta_i = 0}$$

→ $\Sigma F_y = 0$

$$\boxed{\sum_{i=1}^{n} m_i r_i \sin\theta_i = 0}$$

→ $m_1 r_1 \omega^2 l_1 \cos\theta_1 + m_2 r_2 \omega^2 l_2 \cos\theta_2 + \dots = 0$

$m_1 r_1 \omega^2 l_1 \sin\theta_1 + m_2 r_2 \omega^2 l_2 \cos\theta_2 + \dots = 0$

$$\boxed{\sum_{i=1}^{n} m_i r_i l_i \cos\theta_i = 0}$$ & $$\boxed{\sum_{i=1}^{n} m_i r_i l_i \sin\theta_i = 0}$$

be opposite in direction.

(iii) complete balancing

- If the system is statically as well as dynamically balanced it is said to be completely balance
- Reaction will be different in magnitude as well as direction.

CRO
-[1]

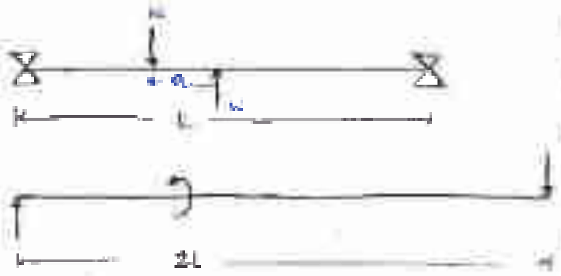
case

$R \cdot L = W \cdot a$

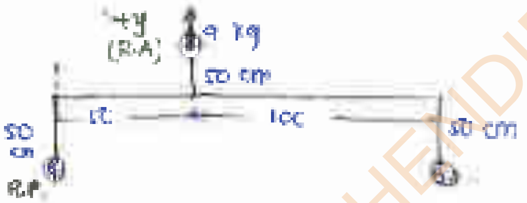
$\boxed{R = \frac{Wa}{L}}$

→ $R' = \frac{Wa}{2L}$

$\boxed{R' = R/2}$



[2]



mass	r	θ	mr cosθ	mr sin θ	ℓ	mrℓ cosθ	mrℓ sinθ
B_1	50	180	$B_1(50)\cos180$	$B_1(50)\sin180$	0	0	0
9	50	0	$9(50)\cos0$	$9(50)\sin0$	50	$9(50)(50)$	0
B_2	50	180	$B_2(50)\cos180$	$B_2(50)\sin180$	150	$B_2(50)\cos(180)(150)$	0

→ $\Sigma mr\ell\cos\theta = 0 \Rightarrow 9(50)(50) = B_2(50)(150)$

$\boxed{B_2 = 3\ kg}$

→ $\Sigma mr\cos\theta = 0 \Rightarrow -B_1(50) + 9(50) - B_2(50) = 0$

$-150 + 450 = B_2(50)$

$\boxed{B_2 = 6\ kg}$

⇒ Shortcut: when system is completely balanced, then ..

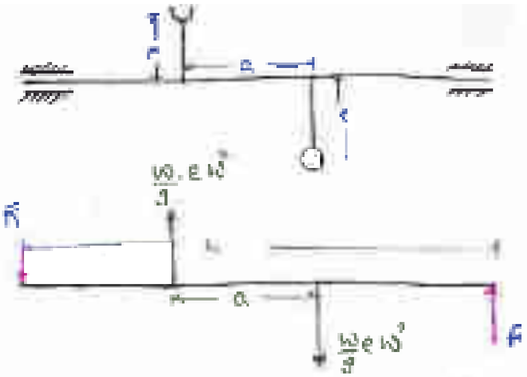
$B_1 = \frac{9 \times 100}{150}$ & $B_2 = \frac{9 \times 50}{150}$

$\boxed{B_1 = 6\ kg}$ $\boxed{B_2 = 3\ kg}$

[4] Dynamic reactions are opposite in nature.

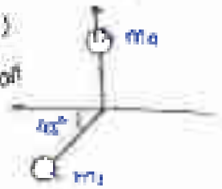
$R \cdot L = \frac{W}{g} e \omega^2 a$

$$\boxed{R = \frac{W}{g} e \omega^2 \frac{a}{L}}$$



[6] & [7]

$m_A \rightarrow$ R.A
m_C or $m_D \rightarrow$ R.P (because total [illegible] entity [illegible])



mass	r	θ	$mr\cos\theta$	$mr\sin\theta$	l	$mrl\cos\theta$	$mrl\sin\theta$
5	20	0	(5)(20)	0	−20	−(5)(20)(20)	0
(R.P) m_C	20	θ_C	$m_C(20)\cos\theta_C$	$m_C(20)\sin\theta_C$	0	0	0
m_D	20	θ_D	$m_D(20)\cos\theta_D$	$m_D(20)\sin\theta_D$	40	$m_D(20)\cos\theta_D$ (40)	$m_D(20)\sin\theta_D$
6	20	135°	(6)(20) cos135°	(6)(20) sin135°	60	(6)(20) cos135° (60)	(6)(20) sin135°

$\Rightarrow \quad \Sigma mrl\cos\theta = 0 \qquad \Sigma mrl\sin\theta = 0$

$-(5)(20)$

$0 + 0 + m_D(20)(\sin\theta_D)(40) + (6)(20)\sin 135°(60) = 0$

$800\, m_D \sin\theta_D = -5091.1$

$\boxed{m_D \sin\theta_D = -6.364} \quad \text{--- (i)}$

$\Sigma mrl\cos\theta = 0$

$-(5)(20)(20) + m_D(20)\cos\theta_D(40) + (6)(20)(\cos 135°)(60) = 0$

$-2000 + 800\, m_D\cos\theta_D - 5091.168 = 0$

$\boxed{m_D\cos\theta_D = 8.864} \quad \text{--- (ii)}$

$\tan\theta_D = -\frac{6.364}{8.863}$

$\boxed{\theta_D = -35.69°}$ from $\theta_D = 324.30°$

$m_D^2\cos^2\theta_D + m_D^2\sin^2\theta_D = (6.364)^2 + (8.863)^2$

$\boxed{m_D = 10.919 \text{ kg}}$

→ $20 m_c \sin\theta_c + (20)(10.99)\sin(-35.67) + (8)(20)(0.707) = 0$

$m_c \sin\theta_c = 2.119$ — (iii)   ~-13.16

$\Sigma m r \cos\theta = 0$

→ $20 m_c \cos\theta_c + (20)(10.41)\cos(-35.67) - (8)(20)(0.707) = 0$

$m_c \cos\theta_c = -9.621$ — (iv)   2.12

~~$m_c^2 = (-2.119)^2 + (-4.621)^2$~~ $m_c^2 = (2.119)^2 + (-9.621)^2$

~~$\boxed{m_c = 5.06 \text{ kg}}$~~ $\boxed{m_c = 9.85 \text{ kg}}$

~~$\tan\theta_c = \frac{-2.119}{-4.621}$~~ $\tan\theta_c = \frac{+2.119}{-9.621}$

~~$\theta_c = 24.63$~~ $\boxed{\theta_c = -12.45}$, 167.5

i.e. $\underline{167.5^\circ}$

→ time period for tan is 180° & repeat after a 180°

Note →

$\sin\theta \to 2\pi$ $\cos\theta \to 2\pi$ $\tan\theta \to \pi$

$\sin n\theta \to \frac{2\pi}{n}$ $\cos n\theta \to \frac{2\pi}{n}$ $\tan n\theta \to \frac{\pi}{n}$

$T = 500 + 150\sin\theta + 150\sin 2\theta - 100\cos 3\theta$

time period $= \frac{\text{LCM of } N^r}{\text{HCF of } D^r}$

time period $\frac{2\pi}{1}$ $\frac{2\pi}{2}$ $\frac{2\pi}{3}$

$= \frac{2\pi}{1}$

8/9

mass	r	θ	mr cosθ	mr sinθ	ℓ
10	10	90°	0	100	
5	20	330	86.60	-50	
m_c	10	θ_p	$10 m_p \cos\theta_p$	$10 m_p \sin\theta_p$	

$m_D \sin\theta_D = -5$ — (1)

$86.60 + m_D(10)\cos\theta_D = 0$

$m_D \cos\theta_D = -8.66$ — (2)

$m_D = \sqrt{(-5)^2 + (-8.66)^2}$

$\boxed{m_D = 10\ kg}$

$\tan\theta_D = -5/-8.66$

$\boxed{\theta_D = 30}$

$\theta_D = 210°$ from (+x axis)

d)

$F_C = m_D r_D \omega^2 = 3947.84\ N$

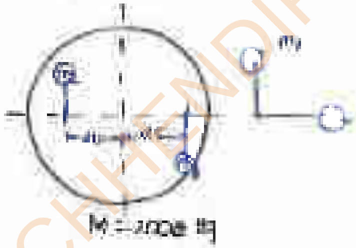
$F_C = F_D$

$3947.84 \times \frac{20}{100} = R \times \frac{40}{100}$

$\boxed{R = 1973.9\ N} \approx 2\ kN$

e)

mass	r	θ	mr cosθ	mr sinθ
52 kg	10	0°	(52)(10)	0
2000	e	θ	(2000)(e)cosθ	(2000)(e)sinθ
75 kg	10	90	0	(75)(10)



$(52)(10) + (2000)(e)\cos\theta = 0$

$\boxed{e\cos\theta = -0.26}$ — (1)

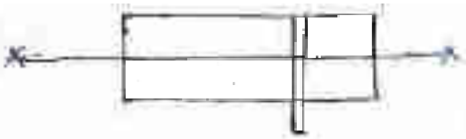
$(75)(10) + (2000)(e)\sin\theta = 0$

$\boxed{e\sin\theta = -0.375}$

$e^2 = (-0.26)^2 + (-0.375)^2 \Rightarrow \boxed{e = 0.456\ cm}$

$\tan\theta = \frac{-0.375}{-0.26}$

$\boxed{\theta = 55.26°} \Rightarrow 235.26°$ (from +x axis)



$r_A = r_B = 0.5$

mass	r	θ	$mr\cos\theta$	$mr\sin\theta$	l	$mrl\cos\theta$	$mrl\sin\theta$
(R.P) m_A	0.5	θ_A	$0.5 m_A \cos\theta_A$	$0.5 m_A \sin\theta_A$	0	0	0
	2	0°	2	0	0.3	0.6	0
m_B	0.5	θ_B	$0.5 m_B \cos\theta_B$	$0.5 m_B \sin\theta_B$	0.5	$0.5^2 m_B \cos\theta_B$	$0.5^2 m_B \sin\theta_B$

$\rightarrow 0.5^2 m_B \sin\theta_B = 0$

$\boxed{\theta_B = 0 \text{ or } 180^\circ}$

$\rightarrow \theta_B = 0 \Rightarrow 0.6 + 0.5^2 m_B \cos 0 = 0 \leftarrow \theta_B = 180^\circ$ only possible

$\boxed{m_B = 2.4 \text{ kg}}$; $\theta_B = 180 \rightarrow \boxed{m_B = 2.4 \text{ kg}}$

$\rightarrow \Sigma m r \sin\theta = 0$

$(0.5)(2.4)\sin(180) + (0.5) m_A \sin\theta_A = 0$

$m_A \sin\theta_A = 2.4$ — (i)

$\rightarrow \Sigma m r \cos\theta = 0$

$2 + (0.5)(2.4)\cos(180) + (0.5) m_A \cos\theta_A = 0$

$m_A \cos\theta_A = 2.4 \longrightarrow$

$= 0.6 \rightarrow 2 - 1.2 + (0.5) m_A \cos\theta_A = 0$

$m_A \cos\theta_A = -1.6$ — (ii)

$\boxed{m_A = 2.66 \text{ kg}}$

$\Sigma m r \sin\theta = 0$

$0.5 m_A \sin\theta_A + 0 + 0.5 m_B \sin\theta_B = 0$ $\{\theta_B = 0$

$\boxed{\theta_A = 0 \text{ or } 180}$

$\rightarrow \Sigma m r \cos\theta = 0$

$0 = m_A \cos\theta_A + 2 + 0.5 m_B \cos\theta_B = 0$

$\boxed{m_A = 1.6 \text{ kg}}$

contd

mass	r	θ	$m r\cos\theta$	$m r\sin\theta$
20	15	0°	500	0
25	20	135°	-353.55	353.55
M	30	θ	$(20)(M)\cos\theta$	$(30)(M)\sin\theta$

$\Rightarrow$ $353.55 = (20)(M)\sin\theta$

$M\sin\theta = -17.67$ — (i)

$\Rightarrow$ $500 - 353.55 + (20)(M)\cos\theta = 0$

$M\cos\theta = 2.67$ — (ii)

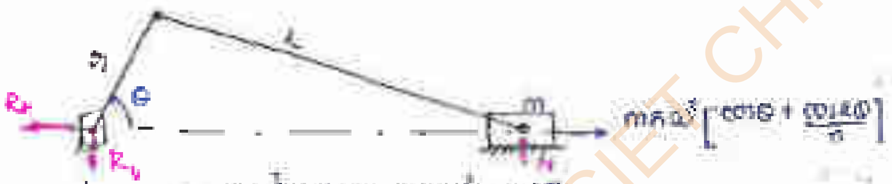
$\boxed{M = 17.87 \text{ kg m}}$

$\tan\theta = \frac{-17.67}{2.67} \Rightarrow \boxed{\theta = -81.40}$

$= 98.61°$ (from -x axis)

$= 278.6$ (from +x axis)

→ Balancing of Reciprocating parts :



→ $(F_i)_{net} = \underbrace{m r\omega^2\cos\theta}_{\text{primary inertial force}} + \underbrace{\frac{m r\omega^2\cos 2\theta}{n}}_{\text{secondary}}$

(which will balance completely) → primary inertial force

→ $n = \frac{l}{r}$; $n > 1$

→ Since n is quite large ∴ so secondary inertia force crosses in magnitude with respect to primary inertial force & hence it can be neglected.

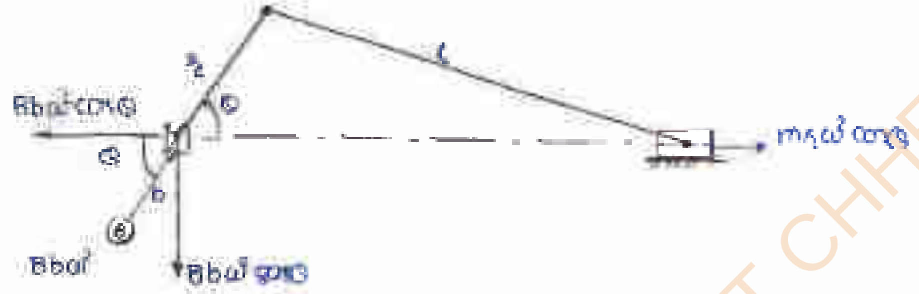
→ The primary inertia force $m r\omega^2\cos\theta$ results in unbalance force which changes its magnitude as well as direction of the crank rotating.

→ R_H is known as shaking force.

→ R_V and lateral thrust acting on cylinder will introduce a couple known as shaking couple.

force with the help of balancing mass or counter mass
- It has been observed that F_V is more harmful for the pin with respect to F_H therefore we require partial balancing

- Vertical force is very danger so balancing of reciprocating parts vertical completely is not possible so we go for partially balance.



Balancing mass = B
balancing radius = b

$$\boxed{B \cdot b = C \cdot m \cdot r}$$

where :- → B = balancing mass
→ b = balancing mass radius
→ c = fraction
→ m = mass of reciprocating part
r = crank radius

⟹ unbalanced force along line of stroke

$$F_{UN,H} = mr\omega^2\cos\theta - Bb\omega^2\cos\theta$$

$$= mr\omega^2\cos\theta - c\,mr\omega^2\cos\theta$$

$$\boxed{F_{UN,H} = (1-c)\,mr\omega^2\cos\theta}$$

$\boxed{F_{un,v} = c\, m_r \omega^2 \sin\theta}$

⇒ net unbalance force

$R_{net} = \sqrt{F_{un,H}^2 + F_{un,v}^2}$

$= \sqrt{((1-c) m_r \omega^2 \cos\theta)^2 + (c\, m_r \omega^2 \sin\theta)^2}$

NOTE: R_{net} will be minimum at $\boxed{c = \frac{1}{2}}$

in steam engine $\boxed{c = \frac{2}{3}}$

③ Effects

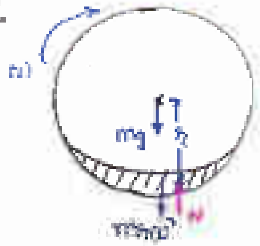
(1) Hammer blow

- The maximum magnitude of unbalanced force perpendicular to the line of stroke is called a hammer blow.

$\boxed{(F_{un,v})_{max} = m_r \omega^2}$ ⇒ in complete balancing

NOTE: In order to reduce the unbalance force perpendicular to the line of stroke we did partial balancing.

Wheel



$Mg \pm m_r \omega^2 = N$

- The Hammer blow puts a limit on the speed of locomotive.

(2) Effect in coupled locomotive

- In coupled locomotive cranks will be connected perpendicular to each other.

⇝ Tractive force

$F_{un,1} = (1-c) m_r \omega^2 \cos\theta$

$F_{un,2} = (1-c) m_r \omega^2 \cos(90 + \theta)$

$F_{un,net} = F_{un,1} - F_{un,2}$

$\boxed{F_{un,net} = (1-c) m_r \omega^2 [\cos\theta - \sin\theta]}$

$F_{un,net} = f(\theta)$

$d\theta$

$-\sin\theta - \cos\theta = 0$

$\tan\theta = -1$

$$\boxed{\theta = 135^\circ \text{ or } 315^\circ}$$

F_{un} @ $\theta = 135^\circ$ $= (1-c)\, m r \omega^2 [\cos 135^\circ - \sin 135^\circ]$

$= -\sqrt{2}\, m r \omega^2 (1-c)$

F_{un} @ $\theta = 315^\circ$ $= +\sqrt{2}\, m r \omega^2 (1-c)$

$$\boxed{\text{Tractive force} = \pm\sqrt{2}(1-c)\, m r \omega^2}$$

→ <u>Swaying couple</u>

$M = (1-c)\, m r \omega^2 \cos\theta \cdot \frac{a}{2} - (1-c)\, m r \omega^2 \cos(90+\theta)\, \frac{a}{2}$

$$\boxed{M = (1-c)\, m r \omega^2 \frac{a}{2} [\cos\theta + \sin\theta]}$$

$M = f(\theta)$

for max^m & min^m

$\frac{dM}{d\theta} = 0$

$\Rightarrow (1-c)\, m r \omega^2 \frac{a}{2} [-\sin\theta + \cos\theta] = 0$

$\tan\theta = 1$

$$\boxed{\theta = 45^\circ \;\&\; 225^\circ}$$

$M @_{\theta = 45^\circ} = (1-c)\, m r \omega^2 \frac{a}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

$= +\frac{1}{\sqrt{2}} (1-c)\, m r \omega^2 a$

$M @_{\theta = 225^\circ} = -\frac{1}{\sqrt{2}} (1-c)\, m r \omega^2 a$

$$\boxed{\text{Swaying couple} = \pm \frac{a}{\sqrt{2}} (1-c)\, m r \omega^2}$$

$F_{primary} = m r \omega^2 \cos\theta$

$F_{secondary} = m r \omega^2 \frac{\cos 2\theta}{n}$

m
r
ω
θ

$F_s = m\left(\frac{r}{4n}\right) 4\omega^2 \cos 2\theta$

$= m\left(\frac{r}{4n}\right)(2\omega)^2 \cos 2\theta$

$\boxed{F_s = m r' (\omega')^2 \cos 2\theta}$

for converting primary → secondary →

m → m
$r' = r/4n$
$\omega' = 2\omega$
$\theta' = 2\theta$

→ Rotating can be balance completely whereas Reciprocating masses are partially balance always.

$B \cdot b = m_{rota} \cdot r + c\, m_{rec} \cdot r$

if $b = r$

$\boxed{B = m_{rota} + c\, m_{rec}}$

[15] $m = 10$ kg

$r = 0.1$ m (stroke = 2r = 0.2)

$B = 5$ kg

$b = r = 0.1$

$\theta = 30$

$F_{unb} = \underline{c m r}\, \omega^2 \sin\theta$

$= B b \omega^2 \sin 30 = 5 \times 0.1 \times 10^2 \times \frac{1}{2}$

$\boxed{F_{un} = 30\ N}$

$Bb = cmr$

[16] $m = 10$ kg

$r = 15$ cm

$c = 0.6$

$\theta = 60$

$\omega = \pi$ rad/s

along line of axis

$F_{un,H} = (1-c) m r \omega^2 \cos\theta$ (portion of mass balanced)

$= (1-0.6)(10)(150 \cdot 10^{-3})(\pi)^2 \cos 60$

$\boxed{F_{un,H} = 4.5\ N}$

⇒ Classification of vibration

1) on the basis of excitation

(i) Natural vibration

- If the system vibrates due to inherent forces or self weight

Does not require any external forces, is defined as Natural vibration.

(ii) Free vibration

- If an external force is required to produce the vibration on the system & force will not be considered during the analysis of motion is called free vibrations.
- In free vibration ; the energy of system, may or may not remain conservative.

(iii) Forced vibration

- The vibration due to external excitation force is defined as forced vibration.

(iv) parametrically excited vibration

(v) self excited vibration

ex. vocal chord of human body

2) on the basis of degree of freedom

(i) single D.O.F

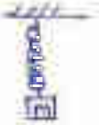
(ii) multi D.O.F

(iii) infinite DOF

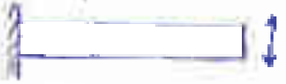
or elastic bodies

3) on the basis of direction of motion

(i) Longitudinal vibration



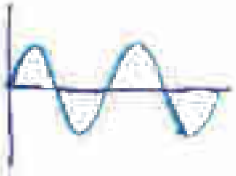
(ii) Transverse vibration



(iii) Torsional vibration

Vibration

Undamped
(Energy = const.)



Damped vibration
- viscous damping
- friction (or) columb damping
- Gravitational damping
- Hysteresis

Linearization of parameter

⟹ **System Parameter**

i) Inertia

- The ability of any body to resist the change is known as inertia.
- Measure of inertia in pure translation is mass when in pure rotation it is mass moment of inertia.

pure translation

$$F_{ext} = \frac{d}{dt}(mv)$$

$$F_{ext} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

If m = const.

$$F_{ext} = m\frac{dv}{dt}$$

$$F_{ext} = ma_{cm}$$

$$\boxed{\begin{array}{l} F_I = m\cdot a_{cm} \\ F_I = m\cdot\ddot{x} \end{array}}$$

in translation

If 'x' is displacement

$\frac{dx}{dt} = \dot{x}$ velocity

$\frac{d^2x}{dt^2} = \ddot{x}$ accⁿ

in rotation

θ

$\frac{d\theta}{dt} = \dot{\theta}$

$\frac{d^2\theta}{dt^2} = \ddot{\theta}$

of inertia (MOI) (which is distribution of mass about axis of rotation)

Pure rotation

$T_{ext} = \frac{d}{dt}(mvr)$

$\boxed{T_i = I\ddot{\theta}}$

external torque   Mass M.O.I

⇒

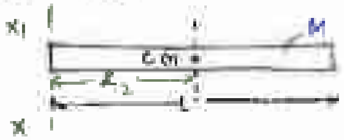
1) Disc

$I_{CG} = \frac{MR^2}{2}$

2) concentrated mass

$I_P = ml^2$

3) Rod



$I_{CG} = \frac{Ml^2}{12}$

$I_{x_1x_1} = I_{CG} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12} + \frac{ml^2}{4}$

$I_{x_1x_1} = \frac{ml^2}{3}$

4)

$I_{x_1x_1} = I_{CG} + Mr^2$

$= \frac{mr^2}{2} + mr^2$

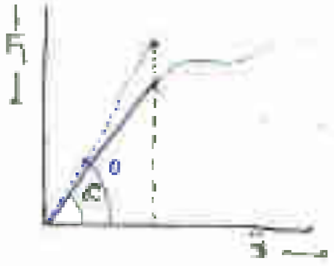
$I_{x_1x_1} = \frac{3}{2}mr^2$

5) ✓

$\boxed{I_{x_2x_2} = I_{CG} - ma^2}$

$\vec{x}$

$\boxed{\tan\theta = m}$



- $m_1 > m$
- $\theta_1 > \theta$
- $F_1 > F$

MOU : MOU is the slope of Fi vs x diagram upto linear region

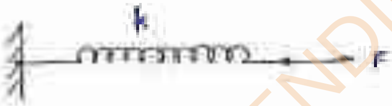
if MOU ↑ θ ↑ Fi ↑

→ This is a region we always have smaller link of drive in belt drive, in gear - pinion mechanism, pinion is driven → having lower inertia → low starting torque

2) Calibration

- Every body comes back to its original position, this ability is known as restoration
- To represent restoration characteristic of any system ; we use springs

Hooke's law ;



$F_{spring} \propto -x$
(-ve) sign

Translation ·

→ Negative sign indicates that spring force will always be opposite to the displacement

$\boxed{F_s = -kx}$ → $k = \frac{F_s}{x}$

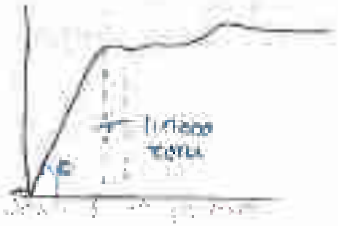
if $x = 1$ unit

$\boxed{k = F_s}$

the amount of force req^d to produce unit deflection is known as stiffness.

→ k ↑ F_s ↑ chances of failure ↓

$\boxed{\tan\theta = k}$

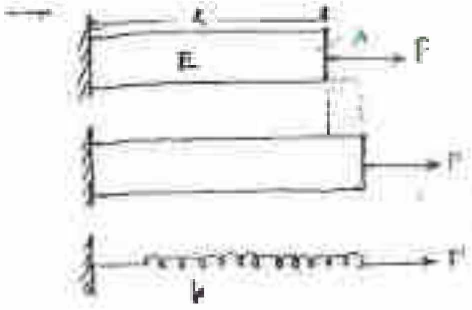


Where k = stiffness
= spring const
= spring rate

unit = N/m

$$\boxed{Q.\ Or\ K_{eq} = \frac{T_1}{\theta}}$$

unit - $\frac{N\text{-}m}{rad}$



In solids

$\sigma \propto \epsilon$ (P-L)

$$\boxed{\sigma = E\epsilon}$$

$\frac{P}{A} = E\,\frac{\delta L}{L}$

$P = \left(\frac{AE}{L}\right)\delta L$

$$\boxed{\text{Axial stiffness} = \frac{AE}{L}}$$

In spring

$F_s \propto x$

$F_s = kx$

Loading	Geometrical properties	Material properties	Rigidity	Stiffness = $\frac{\text{Rigidity}}{\text{length}}$
Axial	A	E	AE axial rigidity	Axial stiffness = $\frac{AE}{L}$
flexural	I	E	EI flexural rigidity	flexural stiffness = $f\left(\frac{EI}{L}\right)$
Torsion	J	G	GJ	$\frac{GJ}{L}$

IMP → stiffness $\propto \frac{1}{\text{length}}$

eg length ↑
k ↓
rigidity ↓
chances of failure ↑

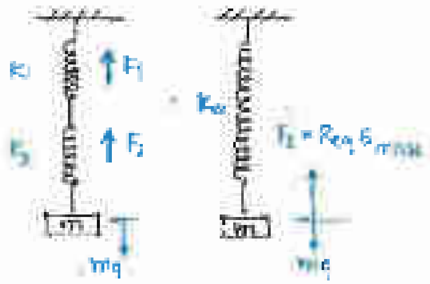
→ If the length of k are into ratio of m : n then their stiffness will be in a ratio n : m

No. of Turns

equal then springs of eqv. stiffness '2k' & '3k'.

⇒ Spring connection :

(i) series connection



$F_1 = F_2 = mg$ (forces are same)

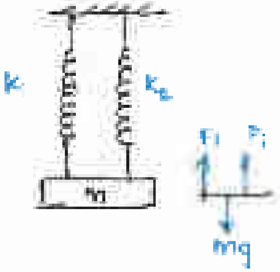
$\delta_{max} = \delta_1 + \delta_2$ (deflection is cumulative)

$$\frac{mg}{K_{eq}} = \frac{F_1}{K_1} + \frac{F_2}{K_2}$$

$$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}}$$

$$\boxed{\frac{1}{K_{eq}} = \sum_{i=1}^{n} \frac{1}{K_i}}$$

(ii) parallel connection



1) Mass remains Horizontal

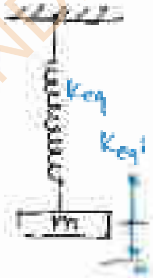
$F_1 + F_2 = mg$ (forces are cumulative)

$\delta_1 = \delta_2 = \delta_{max}$

$K_1\delta_1 + K_2\delta_2 = K_{eq}\delta_{max}$

$$\boxed{K_{eq} = K_1 + K_2}$$

$$\boxed{K_{eq} = \sum_{i=1}^{n} K_i}$$



iii)

→ parallel connection

parallel conn are more rigid → because combined stiffness incr

(3) ⇒ Damping :

i) Viscous damping :

Newton's law of viscosity

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \left[\frac{u_2 - u_1}{y_2 - y_1}\right]$$

at no slip condition $\boxed{u_1 = 0}$

y_2

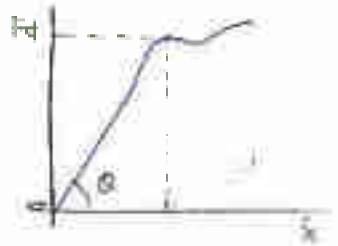
$\tau \propto \frac{u}{y}$

$\tau \propto$ velocity of fluid

$\boxed{F_d \propto \text{velocity of fluid}}$

→ $F_d \propto \dot{x}$

$\boxed{F_d = C\dot{x}}$ where C = damping coeficient



In translation :

$C = \frac{F_d}{\dot{x}}$ unit $\frac{N}{(m/s)} = \frac{N\text{-}s}{m}$

In Rotation

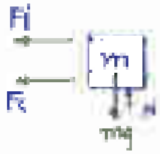
$C_{eq} = \frac{T_d}{\dot{\theta}}$ unit $\frac{N\text{-}m}{rad/s} = \frac{N\text{-}m\text{-}sec}{rad}$

(b) ⟹ Equilibrium position :

- The position about which system vibrate

[■] Equation of motion in single d.o.f. / undamped free vibrate

case-(i)



Assumption!
spring is massless

$F_i + F_s = 0$

$m\ddot{x} + Kx = 0$

$\boxed{\ddot{x} + \frac{K}{m}x = 0}$ compare with $\ddot{x} + \omega_n^2 x = 0$

→ $\boxed{\omega_n = \sqrt{\frac{K}{m}}}$

where ω_n = natural angular frequency of undamped system

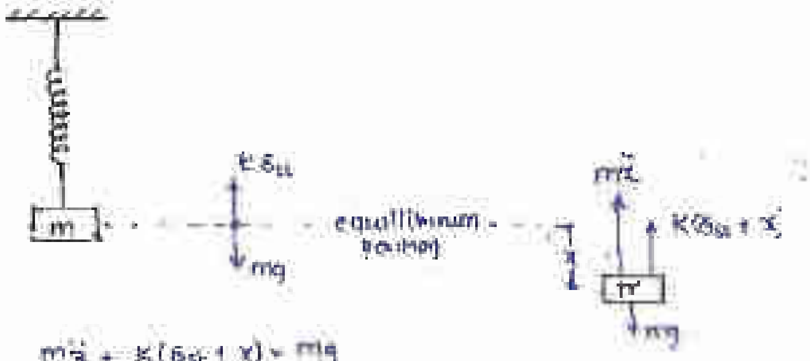
→ $\boxed{\omega_n = 2\pi f_n}$

f_n = linear frequency Hz or sec

→ Time period $\boxed{T = \frac{1}{f_n}}$ unit : T → sec

Time period : Time required to complete one cycle is called time pe

Case-(ii) :



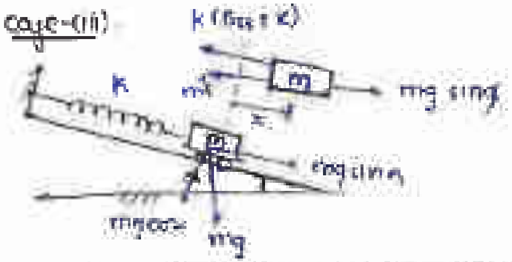
$m\ddot{x} + K(\delta_{st} + x) = mg$

$m\ddot{x} + \cancel{K\delta_{st}} + Kx = \cancel{mg}$

$$\boxed{\ddot{x} + \frac{K}{m}x = 0}$$

where $\boxed{\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\delta_{st}}}}$

Case-(iii)



at initially $\boxed{mg \sin\alpha = K\delta_{st}}$

$\rightarrow m\ddot{x} + K(\delta_{st} + x) = mg \sin\alpha$

$\rightarrow m\ddot{x} + K\delta_{st} + Kx = mg \sin\alpha$

$$\boxed{\ddot{x} + \frac{K}{m}x = 0}$$

where $\boxed{\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g\sin\alpha}{\delta_{st}}}}$

$\sqrt{\frac{K\delta}{m}} = \sqrt{\frac{mg \sin\alpha}{m}}$

→ solution of equation of motion

$\ddot{x} + \omega_n^2 x = 0$

$\rightarrow \frac{d^2x}{dt^2} + \omega_n^2 x = 0$

$[D^2 + \omega_n^2]x = 0$

→ soln of x = (CF,

soln = CF + P.I = 0 because (right part is zero)

<u>CF.</u>

$D^2 + \omega_n^2 = 0$

$\Rightarrow D^2 = -\omega_n^2$

$\boxed{D = \pm i\omega_n}$

→ $x = A\cos\omega_n t + B\sin\omega_n t$ { in vibration it is still follow

$A = X\sin\phi$

$B = X\cos\phi$

$x = X\sin\phi\cos\omega_n t + X\cos\phi\sin\omega_n t$

$\boxed{x = X\sin(\omega_n t + \phi)}$ ⇐ Displacement eqn

velocity eqn

$\dot{x} = X\omega_n\cos(\omega_n t + \phi)$

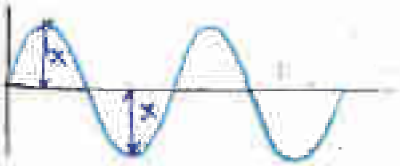
<u>Accn eqn</u>

$\ddot{x} = -X\omega_n^2\sin(\omega_n t + \phi)$

→ Analogy b/w translation & rotation

Pure translation	Rotation
→ $m\ddot{x} + Kx = 0$	→ $I\ddot{\theta} + K_{eq}\theta = 0$
$\boxed{\omega_n = \sqrt{\frac{K}{m}}}$	K_{eq} $\boxed{\omega_n = \sqrt{\frac{K_{eq}}{I}}}$ about centre of rotation
→ Displacement $x = X\sin(\omega_n t + \phi)$	→ $\theta = \Theta\sin(\omega_n t + \phi)$
velocity $\dot{x} = X\omega_n\cos(\omega_n t + \phi)$	$\dot{\theta} = \Theta\omega_n\cos(\omega_n t + \phi)$
Accn $\ddot{x} = -X\omega_n^2\sin(\omega_n t + \phi)$	$\ddot{\theta} = -\Theta\omega_n^2\sin(\omega_n t + \phi)$
$x_{max} = X$	$\theta_{max} = \Theta$
$\dot{x}_{max} = X\omega_n$	$\dot{\theta}_{max} = \Theta\omega_n$
$\ddot{x}_{max} = -X\omega_n^2$	$\ddot{\theta}_{max} = \Theta\omega_n^2$

$x = X \sin(\omega_n t + \phi)$



$\dot{x} = X \omega_n \cos(\omega_n t + \phi)$

NOTE

- There is 90 phase lag between displacement velocity vector, acc^n ∝ − displacement ; which indicate the motion is SHM

→ **Energy Method** :

T.E of system = const

$$\frac{d}{dt}\left[\text{T.E of system}\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2\right] = 0$$

K.E of mass

P.E of spring

$$\Rightarrow \frac{1}{2}(\cancel{2} m \dot{x} \ddot{x}) + \frac{1}{2}(\cancel{2} k x \dot{x}) = 0$$

$$m\ddot{x} + kx = 0$$

$$\boxed{\ddot{x} + \frac{k}{m} x = 0}$$ ← valid for only undamped system

— **Rayleigh's Method** :

= If the system is undamped, then T.E of system = const

i.e max^n energy at equilibrium = max^n energy at extreme position

$$\Rightarrow \frac{1}{2} m \dot{x}^2_{max} = \frac{1}{2} k x^2_{max}$$

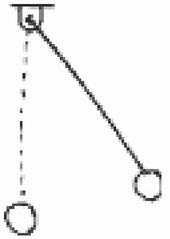
$$\Rightarrow m \dot{x}^2_{max} = k x^2_{max}$$

$$\Rightarrow m(X\omega_n)^2 = k(X)^2$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

at mid position
V = max^n
Extreme position
V = 0

NOTE :



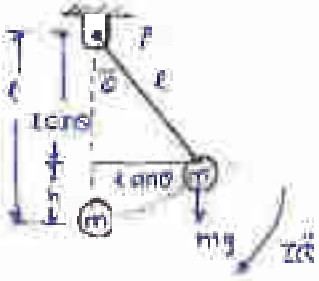
→ $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\Rightarrow \lim_{\theta \to 0} \sin\theta = \theta$

→ $\lim_{\theta \to 0} \cos\theta = 1$

→ $\lim_{\theta \to 0} (1-\cos\theta) = \frac{\theta^2}{2}$ $\left\{ \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \right.$

Case - I :



let C.W couple +ve.

$I_P \ddot{\theta} + mg\ell\sin\theta = 0$

$I_P \ddot{\theta} + mg\ell\theta = 0$

$\ddot{\theta} + \left(\frac{mg\ell}{I_P}\right) = 0$

$\omega_n = \sqrt{\frac{mg\ell}{I_P}}$ at $\boxed{I_P = m\ell^2}$

$\frac{2\pi}{T} = \sqrt{\frac{mg\ell}{m\ell^2}} = \sqrt{\frac{9.81}{\ell}}$ the $\boxed{\omega_n = \frac{2\pi}{T}}$

$\frac{2\pi}{0.5} = \sqrt{\frac{9.81}{\ell}}$

$\boxed{\ell = 62 \text{ mm}}$

Energy method :

$\frac{d}{dt}(\text{TE of system}) = 0$

$\Rightarrow \frac{d}{dt}(KE + PE) = 0$

$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}I_P\dot{\theta}^2 + mgh\right) = 0$

$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}I_P\dot{\theta}^2 + mg(\ell - \ell\cos\theta)\right) = 0$

$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}I_P\dot{\theta}^2 + mg\ell(1-\cos\theta)\right) = 0$

$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}I_P\dot{\theta}^2 + mg\ell\frac{\theta^2}{2}\right) = 0$

— I_P due to mass M ; some displacement is come (happened) then the M will not come.

$$\boxed{I_p\ddot{\theta} + mgl\theta = 0}$$

⇒ **Rayeigh's method**

max KE = max P.E

$$\Rightarrow \frac{1}{2} I_p \dot{\theta}^2_{max} = mgl(1-\cos\theta)_{max}$$

$$\Rightarrow \frac{1}{2} I_p \dot{\theta}^2_{max} = mgl\frac{\theta^2_{max}}{2}$$

$$\Rightarrow I_p(\theta\omega_n)^2 = mgl\,\theta^2$$

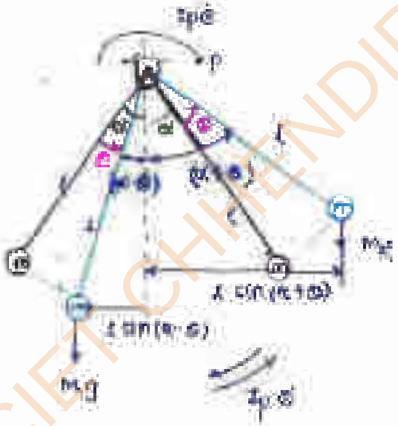
$$\omega_n = \sqrt{\frac{mgl}{I_p}} = \sqrt{\frac{mgl}{ml^2}} = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{g}}$$

$$\boxed{l = 82\_mm}$$

CQ8

$\boxed{2}$ c.w. (+ve)



$$\Rightarrow I_p\ddot{\theta} + m_2g(l\sin(\alpha+\theta))$$
$$- m_1g(l\sin(\alpha-\theta)) = 0$$

$$\Rightarrow I_p\ddot{\theta} + m_2gl(\alpha+\theta) - m_1gl(\alpha-\theta) = 0$$

$$\Rightarrow I_p\ddot{\theta} + \cancel{mgl\alpha} + mgl\theta$$
$$- \cancel{mgl\alpha} + mgl\theta = 0$$

$$I_p\ddot{\theta} + 2mgl\theta = 0$$

$$\Rightarrow I_p\ddot{\theta} - mgl\sin(\alpha-\theta) + mgl\sin(\alpha+\theta) = 0$$

$$\Rightarrow I_p\ddot{\theta} + mgl[\sin(\alpha+\theta) - \sin(\alpha-\theta)] = 0$$

$$\Rightarrow I_p\ddot{\theta} + mgl\left[2\cos\left(\frac{\alpha+\theta+\alpha-\theta}{2}\right)\sin\left(\frac{\alpha+\theta-\alpha+\theta}{2}\right)\right] = 0$$

$$\Rightarrow I_p\ddot{\theta} + 2mgl[\cos\alpha\sin\theta] = 0 \qquad \Big|\ S - S = 2CS$$

$$I_p\ddot{\theta} + 2mgl\cos\alpha\,\theta = 0$$

where $I = m_1l^2 + m_2l^2$
$= 2ml^2$

$$\ddot{\theta} + \frac{2mgl\cos\alpha}{2ml^2}\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{g\cos\alpha}{l}}}$$

12)

$F = 5000$ N
$\delta t = 10^{-4}$ sec.
$m = 1$ kg
$K = 10$ kN/m
initially rest

→ impulse force = change in momentum

$$F\,\delta t = (mv)_f - \cancel{(mv)_i} \qquad \left\{ v_i = 0 \right.$$
initially rest

$$5000 \times 10^{-4} = 1 \times v_f$$

$$\boxed{v_f = 0.5\ m/s} \leftarrow V_{max}$$

→ $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10 \times 10^3}{1}}$

$$\boxed{\omega_n = 100\ rad/s}$$

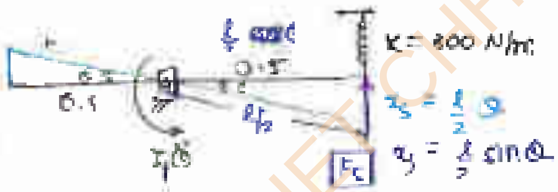
→ $x_{max} = X\,\omega_n$

$0.5 = 100\,X$

$$\boxed{X = 5\ mm}$$

4 ω_n same at moon and earth.

5



$$I_p\ddot{\theta} + F_s\left(\frac{l}{2}\cos\theta\right) = 0$$

$$\Rightarrow I_p\ddot{\theta} + \left(K\frac{l}{2}\sin\theta\right)\left(\frac{l}{2}\cos\theta\right) = 0$$

$$\Rightarrow I_p\ddot{\theta} + K\left(\frac{l}{2}\theta\right)\frac{l}{2} = 0$$

$$\Rightarrow I_p\ddot{\theta} + \frac{Kl^2}{4}\theta = 0$$

$$\ddot{\theta} + \frac{Kl^2}{4I_p} = 0$$

$I_p = \frac{ml^2}{12}$

$$\omega_n = \sqrt{\frac{Kl^2}{4I_p}} = \sqrt{\frac{Kl^2}{\frac{ml^2}{12}\cdot 4}}$$

$$= \sqrt{\frac{3 \times 300}{1}}$$

ω_n = [illegible] rad/s

→

~~$\frac{d}{dt}(KE + PE) = 0$~~

~~$\frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2\right) = C$~~

~~$\frac{d}{dt}\left(\frac{1}{2} m \dot{x}\right.$~~

$$\frac{d}{dt}\left[\frac{1}{2} I_P \dot{\theta}^2 + \frac{1}{2} K x_c^2\right] = 0$$

$$\frac{d}{dt}\left[I_P \dot{\theta}^2 + K\left(\frac{l}{2}\theta\right)^2 \right] = 0$$

$$\Rightarrow \frac{d}{dt}\left[I_P \dot{\theta}^2 + \frac{K \theta^2 l^2}{4} \right] = 0$$

$$\Rightarrow I_P (2\dot{\theta}\ddot{\theta}) + \frac{K l^2}{4} (2\theta\dot{\theta}) = C$$

$$\boxed{I_P \ddot{\theta} + \frac{K l^2}{4} \theta = 0}$$

→ Rayleigh method

$$(K.E)_{max} = (P.E)_{max}$$

$$\frac{1}{2} I_P \dot{\theta}^2_{max} = \frac{1}{2} K x^2_{max}$$

$$\frac{1}{2} I_P \dot{\theta}^2_{max} = \frac{1}{2} K \left(\frac{l}{2}\theta\right)^2$$

$$I_P (\dot{\theta}_{max})^2 = \frac{K l^2}{4} \theta^2$$

$$I_P (\theta \omega_n)^2 = \frac{K l^2}{4} \theta^2$$

$$\boxed{\omega_n = \sqrt{\frac{K l^2}{4 I_P}}}$$

6 $f_n = \frac{1}{2\pi}\sqrt{\frac{Ka^2 - Wb}{I_0}}$

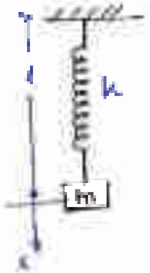
for fn $\boxed{f_n > 0}$

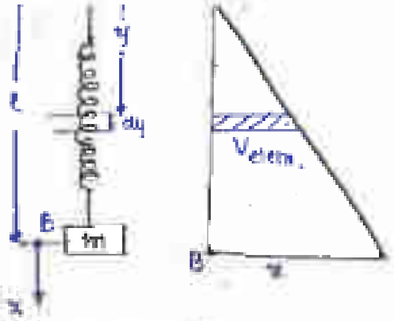
$$\sqrt{\frac{Ka^2 - Wb}{I_0}} > 0$$

$$Ka^2 - Wb > 0$$

$$Wb < Ka^2 \rightarrow \boxed{b < Ka^2/W}$$

7





$$\frac{V_{elem}}{\dot{x}} = \frac{y}{l}$$

let mass per unit length of spring = γ

$M_{spring} = \gamma L$

Energy method :

TE of system = KE of mass + KE of spring + PE of spring

K.E of spring = $\int d(KE)_{element}$

$dm\Big|_{element} = \gamma\, dy$

$V_{element} = \frac{\dot{x}y}{l}$

$$\text{KE of spring} = \int \frac{1}{2}(dm)_{element}(V_{element})^2$$

$$= \int \frac{1}{2}\gamma\, dy \left(\frac{\dot{x}y}{l}\right)^2$$

$$= \frac{\gamma \dot{x}^2}{2l^2}\int_0^l y^2\, dy$$

$$= \frac{\gamma\dot{x}^2}{2l^2}\left[\frac{y^3}{3}\right]_0^l = \frac{\gamma}{2}\dot{x}^2 \cdot \frac{l}{3} \qquad \left\{ \text{here } \gamma L = M_{spring} \right.$$

$$\boxed{\text{KE of spring} = \frac{1}{6} m_s \dot{x}^2}$$

→ TE of system $= \frac{1}{2}m\dot{x}^2 + \frac{1}{6}m_s\dot{x}^2 + \frac{1}{2}kx^2$

$$\frac{d}{dt}(TE) = \frac{1}{2}m(2\dot{x}\ddot{x}) + \frac{1}{6}m_s(2\dot{x}\ddot{x}) + \frac{1}{2}(k x \dot{x}) = 0$$

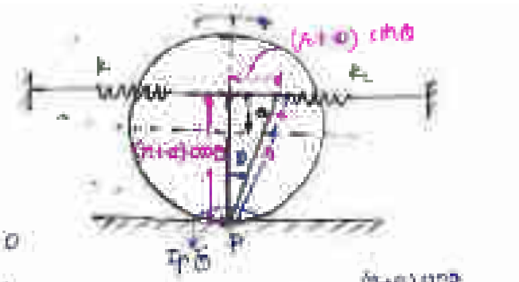
$$m\ddot{x} + \frac{m_s}{3}\ddot{x} + kx = 0$$

$$\left(m + \frac{m_s}{3}\right)\ddot{x} + kx = 0$$

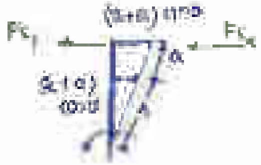
$$\ddot{x} + \frac{k}{\left(m + \frac{m_s}{3}\right)}x = 0$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m + \frac{m_s}{3}}}}$$

→ Rolls w/o slipping
so at centre of
rotation about P.



$$I_P\ddot{\theta} + F_{s_2}(r+a)\cos\theta + F_{s_1}(r+a)\cos\theta = 0$$

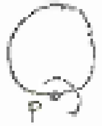


$$\rightarrow I_P\ddot{\theta} + [k_2(r+a)\theta](r+a)(1) + [k_1(r+a)\theta](r+a)(1) = 0$$

$$\rightarrow I_P\ddot{\theta} + k(r+a)^2\theta + k(r+a)^2\theta = 0$$

$$\rightarrow I_P\ddot{\theta} + 2k(r+a)^2\theta = 0$$

$$\ddot{\theta} + \left[\frac{2k(r+a)^2}{I_P}\right] = 0$$



$I_P = \frac{3}{2}mr^2$

$mr^2 + \frac{mr^2}{2}$

$$\omega_n = \sqrt{\frac{2k(r+a)^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{4k(r+a)^2}{3mr^2}}$$

$$\boxed{\omega_n = 500 \text{ rad/s}} \quad \& \quad f =$$

→ Energy method

$$\frac{d}{dt}\left[\frac{1}{2}I_P\dot{\theta}^2 + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}I_P\dot{\theta}^2 + k_1x^2\right] = 0$$

$$\frac{d}{dt}\left[I_P\dot{\theta}^2 + 2k_1x^2\right] = 0$$

$$I_P\dot{\theta}^2 + 2k[(r+a)\theta]^2 = 0$$

$$2I_P\cancel{\dot{\theta}}\ddot{\theta} + 2k(r+a)^2\theta(2)\cancel{\dot{\theta}} = 0$$

$$\ddot{\theta} + \frac{4k(r+a)^2}{2I_P} = 0$$

(a) Below pt.

$I_P = \frac{3}{2}mr^2$

$$\omega_n = \sqrt{\frac{4k(r+a)^2}{2\left(\frac{3}{2}mr^2\right)}}$$

$$\boxed{\omega_n = \sqrt{\frac{4k(r+a)^2}{3mr^2}}}$$

10

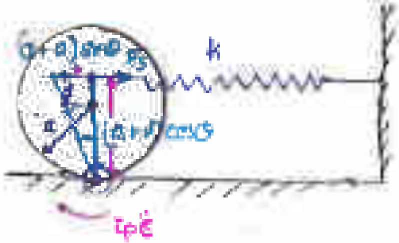
displacement = a+d

$I_p \ddot{\theta} + F_s (a+d) \cos\theta = 0$

$I_p \ddot{\theta} + k (a+d)\theta (a+d) = 0$

$I_p \ddot{\theta} + k (a+d)^2 \theta = 0$

$$\ddot{\theta} + \frac{k(a+d)^2}{I_p} = 0$$



$I_p = \frac{3}{2} ma^2$

$$\omega_n = \sqrt{\frac{2k(a+d)^2}{5ma^2}}$$

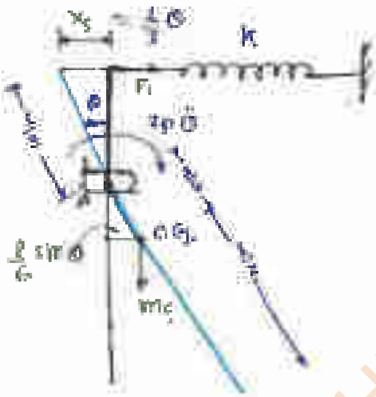
9

$\therefore I_p \ddot{\theta} + F_s \frac{l}{3} \cos\theta + mg \frac{l}{6} \sin\theta = 0$

$\Rightarrow I_p \ddot{\theta} + (k x_s) \frac{l}{3} + mg \frac{l}{6} \theta = 0$

$\Rightarrow I_p \ddot{\theta} + k\left(\frac{l}{3}\theta\right) \frac{l}{3} + mg \frac{l}{6} \theta = 0$

$\Rightarrow I_p \ddot{\theta} + \frac{kl^2}{9}\theta + \frac{mgl}{6} \theta = 0$



$$\omega_n = \sqrt{\frac{\frac{kl^2}{9} + \frac{mgl}{6}}{I_p}}$$

$I_p = I_{c.m} + m\left(\frac{l}{6}\right)^2$ — dist^n from (c.g to where we have to find)

$I_{cg} = \frac{ml^2}{12}$

$= \frac{ml^2}{12} + \frac{ml^2}{36}$

$= \frac{3ml^2}{36} + \frac{ml^2}{36} = \frac{ml^2}{9}$

$$\omega_n = \sqrt{\frac{kl^2}{9\left(\frac{ml^2}{9}\right)} + \frac{mgl}{6\left(\frac{ml^2}{9}\right)}}$$

$$\boxed{\omega_n = \sqrt{\frac{3g}{2l} + \frac{k}{m}}}$$

$$\frac{d}{dt}(T.E) = 0$$

$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k x_s^2 + mgh\right] = 0$$

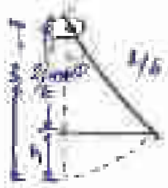
$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k \left(\frac{\ell}{3}\theta\right)^2 + mg\left(\frac{\ell}{6}\frac{\theta^2}{2}\right)\right] = 0$$

$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k \frac{\ell^2}{9}\theta^2 + \frac{1}{2} mg\frac{\ell}{6}\theta^2\right] = 0$$

$$I_p(2\dot{\theta}\ddot{\theta}) + \frac{k\ell^2}{9}(2\theta\dot{\theta}) + mg\frac{\ell}{6}(2\theta)\dot{\theta} = 0$$

$$I_p\ddot{\theta} + \left(\frac{k\ell^2}{9}\right)\theta + \left(\frac{mg\ell}{6}\right)\theta$$

$$\boxed{\omega_n = \sqrt{\frac{\frac{k\ell^2}{9} + \frac{mg\ell}{6}}{I_p}}}$$



$$h = \frac{\ell}{6} - \frac{\ell}{6}\cos\theta$$

$$= \frac{\ell}{6}(1-\cos\theta)$$

$$= \frac{\ell}{6}\frac{\theta^2}{2}$$

$\boxed{10}$ $\omega = 5$ rad/s

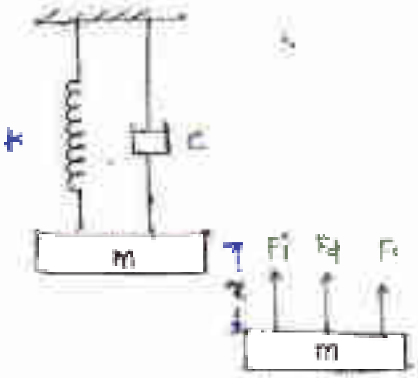
$x = 10$ cm

a = max. displacement at initial

$x = A\sin(\omega_n t + \phi)$

$$\boxed{X = 10 \text{ cm}}$$

Single D.O.F. / Damped / free vibration :



$$F_i + F_d + F_s = 0$$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = 0$$

$$\Rightarrow \boxed{\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0}$$

$$\Rightarrow D^2x + \frac{c}{m}Dx + \frac{k}{m}x = 0$$

where $D = \frac{d}{dt}$

Let $x = f(t)$

So $x = c \cdot e^{pt}$

$$\boxed{D^2 + \frac{c}{m}D + \frac{k}{m} = 0}$$

$$D_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - \frac{4k}{m}}}{2}$$

$$= -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - \frac{4k}{m}}$$

$$\boxed{D_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}}$$

→ Damping ratio / Damping factor (ξ)

$$\because \quad \xi = \sqrt{\frac{(c/2m)^2}{(k/m)}} = \sqrt{\frac{(c/2m)^2}{\omega_n^2}}$$

$$\boxed{\xi = \frac{c}{2m\omega_n}} \quad ; \quad \boxed{\xi = \frac{c}{2\sqrt{km}}}$$

— $\xi = 0$: undamped system

$\xi > 0$; damped system

$\xi < 1$ — under-damping

$\xi = 1$ — critical damping

$\xi > 1$ — over damping

→ $$\boxed{\xi = \frac{c}{c_c} = \frac{\text{Actual damping coefficient}}{\text{critical damping coefficient}}}$$

– If $\xi = 0.4$ then it says actual damping is 40% of critical damping

$$\because \quad \xi = \frac{c}{2m\omega_n}$$

$$\therefore \quad 1 = \frac{c_c}{2m\omega_n}$$

$$\boxed{c_c = 2m\omega_n} \quad \bullet \quad \boxed{c_c = 2\sqrt{km}}$$

so. $$D_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$= -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$\xi = \frac{c}{2m\omega_n}$

$\frac{c}{2m} = \xi\omega_n$

→ $D_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$

$D_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{(-1)(1-\xi^2)}$

$$\boxed{D_{1,2} = -\xi\omega_n \pm i\,\omega_n\sqrt{1-\xi^2}}$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

damped natural angular frequency

$$\boxed{D_{1,2} = -\xi\omega_n \pm i\omega_d}$$

Soln · $x = e^{-\xi\omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$

let $A = X\sin\phi$

$B = X\cos\phi$

$x = e^{-\xi\omega_n t}\left[X\sin\phi\cos\omega_d t + X\cos\phi\cdot\sin\omega_d t\right]$

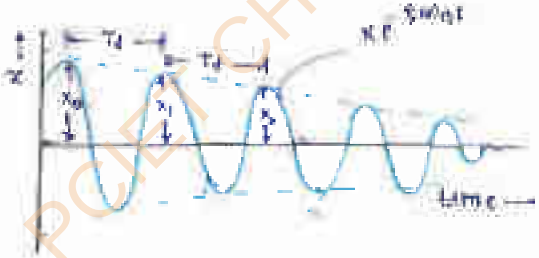
$$\boxed{x = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)}$$ ← Displacement eqn

repetitive term

→ $\boxed{\text{Amplitude} = X e^{-\xi\omega_n t}}$

exponential decrement

$\therefore x = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)$



@ $t = 0$

$X_0 = X e^{-0}\sin(0+\phi)$

$$\boxed{X_0 = X\sin\phi}$$

@ $t = T_d$

$X_1 = X e^{-\xi\omega_n T_d}\sin(\omega_d T_d + \phi)$

$= X e^{-\xi\omega_n \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}}\sin\left(\frac{2\pi}{\omega_d}\omega_d T_d + \phi\right)$

$T_d = \frac{2\pi}{\omega_d}$

$= \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$

$$\boxed{X_1 = X e^{-\delta}\sin\phi}$$

$$\boxed{\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

@ $t = 2T_d$

$x = X e^{-\xi\omega_n 2T_d}\sin[\omega_d(2T_d) + \phi]$

$= X e^{-\frac{\xi\omega_n(2\pi)2}{\omega_n\sqrt{1-\xi^2}}}\sin(4\pi + \phi)$

$$\boxed{x_2 = X e^{-2\delta} \sin\phi}$$

—
$$x_0 = X \sin\phi$$
$$x_1 = X e^{-\delta} \sin\phi$$
$$x_2 = X e^{-2\delta} \sin\phi$$
$$x_3 = X e^{-3\delta} \sin\phi$$
$$\vdots$$
$$x_n = X e^{-n\delta} \sin\phi$$

⇒ Ratio of successive Amplitude

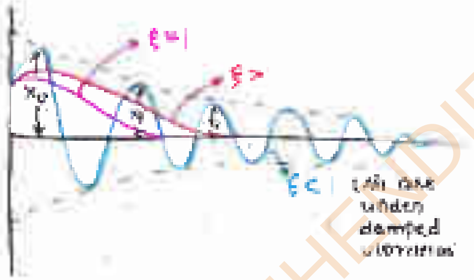
$$\frac{x_0}{x_1} = \frac{X \sin\phi}{X e^{-\delta} \sin\phi} = e^{\delta}$$

$$\frac{x_1}{x_2} = \frac{X e^{-\delta} \sin\phi}{X e^{-2\delta} \sin\phi} = e^{\delta}$$

$$\frac{x_2}{x_3} = \frac{X e^{-2\delta} \sin\phi}{X e^{-3\delta} \sin\phi} = e^{\delta}$$

$$\vdots$$

$$\frac{x_n}{x_{n+1}} = \frac{X e^{-n\delta} \sin\phi}{X e^{-(n+1)\delta} \sin\phi} = e^{\delta}$$



→ $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ = const [∴ ξ const]

- The ratio of two successive amplitude is const'

⇒ Logarithmic decrement

$$\frac{x_0}{x_n} = \frac{x_0}{x_1} \cdot \frac{x_1}{x_2} \cdot \frac{x_2}{x_3} \cdots \frac{x_{n-1}}{x_n}$$
$$= e^{\delta} \cdot e^{\delta} \cdots e^{\delta}$$

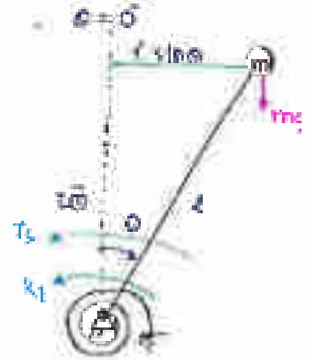
$$\boxed{\frac{x_0}{x_n} = e^{n\delta}}$$

$$\log_e\left(\frac{x_0}{x_n}\right) = \log_e e^{n\delta}$$

$$\boxed{\delta = \frac{1}{n} \log_e\left(\frac{x_0}{x_n}\right)}$$

A system will vibrate periodically & it will take infinite time in that amplitude become zero

→ Critical damping
- It is smallest possible damping for this system will be not vibrate at all



CRD - [11]

$I\ddot\theta + T_s - mgl\sin\theta = 0$

$I\ddot\theta + k_t\theta - mgl\sin\theta = 0$

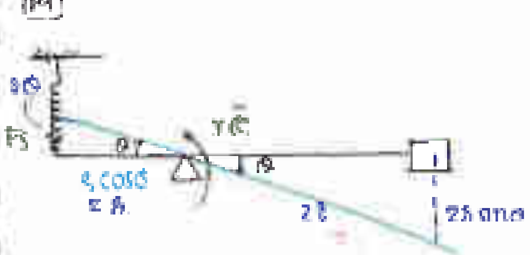
$$\boxed{I\ddot\theta + (k_t - mgl)\sin\theta = 0}$$

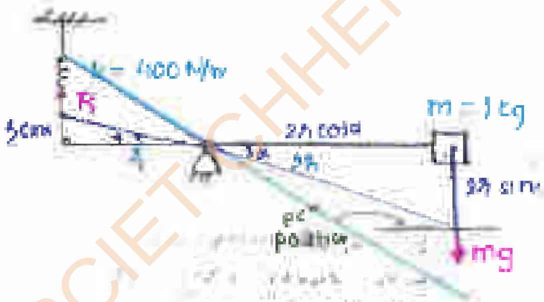
$I\ddot\theta + (k_t - mgl)\theta = 0$

$I_{eq} = ml^2$

$k_{eq} = k_t - mgl$

[14]





$I\ddot\theta + F_s(h\cos\theta) = 0$

$I\ddot\theta + k(h\theta)(h) = 0$

$I\ddot\theta + kh^2\theta = 0$

$$\omega_n = \sqrt{\frac{kh^2}{m(2h)^2}}$$

$$= \sqrt{\frac{k}{4m}} = \sqrt{\frac{400}{4}}$$

$F_s h = mg\,2h$

$F_s = 2mg$

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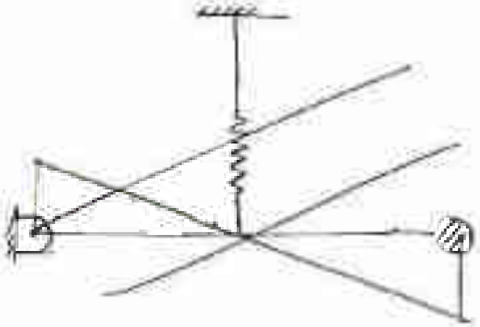

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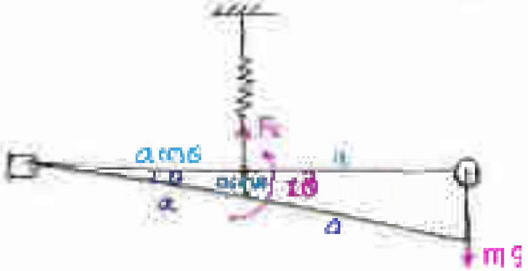
$$\frac{mg}{2a} = \frac{F_s}{a}$$

$$F_s = \frac{mg}{2}$$



$\Rightarrow I\ddot{\theta} + F_s \cdot (a\cos\theta) = 0$

$\Rightarrow I\ddot{\theta} + K(a\theta)(a) = 0$

$\Rightarrow I\ddot{\theta} + Ka^2\theta = 0$

$$\boxed{I = m(2a)^2}$$

$$\omega_n = \sqrt{\frac{Ka^2}{I}}$$

$$= \sqrt{\frac{Ka^2}{4ma^2}} = \sqrt{\frac{K}{4m}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{4m}}$$

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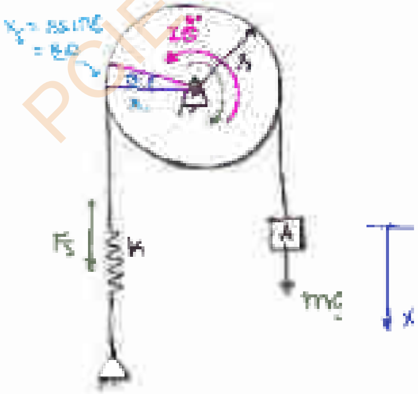
If some displacement is coming due to weight M then in final eqn mg will not come.



$\Rightarrow I\ddot{\theta} + F_s(R) = 0$

$\Rightarrow I\ddot{\theta} + (Kx_1)(R) = 0$

$\Rightarrow I\ddot{\theta} + (KR\theta)R = 0$

$\Rightarrow I\ddot{\theta} + KR^2\theta = 0$

$$\ddot{\theta} + \frac{KR^2}{I} = 0$$

$$\cancel{\omega_n = \sqrt{\frac{KR^2}{I}} = \sqrt{\frac{KR^2}{(mR^2 + MR^2)}}}$$

I_{total} = due to mg + due to pulley

$$= mR^2 + \frac{MR^2}{2}$$

$$= 15\pi^2$$

$$\omega_n = \sqrt{\frac{K\pi^2}{15\pi^2}} = \sqrt{\frac{1500}{15}} \Rightarrow \boxed{\omega_n = 10 \text{ rad/s}}$$

Shortcut method

$$\omega_n = \sqrt{\frac{K\,(\text{distance from pivot point})^2}{m\,(\text{dist}^n \text{ from pivot point})^2}}$$

or $$\omega_n = \sqrt{\frac{K\,(\text{dist}^n \text{ from pivot pt})^2}{\text{Inertia of system}}}$$

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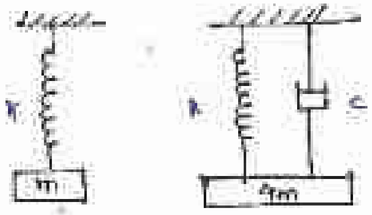

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$
$$\omega_d = 20 \text{ rad/sec}$$
$$\xi = (?)$$

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \omega_n \propto \frac{1}{\sqrt{m}}$$

$$\frac{\omega_{n_1}}{\omega_{n_2}} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{10}{\omega_{n_2}} = \sqrt{\frac{4m}{m}}$$

$$\boxed{\omega_{n_2} = 25 \text{ rad/s}}$$

$$\rightarrow \omega_d = \omega_{n_2}\sqrt{1-\xi^2}$$
$$20 = 25\sqrt{1-\xi^2}$$
$$\left(\frac{4}{5}\right)^2 = 1-\xi^2 \Rightarrow \xi^2 = \frac{9}{25} \Rightarrow \boxed{\xi = 3/5} = 0.6 = 60\%$$

20

$$M = 240 \text{ kg}$$
$$K_{eq} = K_1 + K_2 + K_3 + K_4$$
$$= 16 + 16 + 32 + 32$$
$$= 96 \text{ kN/m}$$

Resonance

$$\omega = \omega_n \Rightarrow \omega_n = \sqrt{\frac{K_{eq}}{M}} = \sqrt{\frac{96\times10^3}{240}}$$

$$\boxed{\omega_n = 632.5 \text{ rad/s}}$$

$$\frac{2\pi N}{60} = 632.5$$

$$\boxed{N = 6040 \text{ rpm}}$$

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$$= I_P\ddot{\theta} + F(\ell\cos\theta) + F_d(2\ell\cos\theta) = 0$$

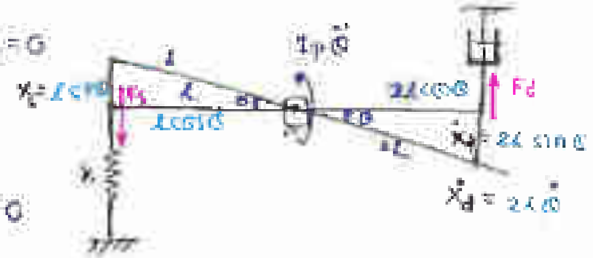
$$\Rightarrow I_P\ddot{\theta} + F_s(\ell) + F_d(2\ell) = 0$$

$$\Rightarrow I_P\ddot{\theta} + Kx_s(\ell) + C\dot{x}_d(2\ell) = 0$$

$$\Rightarrow I_P\ddot{\theta} + K(\ell\theta)(\ell) + C(2\ell\dot{\theta})(2\ell) = 0$$

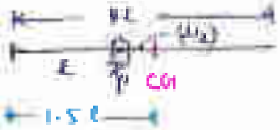
$$\Rightarrow I_P\ddot{\theta} + K\ell^2\theta + C(4\ell^2)\dot{\theta} = 0$$

$$\Rightarrow I_P\ddot{\theta} + 4\ell^2C\dot{\theta} + K\ell^2\theta = 0$$



comparing $I_{eq}\ddot{\theta} + C_{eq}\dot{\theta} + K_{eq}\theta = 0$

$I_{eq} = I_P$ | $\boxed{C_{eq} = 4\ell^2C}$ | $\boxed{K_{eq} = K\ell^2}$



$$I_{CG} = \frac{m(3\ell)^2}{12}$$

$$I_P = I_{CG} + m\left(\frac{\ell}{2}\right)^2$$

$$= \frac{9m\ell^2}{12} + \frac{m\ell^2}{4}$$

$$\boxed{I_P = m\ell^2}$$

$$\zeta = \frac{c}{c_c}$$

$$= \frac{C_{eq}}{2m\omega_n} \text{ (in translation)} = \frac{C_{eq}}{2I_{eq}\omega_n} \text{ (in rotation)}$$

$c \to C_{eq}$
$m \to I$

$$= \frac{C_{eq}}{2\sqrt{K_{eq}m}} = \frac{C_{eq}}{2\sqrt{I_{eq}K_{eq}}} \text{ (in rotation)}$$

$$= \frac{4\ell^2c}{2\sqrt{m\ell^2(K\ell^2)}} = \frac{4c}{2\sqrt{km}}$$

$$\boxed{\zeta = \frac{2c}{\sqrt{km}}}$$

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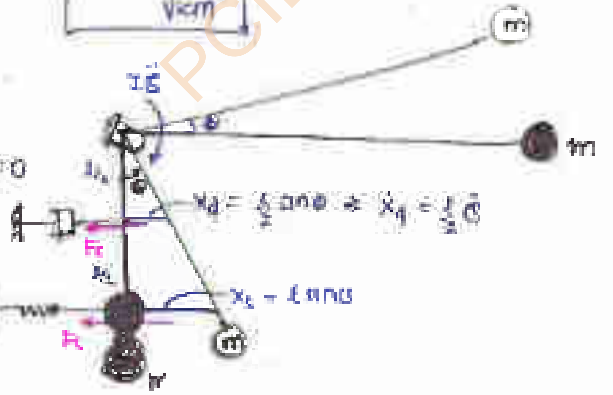


$$\Rightarrow I_P\ddot{\theta} + F_d\left(\frac{\ell}{2}\cos\theta\right) + F_s(\ell\cos\theta) = 0$$

$$\Rightarrow I_P\ddot{\theta} + c\left(\frac{\ell}{2}\dot{\theta}\right)\frac{\ell}{2} + K(\ell\theta)\ell = 0$$

$$\Rightarrow I_P\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + K\ell^2\theta = 0$$

Compare

$$I_P\ddot{\theta} + c_{eq}\dot{\theta} + K_{eq}\theta = 0$$

eq. $I_{eq} = I_P$ | $C_{eq} = \frac{c\ell^2}{4}$ | $K_{eq} = K\ell^2$

$I_{eq} = m\ell^2 + m(2\ell)^2$

$\boxed{I_{eq} = 5m\ell^2}$

$$= \sqrt{\frac{K_{eq}}{I_{eq}}} = \sqrt{\frac{K\theta^2}{5m\theta^2}} = \sqrt{\frac{K}{5m}}$$

$$= \sqrt{\frac{400}{5(10)}}$$

$$\boxed{\omega_n = 2.83 \text{ rad/s}}$$

→ $\xi = (?)$

$\xi = \frac{C}{C_c} = \frac{C}{2m\omega_n}$ (or) $\frac{C}{2\sqrt{km}}$ (in translation)

$\xi = \frac{C_{eq}}{2I_{eq}\omega_n}$ (or) $\frac{C_{eq}}{2\sqrt{K_{eq}I_{eq}}}$ (in rotation)

$$\xi = \frac{C\ell^2/4}{2\sqrt{K\ell^2 \cdot 5m\ell^2}} = \frac{C}{8\sqrt{5Km}} = \frac{400}{8\sqrt{5 \times 400 \times 10}}$$

$$\boxed{\xi = 0.35}$$

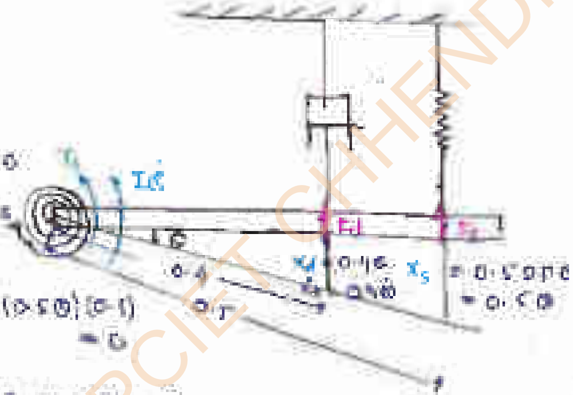
$\boxed{Q5}$

$\Rightarrow I\ddot{\theta} + T_t + F_d\,(0.4\cos\theta)$
$\quad + F_s(0.5\cos\theta) = 0$

$\Rightarrow I\ddot{\theta} + T_t + F_d(0.4) + F_s(0.5) = 0$

$\Rightarrow I\ddot{\theta} + T_t + C\dot{x}_4(0.4) + Kx_5(0.5) = 0$

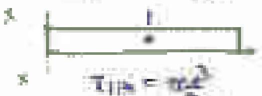
$\Rightarrow I\ddot{\theta} + T_t + C(0.4\dot{\theta})(0.4) + K(0.5\theta)(0.5) = 0$



$\Rightarrow I\ddot{\theta} + K_t\theta + 0.16C\dot{\theta} + 0.25K\theta = 0$

$\Rightarrow \underline{I\ddot{\theta}} + \underline{0.16C\dot{\theta}} + \underline{(K_t + 0.25K)} = 0$

$I_P = I_{eq}$



$I_{GG} = \frac{m\ell^2}{12}$

$I_{xx} = \frac{m\ell^2}{3}$

$I_P = \frac{m\ell^2}{3} = 0.433$ kg·m²

$C_{eq} = 0.16\,C$

$C_{eq} = 0.16(500)$

$C_{eq} = 80 \ \frac{N\text{-}m\text{-}s}{rad}$ for rotation

$C = \frac{\text{Torque}}{\dot{\theta}}$

$K_{eq} = K_t + 0.25K$

$K_{eq} = 1 + (0.25)(2)$

$= 1.5 \ \frac{kN\text{-}m}{rad}$ for rotation

$K = \frac{\text{Torque}}{\theta}$

$$\omega_n = \sqrt{\frac{k_t}{I_m}} = \sqrt{\frac{1500}{0.833}}$$

$$\boxed{\omega_n = 42.43 \ \text{Rad/s}}$$

[Q4] $$\frac{d^2x}{dt^2} + 2\xi\omega_n\frac{dx}{dt} + \omega_n^2 x = 0$$

$$\rightarrow x = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi) \Rightarrow x(t) = X e^{-\xi\omega_n t}$$

$$\rightarrow x(nT_d) = X e^{-\xi\omega_n n\left(\frac{2\pi}{\omega_n\sqrt{1-\xi^2}}\right)}$$

$$\boxed{x(nT_d) = X e^{-2n\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}}$$

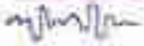
$$\left\{ t = nT_d = n\frac{2\pi}{\omega_d} = \frac{n2\pi}{\omega_n\sqrt{1-\xi^2}} \right.$$

[Q5] $$\xi = \frac{c}{2\sqrt{km}} = \frac{25}{2\sqrt{100\times1}}$$

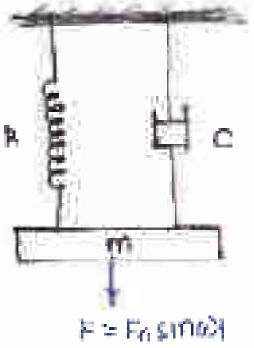
$$= \frac{25}{20} \Rightarrow \boxed{\xi = 1.25}$$

→ **Forced Vibration :**

- (i) const. forcing $\boxed{F = F_0}$
- (ii) Harmonic forcing $\boxed{F = F_0 \sin\omega t}$
- (iii) Random



- (iv) Chaos



$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$$

$$\Rightarrow \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\sin\omega t$$

Sol^n $x = f(t)$

$x = C.F + P.I$

C.F : $$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\Rightarrow D^2x + \frac{c}{m}Dx + \frac{k}{m}x = 0$$

$$\Rightarrow \left[D^2 + \frac{c}{m}D + \frac{k}{m}\right]x = 0$$

$$\boxed{C.F = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)}$$

$$\frac{\quad}{D^2 + \frac{c}{m} D + \frac{k}{m}}$$

$$\left\{ D^2 = -\omega^2 \right.$$

$$= \frac{F_0/m \; \sin \omega t}{-\omega^2 + \frac{c}{m} D + \omega_n^2}$$

$$= \frac{(F_0/m) \; \sin \omega t}{(\omega_n^2 - \omega^2) + \frac{c}{m} D} \times \frac{(\omega_n^2 - \omega^2) - c/m\, D}{(\omega_n^2 - \omega^2) - c/m\, D}$$

$$= \frac{(F_0/m) \left[(\omega_n^2 - \omega^2) \sin \omega t - c/m\, D \sin \omega t \right]}{(\omega_n^2 - \omega^2)^2 - \left(\frac{c}{m}\right)^2 D^2}$$

$$\left\{ D = \frac{d}{dt} \right.$$

$$= \frac{(F_0/m) \left[(\omega_n^2 - \omega^2) \sin \omega t - \frac{c\omega}{m} \cos \omega t \right]}{(\omega_n^2 - \omega^2)^2 - \left(\frac{c}{m}\right)^2 (-\omega^2)}$$

$$= \frac{(F_0/m) \left[(\omega_n^2 - \omega^2) \sin \omega t - \frac{c\omega}{m} \cos \omega t \right]}{(\omega_n^2 - \omega^2) + \left(\frac{c\omega}{m}\right)^2}$$

let, $\omega_n^2 - \omega^2 = R \cos\phi$

$\frac{c\omega}{m} = R \sin\phi \Rightarrow R^2 \cos^2\phi + R^2 \sin^2\phi = (\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2$

$$\boxed{R^2 = (\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2}$$

$$P.I. = \frac{F_0/m \left[R \cos\phi \sin \omega t - R \sin\phi \cos \omega t \right]}{R^2}$$

$$= \frac{F_0/m \; \sin(\omega t - \phi)}{R}$$

$$\boxed{P.I. = \frac{F_0/m \; \sin(\omega t - \phi)}{\sqrt{(\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2}}}$$

$$\Rightarrow P.I. = \frac{F_0/m \; \sin(\omega t - \phi)}{\sqrt{\omega_n^4 \left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{c\,\omega\,\omega_n^2}{m\,\omega_n^2}\right)^2}} = \frac{F_0/m \; \sin(\omega t - \phi)}{\omega_n^2 \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{c}{m\omega_n} \cdot \frac{\omega}{\omega_n}\right)^2}}$$

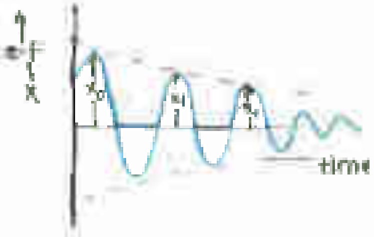
$$P.I. = \frac{F_0/m \; \sin(\omega t - \phi)}{k/m \sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

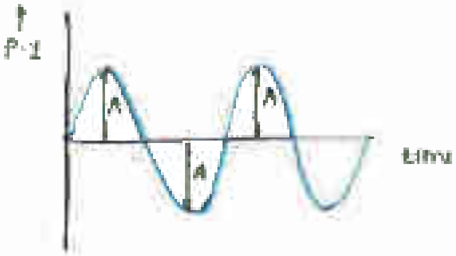
$$P.I. = \frac{F_0/k \; \sin(\omega t - \phi)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}}$$

$$\boxed{x = X e^{-\xi \omega_n t} \sin(\omega_d t + \phi) + \frac{F_0/K \,\sin(\omega t - \phi)}{\sqrt{\left[\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2\right]}}}$$

Amplitud = $X e^{-\xi \omega_n t}$

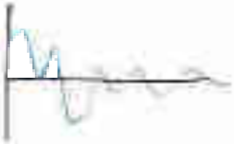
Steady State respon = $\dfrac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$





total sol

$x = CF + PF$



⇨ Steady State Response (or) Dynamic Amplitude (A)

→ The amplitude of P.I is always constant with respect to time therefore it is known steady state response.

$$A = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

[3] assume $\xi = 0.1$

A_1 amplitude @ resonance

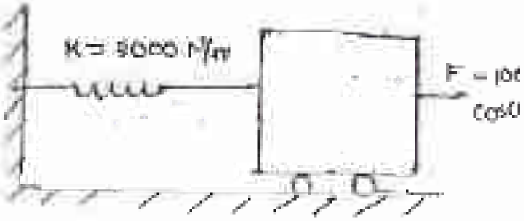
$\frac{\omega}{\omega_n} = 1 \rightarrow \boxed{\omega = \omega_n}$ $\boxed{\gamma_1 = 1}$

at $\frac{\omega}{\omega_n} = 0.5 \rightarrow (?)$ $\boxed{\gamma_2 = 0.5}$

$$A_1 = \frac{F_0/K}{\sqrt{(1-\gamma_1^2)^2 + (2\xi\gamma_1)^2}} = \frac{F_0/K}{2\xi} = 15 \text{ cm}$$

$$A_2 = \frac{F_0/K}{\sqrt{(1-0.25)^2 + (\xi)^2}} = \frac{F_0/K}{\sqrt{0.5625 + \xi^2}} = X$$

$(2\xi)(10) = X(\sqrt{0.5625 + \xi^2}) \Rightarrow 1 = X(0.7586$



A = 50 mm

K = 3000 N/m

$F(t) = 100 \cos(100t)$

$= F_0 \cos(\omega t)$

$F_0 = 100$, $\omega = 100$

— dashpot is absent

$\boxed{\xi = 0}$

$$A = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{100/3000}{(1-r^2)} = 0.05$$

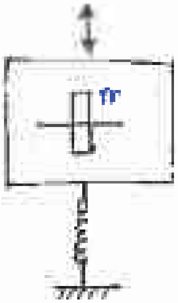
$\boxed{r = 0.577} = \omega/\omega_n$

$\rightarrow \frac{\omega}{\omega_n} = 100/0.577$

$\boxed{\omega_n = 173.20 \text{ rad/s}}$ $\rightarrow$ $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3000}{m}} = 173.20$

$\boxed{m = 0.1 \text{ kg}}$

[36]



M = 100 kg ✓

m = 20 kg

e = 0.5 mm

k_{eq} = 85 kN/m

$\omega = 20\pi$

$\xi = 0 \rightarrow$ Because damping is negligible

→ Rotating unbalance

$F_0 = m e \omega^2$

→ Reciprocating unbalance

$F_I = m r \omega^2 \cos\theta$

$F = F_0 \cos\omega t$

$\boxed{F_0 = m r \omega^2}$

Total body of foundation's mass M is responsible M take in ω_n that

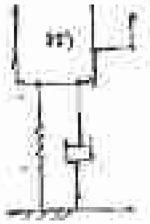
$$A = \frac{F_0/K}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{F_0}{K(1-r^2)}$$

$\left\{ r = \frac{\omega}{\omega_n} = \frac{20\pi}{\sqrt{\frac{K}{100}}} \right.$

now $r = \frac{\omega}{\omega_n} = \frac{\omega}{\sqrt{\frac{K}{M}}} = \frac{20\pi}{\sqrt{\frac{85000}{100}}} = 4.155$

$\therefore F_0 = m e \omega^2 = 39.48$ $\rightarrow$ $\boxed{A = -1.279 \times 10^{-4} \text{ m}}$

Q5



F = 10 cos25t

$m = 10$ kg

$k = 6250$ N/m

$F = F_0 \cos \omega t$

$F_0 = 10$, $\omega = 25$

$A = 40$ mm $= 0.04$ m

$c = ?$

$$r = \frac{\omega}{\omega_n} = \frac{25}{\sqrt{\frac{6250}{10}}} = 1 \rightarrow \boxed{r = 1}$$

Resonance

$$A = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{10}{6250\,(2 \times \xi \times 1)} = 0.04$$

$$\boxed{\xi = 0.02}$$

$$\rightarrow \quad \xi = \frac{c}{2m\omega_n} \rightarrow 0.02 = \frac{c}{2(10)\sqrt{\frac{6250}{10}}}$$

$$\boxed{c = 10 \ \text{Ns/m}}$$

Q3

$A = 10$ N

$k = 100$ N/m

$\xi = 0.2$

$\omega_n = 10\,\omega$

$x = ?$

$$\frac{\omega}{\omega_n} = r = 0.1$$

$$x = \frac{\frac{F_0}{k}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{10}{100\sqrt{(1-0.1^2)^2 + (2 \times 0.2 \times 0.1)^2}}$$

$$\boxed{x = 0.067 \text{ m}} = 0.07 \text{ m}$$

Q5

$F_0 = 100$ N

$\xi = 0.25$

$k = 10000$ N/m

→ Resonance $\omega = \omega_n$

$$\boxed{r = 1}$$

$$x = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$= \frac{100}{10000\,(2 \times 0.25 \times 1)}$$

$$= 0.02 \text{ m}$$

$$\boxed{x = 20 \text{ mm}}$$

Steady state Response $A = \frac{F_0/K}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

→ $F = F_0 \cos \omega t$

if $\omega = 0 \rightarrow F = F_0$

Static deflection $\delta = \frac{F_0}{K}$

Magnification factor = $\frac{\text{Dynamic Amplitude}}{\text{Static deflection}}$

$$\boxed{M.F. = \frac{A}{F_0/K}}$$

$$\Rightarrow \boxed{M.F = \frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}}$$

$M.F = f(r) = f(\xi, r)$

if ξ is constant

$MF = f(r)$

→ for max^m or min^m

$\frac{d}{dr}(MF) = 0$

$$\Rightarrow \frac{d}{dx}\left[\frac{1}{\sqrt{(1-r^2)^2+(2\xi x)^2}}\right] = 0$$

$$\Rightarrow \frac{d}{dx}\left[\left((1-x^2)^2+(2\xi x)^2\right)^{-1/2}\right] = 0$$

$$-\frac{1}{2}\left[(1-x^2)^2+(2\xi x)^2\right]^{-3/2}\left[2(1-x^2)(0-2x)+2(2\xi x)(2\xi)\right] = 0$$

$4x(1-x^2) = 8\xi^2 x \rightarrow 1-x^2 = 2\xi^2$

$\xi = \sqrt{\frac{1-x^2}{2}}$

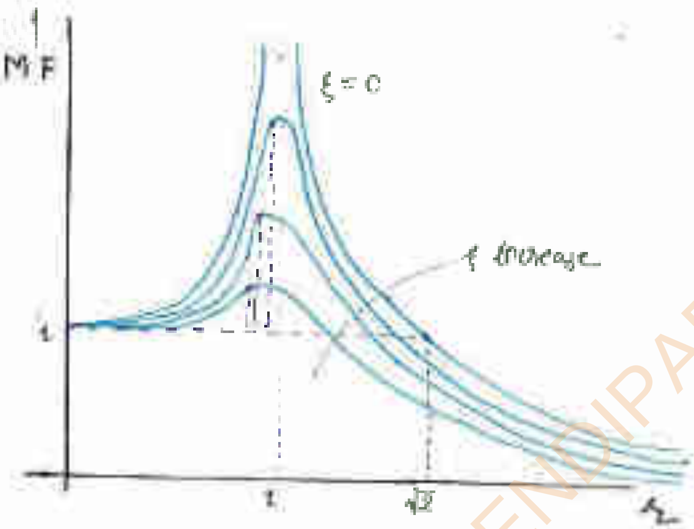
$$\boxed{r_{opt} = \sqrt{1-2\xi^2}}$$

ξ	0	0.1	0.2	0.4	0.5
r_{opt}	1	0.989	0.959	0.824	0.707

$\sqrt{(1-r^2)^2+(2\xi r)^2}$

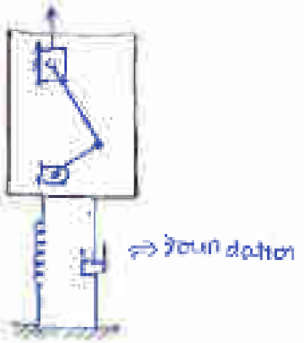
If $r = 0$ $\boxed{MF = 1}$ | $r = 1$ @ Resonance $MF = \frac{1}{2\xi}$ | @ $r = 1$, $\xi = 0$ $\boxed{MF = \infty}$ | If $\xi = 0$ $MF = \pm\frac{1}{1-r^2}$

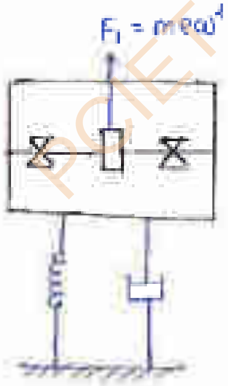
ξ increases → r_p slightly increase

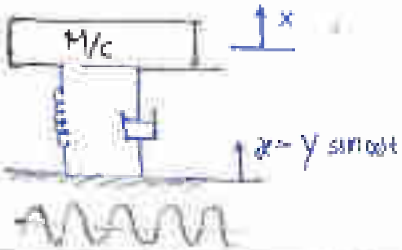


⇒ <u>Vibration</u> <u>Isolation</u> :

$F_t = m e\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$







Force transmissibility　　　　　　Motion transmissibility

→ $m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$

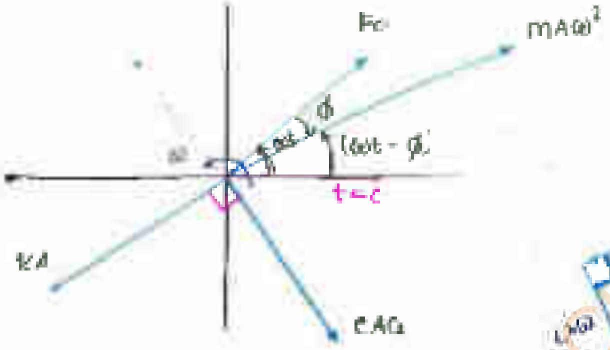
sol^n ⇒ $x = A\sin(\omega t - \phi)$

$\dot{x} = A\omega\cos(\omega t - \phi) = A\omega\sin[(\omega t - \phi) + \pi/2]$

$\ddot{x} = -A\omega^2\sin(\omega t - \phi)$

⇒ $-mA\omega^2\sin(\omega t - \phi) + cA\omega\sin[(\omega t - \phi) + \pi/2] + kA\sin(\omega t - \phi)$
$= F_0\sin\omega t$

⇒ $F_0\sin(\omega t) + mA\omega^2\sin(\omega t - \phi) - cA\omega\sin[(\omega t - \phi) + \pi/2] - kA\sin(\omega t -$
$= 0$



$\tan\phi = \frac{cA\omega}{kA - mA\omega^2}$

→ $\tan\phi = \frac{c\omega}{k - m\omega^2}$

⇒ $\tan\phi = \frac{\frac{c\omega}{m}}{\frac{k}{m} - \omega^2} = \frac{\frac{c\omega}{m}}{\omega_n^2 - \omega^2} = \frac{\frac{c\omega}{m\omega_n^2}}{\frac{\omega_n^2}{\omega_n^2} - \frac{\omega^2}{\omega_n^2}}$

$= \frac{\frac{2c}{2m\omega_n}\cdot\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$

$$\boxed{\tan\phi = \frac{2\xi r}{1 - r^2}}$$

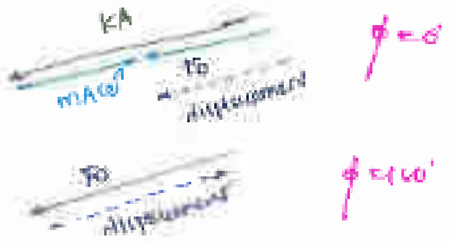
∴ ϕ = phase angle bet^n displacement vector and F_0

$r \quad \overline{1 - r^2}$

$\xi = 0 \Rightarrow$ undamped

$\tan\phi = 0$

$\boxed{\phi = 0^\circ \text{ or } 180^\circ}$



→ If $\phi \to 0^\circ$ the External force & displacement are in same direction, if $\phi \to 180^\circ$ the displacement will be opposite to the direction applied force.

@ $\xi = 1$

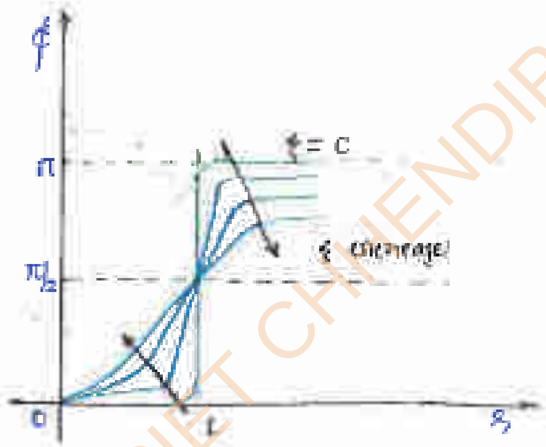
$\tan\phi = \infty$

$\boxed{\phi = \pi/2}$

→ GATE-Q.

→ When ϕ tends to 0° angle the F_0 & displacement are in same direction

→ If ϕ tends to 180° then Fo & displacement in opposite dir^n



⇒ Transmission ratio or Transmissibility (ϵ)

(T·R)

a] Force transmissibility

$\epsilon = \frac{o/p}{i/p} = \frac{\text{effect}}{\text{cause}}$

ϵ = Force transmitted to foundation / Impressing Disturbing force

$\boxed{\epsilon = \frac{F_T}{F_0}}$

→ $F_T = \sqrt{(F_s)^2 + (F_d)^2} = \sqrt{(KA)^2 + (c\omega A)^2}$

$\boxed{F_T = A\sqrt{K^2 + c^2\omega^2}}$

$$\frac{F_T}{F_0} = \frac{[illegible]}{F_0}$$

$$= \frac{F_0}{k\sqrt{(1-r^2)^2+(2\xi r)^2}} \times \frac{\sqrt{k^2+(c\omega)^2}}{F_0}$$

$$= \frac{\sqrt{\frac{k^2}{k^2}+\left(\frac{c\omega}{k}\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$= \frac{\sqrt{1+\left(\frac{c\omega}{k}\cdot\frac{\omega_n^2}{\omega_n^2}\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\epsilon = \frac{\sqrt{1+\left(\frac{c\omega}{k\cdot\omega_n\cdot\omega_n}\cdot\frac{k}{m}\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$= \frac{\sqrt{1+\left(\frac{c}{m\omega_n}\cdot r\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\boxed{\epsilon = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}}$$

special case

i) if $r = 0 \rightarrow \boxed{\epsilon = 1}$

Imp ii) $\xi = 0 \rightarrow \boxed{\epsilon = \pm\frac{1}{1-r^2}}$

iii) $r = 1 \rightarrow \omega = \omega_n \Rightarrow$ Resonance

$$\epsilon = \frac{1+(2\xi)^2}{\sqrt{(1-r^2)^2+(2\xi)^2}} = \frac{1+(2\xi)^2}{2\xi}$$

$$\boxed{\epsilon = \frac{1+4\xi^2}{2\xi}}$$

iv) $r = 1$, $\xi = 0$

$$\boxed{\epsilon = \infty}$$

v) $r = \sqrt{2} \rightarrow \epsilon = \frac{\sqrt{1+8\xi^2}}{\sqrt{1+8\xi^2}} \Rightarrow \boxed{\epsilon = 1}$

{At $r = \sqrt{2}$

$\Rightarrow \boxed{\epsilon = 1}$

→ (i) $0 < r < \sqrt{2}$

$\epsilon > 1$

$\frac{F_T}{F_0} > 1$

$\boxed{F_T > F_0}$

$\xi \uparrow$ }
$\epsilon \downarrow$ }
damping Helpful
spring creating problem

damping creating problem
springs are helpful

→ (ii) $r = \sqrt{2}$

$\epsilon = 1$

$\boxed{F_T = F_0}$

→ (iii) $r > \sqrt{2}$

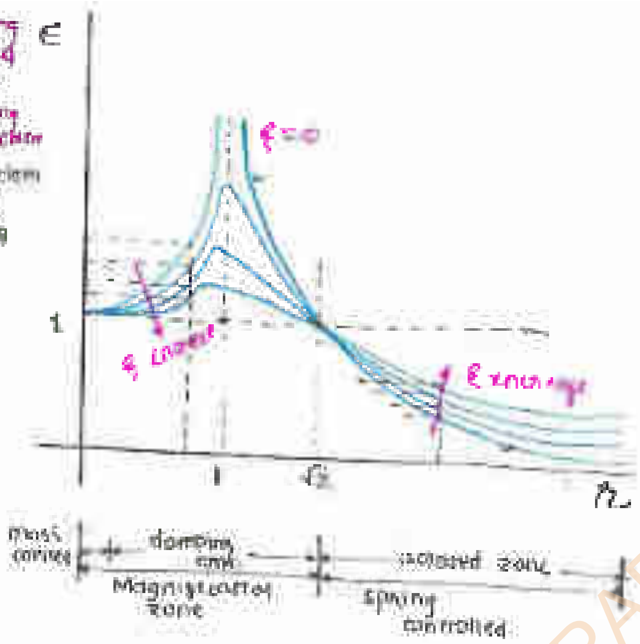
$\epsilon < 1$

$\xi \uparrow$

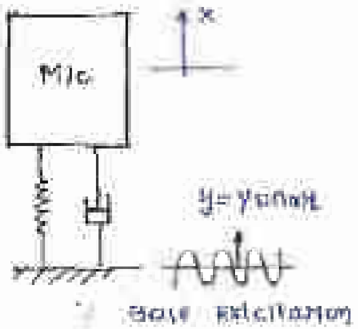
$\epsilon \uparrow$

$\boxed{F_T < F_0}$

damping creating problem
springs are Helpful



Motion Transmissibility :



$\epsilon = \frac{\text{amplitude of mass }(x)}{\text{amplitude of foundation }(y)}$

$y = Y \sin \omega t$

$\dot{y} = Y\omega \cos \omega t$

$\Rightarrow m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$

$\Rightarrow m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

$\Rightarrow m\ddot{x} + c\dot{x} + kx = cY\omega \cos \omega t + kY \sin \omega t$

$cY\omega = R \sin \alpha$

$kY = R \cos \alpha$

$\Rightarrow m\ddot{x} + c\dot{x} + kx = R \sin(\omega t + \alpha)$

$= F_0 \sin \omega_1$

$$\boxed{\epsilon = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}}$$

CLO·183

$m = 1$ kg

$x_1 = x_0/2$

$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_1}\right)$

$\boxed{\delta = 0.6931} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.6931$

$9.06\xi = \sqrt{1-\xi^2}$

$82.08\xi^2 = 1-\xi^2$

$\boxed{\xi = 0.1097}$

→ $\xi = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$

$\boxed{c = 2.19 \text{ N·s/m}}$

(20·31) Static deflection = $F_0/k = 3$ mm

$\omega = 20$ rad/s → $\boxed{r = 2}$

$\omega_n = \sqrt{\frac{k}{m}} = 10$ rad/s ⇒ $\omega_n = 10$ rad/s

$r = \omega/\omega_n = 2$

$A = \frac{F_0/k}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

$A = 0.999 \approx 1$ mm

→ $r > 1$ ⇒ For displacement in opposite dirn

$\tan\phi = \frac{2\xi r}{1-r^2}$ ⇒ $\phi = -8.459$

$\phi = 171.54 \approx 180$ ⇒ (opposite)

95 ~~$\xi = \omega/\omega_n = 20$~~ M.F = 20 ⇒ @ $r = 1$ (resonance)

$M.F = \frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

$20 = \frac{1}{\sqrt{(2\xi)^2}} = \frac{1}{2\xi}$

$\boxed{\xi = 0.025}$

$X = 0.01$ mm

$M = 50$ kg

$y(t) = 0.2 \sin(200\pi t)$

$Y = 0.2$

$\omega = 200\pi$ rad/s

No damping → $\xi = 0$

→ $\epsilon = \frac{X}{Y} = \frac{0.01}{0.2}$

$\boxed{\epsilon = 0.05}$

→ Transmissibility $= \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \pm \frac{1}{1-r^2} = \epsilon$

$0.05 = \pm \frac{1}{1-r^2}$

$1 - r^2 = \pm 20$

$r^2 = 21 \Rightarrow \boxed{r = 4.58}$

→ $\frac{\omega}{\omega_n} = 4.58 \Rightarrow \boxed{\omega_n = 137.04}$ rad/s

$\sqrt{\frac{k}{m}} = 137.04$

$\boxed{k = 939.09 \text{ kN/m}}$

$\boxed{5b}$ $5\ddot{x} + 20\dot{x} + 80x = 8\cos 4t$

$m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega t + \phi)$

$= F_0 \cos \omega t$

$m = 5$

$c = 20$

$k = 80$

$F_0 = 8$

$\omega = 4$

2) $\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$

$= \frac{20}{2\sqrt{5 \times 80}}$

$\boxed{\xi = 0.5}$

1) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{5}} = \sqrt{16} \Rightarrow \boxed{\omega_n = 4}$

4) $M.F = \frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \frac{1}{2 \times 0.5 \times 1}$

$\boxed{M.F = 1}$

B) $A = \frac{F_0/K}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

@ $r = \omega = \omega_n = 1$

$= \frac{F_0}{K \times 2\xi} = \frac{8}{50 \times 2 \times 0.8}$

$\boxed{A = 0.1}$

CRO 39]

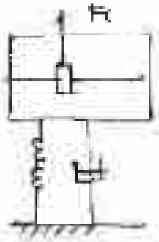
$M = 250$ kg

$K_{eq} = 100$ kN/m

$F_0 = 350$ N

$N = 3600$ rpm

$\xi = 0.15$



→ $\omega = \frac{2\pi N}{60} = 376.8$ rad/s

→ $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \times 10^3}{250}} = 20$

$r = \frac{\omega}{\omega_n} = \frac{376.8}{20} \Rightarrow \boxed{r = 18.84}$

$$\epsilon = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \frac{\sqrt{(1+(2\times0.15\times18.84)^2}}{\sqrt{[1-18.84^2]^2+(2\times0.15\times18.84)^2}}$$

35.41

$= \frac{5.7397}{353.94}$

$\boxed{\epsilon = 0.0162}$

CRO 44] $m = 1$ kg {Base excitation}

$\omega = 2\pi \times 60$

$\epsilon = 0.05$

natural frequency significantly less than 60 Hz

→ ξ neglected

$$\epsilon = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$0.05 = \frac{1}{1-r^2} \rightarrow \boxed{r = 4.58}$

$\frac{\omega}{\omega_n} = 4.58 \Rightarrow \boxed{\omega_n = 82.2 \text{ rad/s}}$

$82.2 = \sqrt{\frac{K}{m}}$

$\boxed{K = 6760 \text{ N/m}}$

3) vibration of beam due to concentrated mass

$$\omega_n = \sqrt{\frac{g}{\delta_{max}}}$$

ω_n doesn't depend on the g

→ if g changes & δ also changes

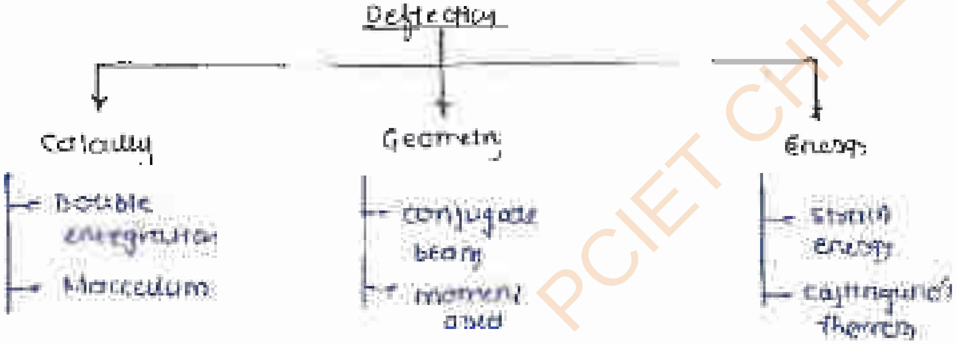
$$\omega_n = \sqrt{\frac{K}{m}}$$ where $K = \frac{3EI}{l^3}$

$y = \frac{Wl^3}{3EI}$ ⇒ $\boxed{K = \frac{3EI}{l^3}}$

Theorem of Moment area

$$\rightarrow \omega_n = \sqrt{\frac{g}{\frac{mgl^3}{3EI}}} \rightarrow \sqrt{\frac{3EI}{ml^3}}$$

$$\boxed{\omega_n = \sqrt{\frac{3EI}{ml^3}}}$$



Q.no- 19

EI = const'
ℓ = 0·01 m
m = 0·05 kg
f_n = 100 Hz

$$\rightarrow \omega_n = \sqrt{\frac{3EI}{ml^3}} \Rightarrow 2\pi(100) = \sqrt{\frac{3EI}{(0.5)(0.01)^3}}$$

$$\boxed{EI = 0.065 \ N m^2}$$

$\ell = 1$ m

$E_s = 200$ GPa

critically damped $\boxed{\xi = 1}$

$$\omega_n = \sqrt{\frac{3EI}{m\ell^3}} = \sqrt{\frac{3\times 200\times 10^3 \times \frac{45^4}{12}}{(20)(1000)^3}\times\frac{1}{(10^{-3})}} \qquad \frac{\frac{N}{mm^2}\times mm^4}{mm^3} = \frac{N}{mm}$$

ω_n = ~~[illegible]~~ rad/s

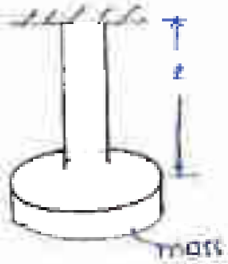
= 91.250 rad/s

$c_c = 2\omega_n m = 2(20)(91.250)$

$$\boxed{c_c = 1950 \text{ Ns/m}}$$

⇒ **Torsional vibration**

1)



mass moment of inertia of rotor

In strength of MI the area MOI

In dynamics the mass M.O.I

→ $I\ddot{\theta} + q\theta = 0$

$\ddot{\theta} + \frac{q}{I}\theta = 0$

$$\boxed{\omega_n = \sqrt{\frac{q}{I}}}$$

where $q = \frac{GJ}{\ell}$

where J = polar area MOI (shaft)

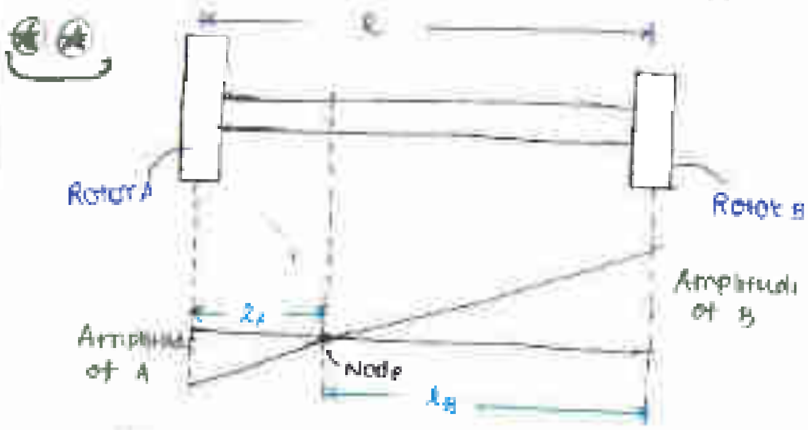
→ If mass MOI of shaft is also considered

$$\boxed{\omega_n = \sqrt{\frac{q}{I_{rotor} + \frac{I_{shaft}}{3}}}}$$

2) When rotor are moving (Amplitude) in same direction

→ The natural frequency of system due to movement of rotor will be zero & system will not vibrate at all.

Case-(ii) : When Rotors are moving in opposite direction



$$\to (\omega_n)_A = (\omega_n)_B$$

$$\left(\sqrt{\frac{q}{I}}\right)_A = \left(\sqrt{\frac{q}{I}}\right)_B$$

{ torsional rigidity GJ = const

$q = \frac{GJ}{l}$

$$\left(\frac{GJ}{lI}\right)_A = \left(\frac{GJ}{lI}\right)_B$$

$$\boxed{l_A I_A = l_B I_B}$$

$$\to \frac{l_A}{l_B} = \frac{I_B}{I_A}$$

$$\boxed{l \propto \frac{1}{\text{Mass MOI of rotor}}}$$

$$\to \boxed{\frac{\text{Amplitude of rotor B}}{\text{Amplitude of rotor A}} = \frac{l_B}{l_A} = \frac{I_A}{I_B}}$$

If $I_A > I_B \to \frac{l_A}{l_B} < 1 \to l_A < l_B$

→ The point on shaft where angular displacement is zero is known as Node.

→ The Node divides the length of shaft in inverse ratio of mass moment of inertia of the rotor mounted at respective ends.

→ At Node the two shaft of different length l_A & l_B are clamped together makes an analogous system as a single shaft carrying two rotor at respective end.

→ Torsionally equivalent system

1) Stepped shaft

$$\theta_{D/A} = \theta_{A/D}$$

$$\frac{T l_{eq}}{G J_{eq}} = \theta_{A/B} + \theta_{B/C} + \theta_{C/D}$$

$$= \frac{T l_1}{G J_1} + \frac{T l_2}{G J_2} + \frac{T l_3}{G J_3}$$

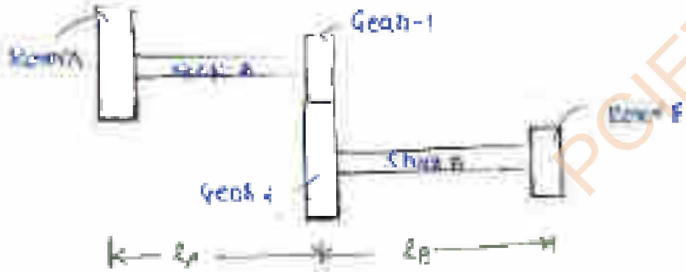
$$= \frac{l_1}{G J_1} + \frac{l_2}{G J_2} + \frac{l_3}{G J_3}$$

let $G_1 = G_2 = G_3 = G_{eq}$

$$\frac{l_{eq}}{J_{eq}} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\boxed{\frac{l_{eq}}{d_{eq}^4} = \frac{l_1}{d_1^4} + \frac{l_2}{d_2^4} + \frac{l_3}{d_3^4}}$$

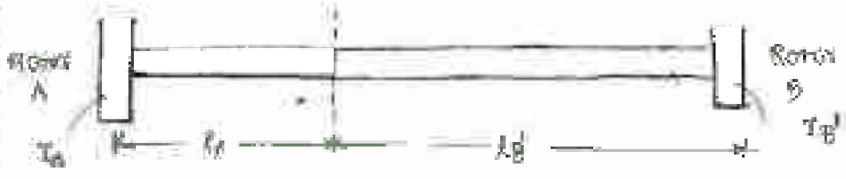
2) Geared system



→ Backlash should not be present

→ The kinetic energy & strain energy of real system & dynamically equivalent system should be same

→ The inertia of the shaft should be negligible



equating the P.E

$$\left(\frac{1}{2}T\theta\right)_{original} = \left(\frac{1}{2}T\theta\right)_{equi}$$

$$(T\theta)_{original} = (T\theta)_{equi}$$

$$\left(\frac{GJ}{\ell}\cdot\theta\cdot\theta\right)_{ori} = \left(\frac{GJ}{\ell}\cdot\theta\cdot\theta\right)_{eq}$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{\ell}$$

if $(GJ)_{orig} = (GJ)_{eq}$

$$\frac{\theta^2_{orig}}{\ell_B} = \frac{\theta^2_{eq}}{\ell_{eq}}$$

$$\boxed{\ell_{eq} = \ell_B\left[\frac{\theta_B'}{\theta_B}\right]^2} \text{ — (1)}$$

$$\boxed{\text{total length of equivalent system} = \ell_A + \ell_B'}$$

since $\theta_{ori} = \theta_B$

$\theta_{equi} = \theta_B'$

$\ell_{eq} = \ell_B'$

⇒ equating the K.E

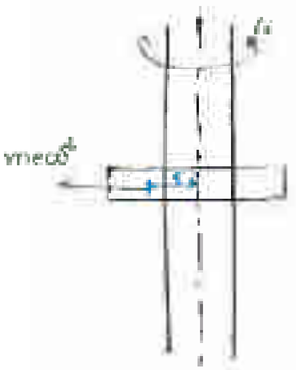
K.E of original system = K.E of equi. system

$$\Rightarrow \frac{1}{2} I_B\omega_B^2 = \frac{1}{2} I_B'(\omega_A)^2$$

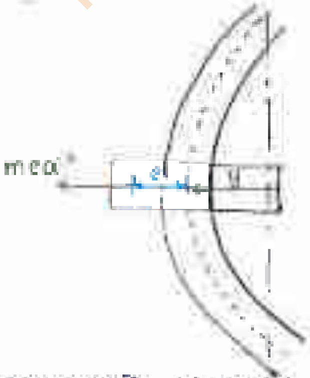
$$\Rightarrow I_B\omega_B^2 = I_B'\omega_A^2$$

$$\boxed{I_B' = I_B\left(\frac{\omega_B}{\omega_A}\right)^2}$$

⇒ Critical / Whirling / Whipping / Rotating Speed of shaft



speed



where e = eccentricity of rotor

m = mass of rotor

K = stiffness of shaft

[illegible] force = [illegible] force

$$\Rightarrow m(y+e)\omega^2 = Ky$$

$$\Rightarrow my\omega^2 + me\omega^2 = Ky$$

$$\Rightarrow my\omega^2 - Ky = -me\omega^2$$

$$\Rightarrow y\omega^2 \; y(m\omega^2 - k) = -me\omega^2$$

$$\Rightarrow y = -\frac{me\omega^2}{m\omega^2 - k}$$

$$\Rightarrow y = \frac{-e}{1 - \frac{k}{m\omega^2}}$$

$$\boxed{y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2}}$$

@ $\omega = \omega_n \rightarrow$ resonance $\boxed{y = \infty}$

⇒ When the speed of shaft becomes eq. to its natural frequency the deflection in shaft is infinite & the shaft vibrates violently & tends to fails.

⇒ Critical speed of the shaft is time dependent phenomenon Hence in order to prevent failure of shaft we accelerate the shaft when it is about to reach near to its critical speed.

Higher critical speeds

- Higher critical speeds of shaft is observed due to its own wt.
- The frequency will depend on the loading condition as well as end condition of shaft that is type of support beam provided.
- If a shaft is supported in short bearings it is analogous to simply supported & if it is supported in long bearing & it analogous to fixed end.

→ Shaft supported in short bearing

→ Let w is wt per unit length of shaft

$\frac{d^2y}{dx^2} = BM_{x-x}$

$\Rightarrow EI\frac{d^3y}{dx^3} = \frac{dBM_{x-x}}{dx}$

$$\left|\begin{array}{l}\frac{dBM}{dx} = SF \\ \frac{dSF}{dx} = load\end{array}\right.$$

$\Rightarrow \boxed{EI\frac{d^3y}{dx^3} = SF_{x-x}}$

$\Rightarrow EI\frac{d^3y}{dx^3} = SF_{x-x} \Rightarrow EI\frac{d^4y}{dx^4} = \frac{dSF_{x-x}}{dx} = load/unit\ length$

$\boxed{EI\frac{d^4y}{dx^4} = \frac{w}{g}y\omega^2}$

$\Rightarrow \frac{d^4y}{dx^4} = \frac{w}{gEI}y\omega^2$ let $n^4 = \frac{w}{gEI}\omega^2$

$\Rightarrow \frac{d^4y}{dx^4} = n^4 y$

$\frac{d^4y}{dx^4} - n^4y = 0$

$\boxed{(D^4 - n^4)y = 0}$

sol^n $\Rightarrow$ $y = f(x)$

sol^n = CF + PI

sol^n = CF

sol^n $D^4 - n^4 = 0$

$D^4 = n^4$

$\boxed{D = \pm n, \pm in}$

~~cf = A.e^{mx}~~

~~y = A cos nx + B sin nx + C sinh(nx) + D cosh(nx)~~

$\Rightarrow y = A\cos nx + B\sin nx + C\sinh(nx) + D\cosh(nx)$

→ since shaft was simply supported (short bearing)

Boundary cond^n

@ $x = 0$; $y = 0$ —(i)

@ $x = L$; $y = 0$ —(ii)

@ $x = 0$; $\frac{d^2y}{dx^2} = 0$ —(iii)

@ $x = L$; $\frac{d^2y}{dx^2} = 0$ —(iv)

(i) → $0 = A + D$

(ii) → $0 = A\cos nL + B\sin nL + C\sinh(nL) + D\cosh(nL)$

(iii) → $\frac{dy}{dx} = -A(n)\sin nx + B(n)\cos nx + C(n)\cosh(nx) + D(n)\sinh(nx)$

$$\frac{d^4y}{dx^2} = -A\eta \cos\eta x - B\eta \sin\eta x + C\eta \sinh(\eta x) + D\eta \cosh(\eta x)$$

(3) → $0 = -A\eta^2 + D\eta^2 \Rightarrow -A + D = 0$ — (3)

(4) → $0 = -A\cancel{\eta^2}\cos(\eta L) - B\cancel{\eta^2}\sin(\eta L) + C\cancel{\eta^2}\sinh(\eta L) + D\cancel{\eta^2}\cosh(\eta L)$

$B\sin(\eta L) + A\cos(\eta L) = C\sinh(\eta L) + D\cosh(\eta L)$

$\cancel{A} + D = 0$ — (1)

$-\cancel{A} + D = 0$ — (3)

$\boxed{2D = 0}$

$\boxed{A = D}$ $\boxed{D = 0}$

$B\sin(\eta L) = C\sinh(\eta L)$ — (5)

−r $B\sin(\eta L) + C\sin(h(\eta L))$ — (6)

$B\sin(\eta L) + C\sin\cancel{h}(\eta L) = 0$

$B\sin(\eta L) - C\sin\cancel{h}(\eta L) = 0$

$2B\sin(\eta L) = 0 \Rightarrow \boxed{\sin(\eta L) = 0}$

so, $\eta L = \pi, 2\pi, 3\pi$

$\eta = \frac{\pi}{l}, \frac{2\pi}{l}, \frac{3\pi}{l}$ ---

$\Rightarrow \left(\frac{\pi}{l}\right)^4 = \frac{w}{gEI}\omega_1^2$

$\omega_1 = \sqrt{\frac{gEI}{w}\cdot\left(\frac{\pi}{l}\right)^4}$

$\boxed{\omega_1 = \left(\frac{\pi}{l}\right)^2\sqrt{\frac{gEI}{w}}}$ ← Critical Speed

gravity = 1

→ Let $\eta^4 = \frac{w}{gEI}\omega^2$

$\left(\frac{2\pi}{l}\right)^4 = \frac{w}{gEI}\omega_2^2$

$\omega_2 = \sqrt{\frac{gEI}{w}\left(\frac{2\pi}{l}\right)^4}$

$\omega_2 = \left(\frac{2\pi}{l}\right)^2\sqrt{\frac{gEI}{w}}$

$\omega_2 = 4\left(\frac{\pi}{l}\right)^2\sqrt{\frac{gEI}{w}}$

$\boxed{\omega_1 = \omega_n}$

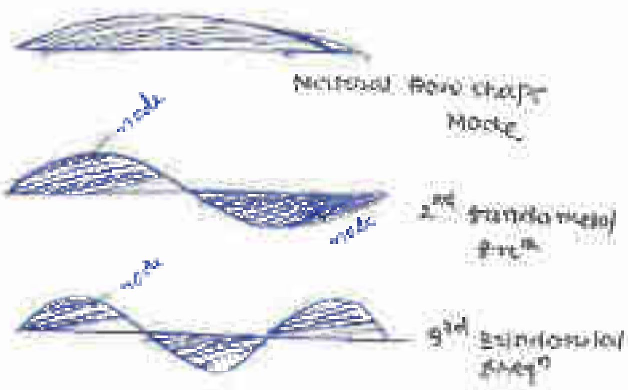
$\omega_2 = 4\,\omega_n$

$= 2^2\,\omega_n$

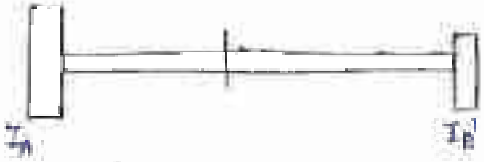
$\boxed{\omega_2 = 2^2\,\omega_n}$ 2-mode

$\omega_3 = 9\,\omega_n$

$\boxed{\omega_3 = 3^2\,\omega_n}$ 3-mode



CRO-46



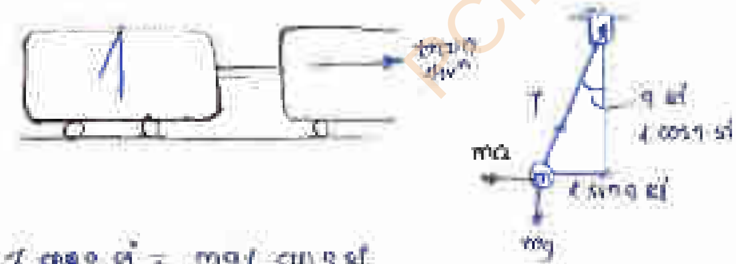
$\frac{\omega_B}{\omega_A} = n$ (given)

$\rightarrow \frac{1}{2} I_B' (\omega_B')^2 = \frac{1}{2} I_B (\omega_B)^2$

$I_B' (\omega_A^2) = I_B (\omega_B^2)$

$\boxed{I_B' = I_B\, n^2}$

CRO-47



$ma \times l \cos 9° = mg\, l \sin 9°$

$a = g \tan 9°$

$\boxed{a = 1.55\ m/s^2}$

CRO-50 two nodes ⟶ 3-mode

$\omega_3 = (3)^2 \omega_1$ (modes) $\Rightarrow 1800 = 3^2 \omega_1$

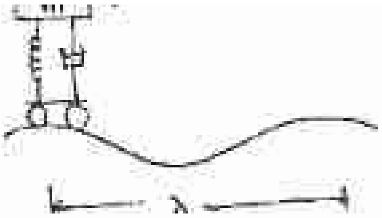
$\boxed{\omega_1 = 200\ \text{rpm}}$

CRO 48

$v = \lambda f$

$\boxed{f = v/\lambda}$

$\rightarrow \omega = 2\pi f \Rightarrow \boxed{\omega = \frac{2\pi v}{\lambda}}$



@ Resonance

$\omega = \omega_n$

$\frac{2\pi v}{\lambda} = \sqrt{\frac{k}{m}}$

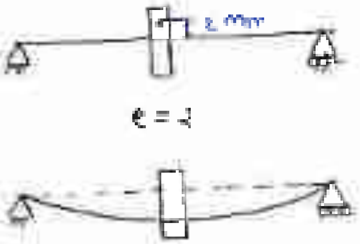
CRO 49

Shaft frequency

→ natural frequency

$\omega_n = 10$ rad/s $= \omega_1$

$e = 2$ mm



$\rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60}$

$= 31.4$ rad/s

$\Rightarrow y = \frac{e}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{2}{\left(\frac{31.4}{10}\right)^2 - 1} = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{2}{\left(\frac{10}{31.4}\right)^2 - 1}$

$\boxed{y = -2.25 \text{ mm}}$

CRO - 51

short bearing = simply supported

long bearing = fixed end

$\rightarrow \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{1.8 \times 10^{-3}}} \Rightarrow \boxed{\omega_n = 705 \text{ rpm}}$

52

Gate -18

$N_{oper} = 1440$ rpm

solid shaft = D

→ critical speed is encountered

$D_o = D$

$D_i = 0.75 D$

→ Let end cond^n are SSB

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$K = \frac{48EI}{l^3}$$

$$\delta = \frac{Wl^3}{48EI} \Rightarrow K\delta = \frac{K\delta l^3}{48EI}$$

$$\boxed{\omega_n \propto \sqrt{I}}$$

$$\Rightarrow \frac{\omega_{n_1}}{\omega_{n_2}} = \sqrt{\frac{I_1}{I_2}} \Rightarrow \frac{1400}{N_2} = \sqrt{\frac{\pi/_{64} D^4}{\pi/_{64}[D^4 - (0.75D)^4]}}$$

$$\frac{1400}{N_2} = \sqrt{\frac{1}{1-(0.75)^4}}$$

$$\boxed{N_2 = 1157.5 \text{ RPM}}$$

→ Hollow shaft use is comfortable

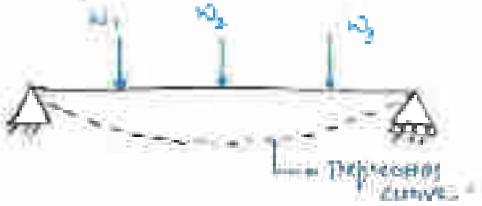
→ since by using Hollow shaft ($N_{op.} - N_{cri}$) is increasing, we can comfortably accelerate the shaft. Therefore using Hollow shaft than solid will be a good alternative.

→ since, critical speed decrease, so you can't to with this method

→ When shaft & beam

② Dunkerley's method :

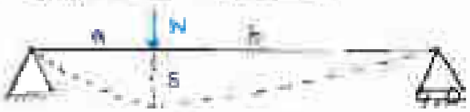
$$\boxed{\frac{1}{\omega_n^2} = \frac{1}{\omega_{n_1}^2} + \frac{1}{\omega_{n_2}^2} + \frac{1}{\omega_{n_3}^2} + \dots}$$



→ $\omega_{n_1} = \sqrt{\frac{g}{\delta_1}}$ → deflection @ the point load W1, when W1 is acting along

→ Simply Supported Beam :

1)



$$\boxed{\delta = \frac{W a^2 b^2}{3EIL}}$$

@ $a = b = \frac{L}{2}$ ⇒ $\boxed{\delta = \frac{WL^3}{48\,EI}}$

2.

UDL

$\boxed{\delta = \frac{5}{384}\frac{wL^4}{EI}}$ @ mid span

3. Fixed beam

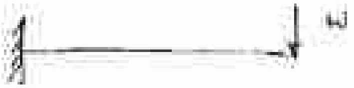
$\boxed{\delta = \frac{W a^3 b^3}{3EIL^3}}$ @ point load

$a = b = L/2$,

$\boxed{\delta = \frac{WL^3}{192\,EI}}$ @ mid point

$$\boxed{\delta = \frac{wl^4}{384\,EI}}$$ @ mid span

→ **Cantilever.**



$$\boxed{\delta_{\text{free end}} = \frac{Wl^3}{3EI}}$$

$$\boxed{\delta_{\text{free end}} = \frac{wl^4}{8EI}}$$

→ Gears and Gear Train ←

① **Gear**: The larger wheel is known as gear.

② **Pinion**: The smaller wheel is known as pinion.

- Due to smaller in the [illegible] of pinion is less than the [illegible] of gear that is that is why it driver. pinion is driver (in general)

→ **Velocity Ratio**

$$V.R = \frac{\omega_{i/p}}{\omega_{o/p}} \quad (>1)$$

$$= \frac{\omega_{o/p}}{\omega_{i/p}} \quad (<1)$$

→ **Gear Ratio**

$$\text{Gear ratio} = \frac{T}{t} \quad (>1)$$

$$= \frac{t}{T} \quad (<1)$$

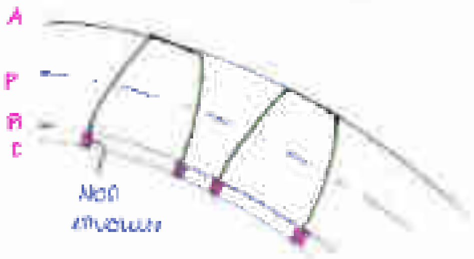
⊛ Gear terminology

① **Pitch circle**: It is an imaginary circle (whose radius can be changed). It is the most important circle in gears as it specifies all the dimensions of gear and the size of gear and all the [illegible] measured along the pitch circle only.

② **Base circle**: It is the smallest circle on gear from where the involute profile is begins.
- It is a real circle and it is actual can not be change.
- common normal in gears which is also known as line of action is tangential to both the base circles (gear and pinion).

③ **Dedendum circle** (m) Root circle: It is smallest circle from where the tooth begins. Small circle

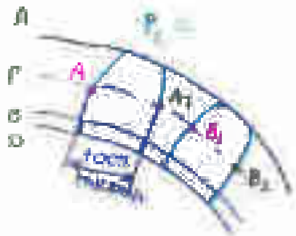
④ **Addendum circle**: A circle which passes through the top of gear tooth.



Centre dist = r + R

[illegible]

→ **Circular pitch** The distance between two similar point on adjacent teeth measured along the pitch circle circumference. (A_1B_1) (A_2B_2)



$$\boxed{P_c = \frac{\text{pitch circle circumference}}{\text{no. of teeth}}}$$

for gear $P_c = \frac{\pi D}{T}$

$P_c = \frac{\pi d}{t}$

→ **Diametrical pitch** No. of teeth per unit diameter (It is FPS unit)

$$\boxed{P_d = \frac{\text{no. of teeth}}{\text{P.C.D}}}$$

for gear $P_d = \frac{T}{D}$

$P_c = \frac{\pi}{\pi} \frac{t}{D}$

→ **module** : It is SI unit of gear defined as

$$\boxed{m = \frac{\text{P.C.D}}{\text{no. of teeth}}}$$

for gear $m_{gear} = \frac{D}{T}$ $\boxed{mm}$

$m_p = \frac{d}{t}$

NOTE : Two gear which are in mesh have same unit

If gear and pinion are in mesh

$m_{gear} = m_{pinion}$

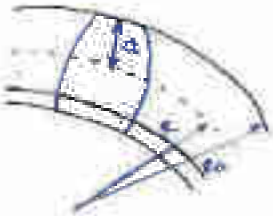
$\frac{D}{T} = \frac{d}{t} \Rightarrow \boxed{\frac{D}{d} = \frac{T}{t}}$

⑥ Dimensions of Gears

i) **tooth thickness** : The thickness of tooth measured along pitch circle circumference (A_1A_2) or (B_1B_2)

ii) **Tooth space** or space width : The distance between two successive teeth measured along pitch circle circumference (A_2B_1)

iii) **addendum / addenda** : [illegible] and pitch circle is known as addenda



for gear $a = R_a - R$

for pinion $a = r_a - r$

$a = f \cdot m$

↳ fraction

$a = 1$ module ⟵ for full depth

$a = 0.8$ module ⟶ for stub tooth

- The wheel with larger addendum always decide the beginning of engagement

iv) **Dedendum / Dedenda** : The radial dist^n between pitch circle & dedendum circle

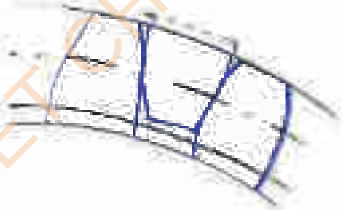
$b = R - R_d$ (for gear)

$b = r_1 - r_d$ (for pinion)

(v) **full depth** : The summation of (a + b) i.e. addendum and dedendum called full depth

(vi) **working depth** : summation of addendum of gear & pinion is known as working depth

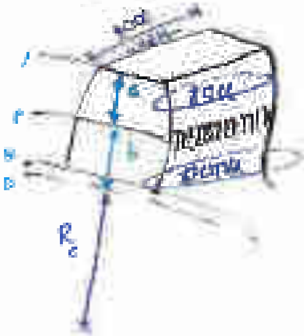
- in order to avoid interference working depth should be less than full depth



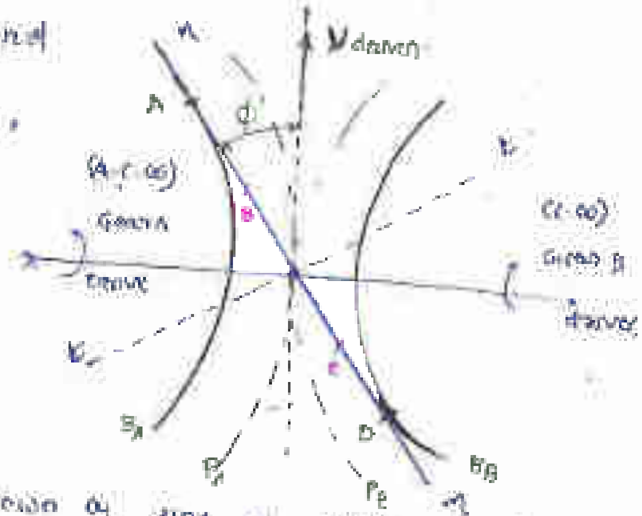
(vii) face : The portion of tooth above the pitch surface known as face

(viii) flank : The portion of tooth below pitch surface is flan

(ix) pressure angle : It is measure of of gear and pinion mechanism



- Pressure angle is defined for a gear as-
- Thus the angle between direction velocity vector of driven gear to the common normal.
- The angle between common tangent to both the pitch circle and common normal is known as pressure angle.



- common normal is known as line of action. It will be tangent to both the base circles.
- Force transmission, path of contact etc. always take place along the common normal.

⇒ Clearance.

Types of clearance

1) circumferential clearance ⇒ Backlash
2) Radial clearance ⇒ clearance

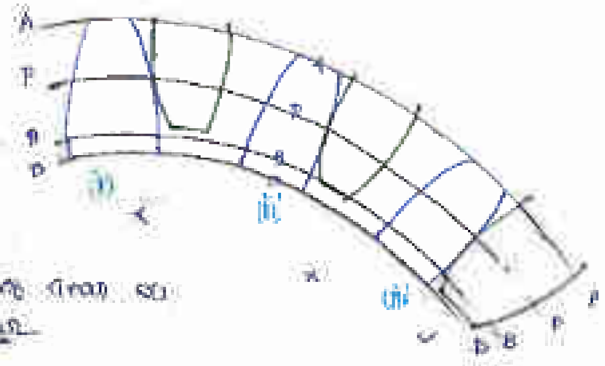
- Backlash appears in mating gears.
- It is the amount by which tooth space is greater than tooth thickness of mating gears.
- Backlash always provided for following reason
  1) During running thermal expansion of gear may take place & backlash take care of this
  2) Backlash take care of machining allowance
  3) Backlash can be increased by increasing the centre distance, it does not affect velocity ratio
- The distance between addendum of gear and dedendum of pinion or vice versa is known as clearance.
- clearance is amount by which full depth exceeds the working depth.

→ clearance is always provided always in order to provide non conjugate action, there is meaning of involute profile and non involute profile, most commonly known as

⇒ Length of path of contact :

→ case (i)

Involute profile of gear - A is meshing with Gear - B so interference will not occur



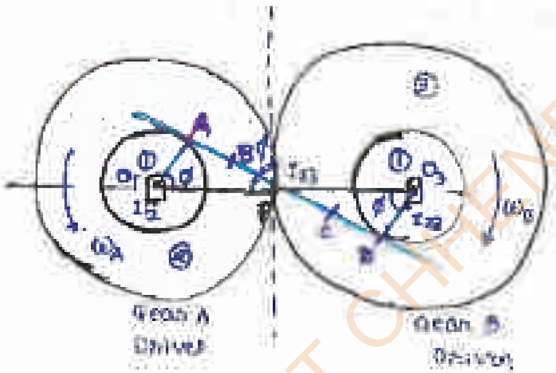
→ case (ii).

Gear - B is just meshing or touching the base circle of Gear so interference will not occur.

→ case (iii)

Involute profile of gear B crosses the base circle boundary of Gear A so then interference occur

- BF → path of approach
- CP → path of recess



Law of gearing :

Gear A → Link 2

B → 3

Angul vel theorm

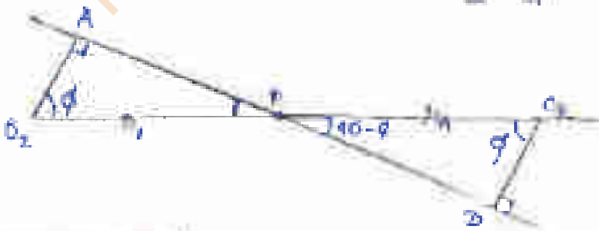
$$\frac{\omega_2}{\omega_3} = \frac{I_{13}I_{23}}{I_{12}I_{23}} = \frac{O_3P}{O_2P}$$

$$\boxed{\frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{t_B}{t_A}} \text{ — (i)}$$

$m_A = m_B \Rightarrow \frac{d_A}{t_A} = \frac{d_B}{t_B}$

$\frac{r_B}{r_A} = \frac{t_B}{t_A}$

In $\Delta\, O_2AP$ & $\Delta\, O_3BP$



$$\frac{O_2P}{O_3P} = \frac{AP}{BP} = \frac{O_2A}{O_3B}$$

since $\frac{O_2A}{O_3B}$ = const

Hence $\boxed{\frac{\omega_B}{\omega_A} = \frac{O_2P}{O_3P} = \frac{AP}{BP} = \frac{O_2A}{O_3B} = \text{const}}$

Statement of Law of gearing

- Velocity ratio in gears always remains constant
- The pitch point P which is the point of contact of pitch circles is a fixed point in a space & it will be ~~a~~ on centres I_{ab}
- The pitch point P divides the centre distance O_1O_2 in a constant ratio.

Imp → The common normal (Line of Action) passes through pitch point and the pitch point divides it in a constant ratio.

- Presence of teeth is not a necessary condition to call an element of gear.
- If two cylinders which does not have teeth on them but satisfy the law of gearing, we will call them as Gears.

@ pitch point Gear A & Gear B are in pure rolling

@ pitch point : velocities are same

$$r_A\omega_A = r_B\omega_B \Rightarrow \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A}$$

@ pitch point velocity of sliding is zero

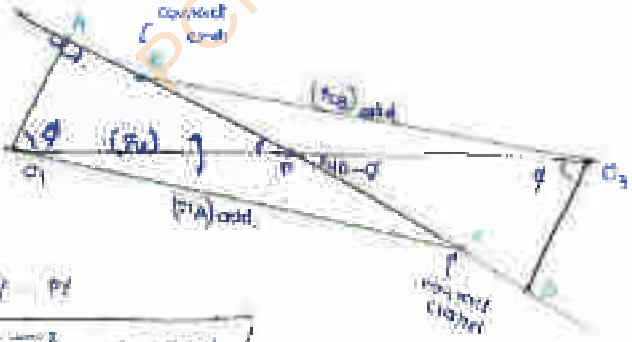
→ $AP = r_A\sin\phi$ & $PD = r_B\sin\phi$

$(r_b)_A = O_1A = r_A\cos\phi$ $O_2D = r_B\cos\phi$

pitch circle radius

$$\boxed{\text{Base circle radius} = \left[\begin{matrix}\text{pitch circle radius}\end{matrix}\right] \times \cos\phi}$$

→ Path of contact = path of approach + path of recess = CP + PB



→ path of approach = CA − PA

$= \sqrt{O_1C^2 - O_1A^2} - PA$

$$\boxed{CP = \sqrt{(r_A)_{add}^2 - (r_A\cos\phi)^2} - r_A\sin\phi}$$

→ path of recess = PB = BD − PD

$$\boxed{PB = \sqrt{(r_B)_{add}^2 - (r_B\cos\phi)^2} - r_B\sin\phi}$$

path of contact $= \left[\sqrt{(r_{A,add})^2 - (r_A\cos\phi)^2} - r_A\sin\phi\right]$
$+ \left[\sqrt{(r_{B,add})^2 - (r_B\cos\phi)^2} - r_B\sin\phi\right]$

⇒ max^m path of contact = AD
⇒ max^m path of approach = DP = $r_B \sin\phi$
⇒ max^m path of recess = PA = $r_A \sin\phi$

max^m path of contact = $(r_A + r_B)\sin\phi$

Arc of contact = $\dfrac{\text{path of contact}}{\cos\phi}$

contact ratio = $\dfrac{\text{Arc of contact}}{\text{circular pitch } (p_c)}$

→ contact ratio predicts the no. of pairs of tooth which are in mesh for a smooth operation contact ratio
$1 < CR < 2$

→ if $\boxed{CR = 1}$ ⇒ it means one pair of teeth in mesh at pitch point.

→ if $\boxed{CR = 1.2}$ ⇒ it means one pair of tooth is in complete meshing at pitch point and another pair of tooth is mesh for 20% of total arc of contact

⇒ Velocity of sliding
@ beginning of engagement = $(I_g C)(\omega_p + \omega_g)$
$= PC\,(\omega_p + \omega_g)$
$=$ path of approach $\times (\omega_p + \omega_g)$

⇒ Velocity of sliding
@ end of engagement = $(I_g A)(\omega_p + \omega_g)$
= path of recess × $(\omega_p + \omega_g)$

→ Velocity of sliding
@ pitch point

@ Angle of action (δ) :

$\delta_{gear} = \dfrac{\text{arc of contact}}{R}$

$\delta_{pinion} = \dfrac{\text{arc of contact}}{r}$

Defn. Angle of action: [illegible] [illegible] during engagement along pitch circle either gear or pinion, then angle always subtends angle at center known as angle of action

Defn Interference: Whenever involute profile mates with non involute profile it result in non conjugate action known as interference.

- Interference leads to several problem like jamming, etc. therefore it must be avoided.

⇒ Methods to avoid interference.

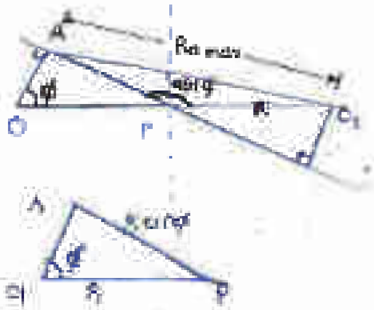
1) By changing the center distance
- If we increase the center distn, pressure angle increases automatically.
- On increases the center distance clearance increases. Hence, interference can be avoided

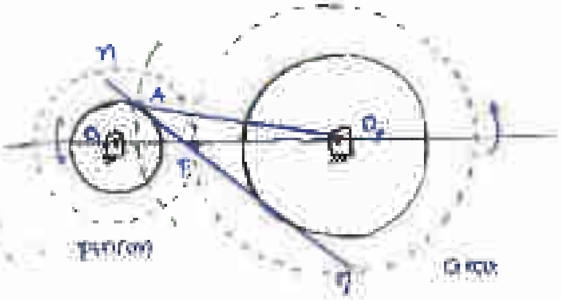
2) Stubbing the tooth
Stubbing means removal of excess portion from the top of gear tooth.
- Stubbing always decreases the strength of gear tooth since moment arm reduces

3) By using cycloidal teeth.
- In cycloidal tooth conjugate action is always maintain due to which interference does not takes place

4) By properly choosing the no. of teeth on pinion

5) Undercutting
- During manufacturing the cutter (tool) removes some material from the flank known as undercutting
- During machining if interference takes place, the involute portion of gear tooth try to remove the non involute portion if it is able to remove it be known as under cutting
- Some time we purposely remove some material from the flank so that extra space becomes available for the gear tooth to mesh properly w/o undergoing in interference is known undercutting
- Undercutting always result in stress concentration, which will make the tooth weaker





In O_2PA,

$$(O_2A)^2 = (O_2P)^2 + (PA)^2 - 2(O_2P)(PA)\cos(90+\phi)$$

$$\Rightarrow R_{r,max}^2 = R^2 + (r\sin\phi)^2 + 2Rr\sin\phi\cdot\sin\phi$$

$$R_{r,max} = \sqrt{R^2 + r^2\sin\phi + 2Rr\sin\phi\sin\phi}$$

$$= \sqrt{R^2\left[1 + \frac{r^2}{R^2}\sin\phi + \frac{2r}{R}\sin^2\phi\right]}$$

$$\frac{r}{R} < 1 \Rightarrow \boxed{\lambda = \frac{r}{R} = \frac{\omega_g}{\omega_p}} \quad (\lambda < 1)$$

$$\Rightarrow R_{a,max} = R\sqrt{1+\lambda(\lambda+2)\sin^2\phi}$$

$$\Rightarrow R_{a,max} - R = R\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - R$$

$$\Rightarrow a_{gear,max} = R\left\{\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right\}$$

(★) $$f m_{min} = R\left[\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right]$$

$$\Rightarrow f\,\frac{2R}{t_{min}} = R\left[\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right]$$

$$\Rightarrow t_{min} = \frac{2Rf}{R\left[\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right]}$$

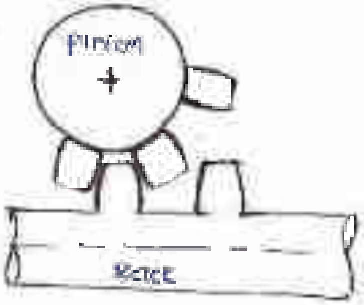
$$\boxed{t_{min} = \frac{\overset{(t_{gear})}{2f}}{\left[\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right]}} \qquad \text{where } \lambda = \frac{r}{R}$$

→ Actual teeth ($t_{act} > t_{min}$) must be greater than t_{min} to avoid interference.

→ Minimum no. of teeth on pinion to avoid interference in Rack & pinion arrangement

$R \to \infty$,

$\lambda = \frac{r}{R} \Rightarrow \boxed{\lambda = 0}$



$$t_{min} = \lim_{\lambda \to 0} \frac{2f\lambda}{\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1} \quad \left(\frac{0}{0}\right)$$

L- Hospital Rule

$$t_{min} = \lim_{\lambda \to 0} \frac{\frac{d}{d\lambda}(2f\lambda)}{\frac{d}{d\lambda}\left[\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right]}$$

$$= \frac{2f}{\frac{1}{2\sqrt{1+\lambda(\lambda+2)\sin^2\phi}}\{0+(2\lambda+2)\sin^2\phi\}} = \frac{2f}{\frac{2\sin^2\phi}{2\sqrt{1+0}}}$$

$$\boxed{t_{min} = \frac{2f}{\sin^2\phi}}$$

NOTE :- If $a = 1$ module i.e. $f =$

V.R $\Rightarrow \boxed{\lambda = 1} \Rightarrow \boxed{r = R}$

gear & pinion are same size.

$$t_{min} = \frac{2f\lambda}{\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1}$$

$$\boxed{t_{min} = \frac{2}{\sqrt{1+3\sin^2\phi} -}}$$

Remember

ϕ	t_{min}
14.5°	23
20°	13
22.5°	11

for gear & pinion $\boxed{f = 1}$

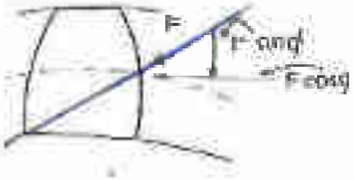
ϕ	t_{min}
14.5°	32
20°	18
22.5°	14

for rack & pinion $\boxed{f = 1}$

⇒ F cosφ component is responsible for power in gear;

$P \propto (F\cos\phi)$

as φ ↑ ⇒ cos φ ↓
power ↓
Interference ↑↓



φ	Interference	power
14.5°	more	max"
20°	moderate	moderate
22.5°	min	min

⇒ Limitation of method to avoid interference :

1) Center distance between gears is the center distance b/w shaft on which they are mounted & distance b/w shaft can not changed

2) By stubbing the teeth length of path of contact decreased which results in decrement of contact ratio because of which operation becomes non smooth

Involute

1) Locus of a point on straight line that rolls without slipping on a circle

2) Involute is also obtained when tightened string is unwrapped from a pulley

3) Due to non conjugate action involute causes interference.

Cycloid

1) It is locus of a point on a circle that rolls w/o slipping on a straight line

2) cycloid = Epicycloid + Hypocycloid

3) GATE

In cycloidal epicycloidal profiles meshes with hypocycloidal & hypocycloidal meshes with epicycloidal which ensures Non conjugate action

- pressure angle in involute is always constant

- Less costly, & easy to manufacture

does not undergo [illegible]

- In cycloidal pressure angle is max at beginning of engagement & zero at pitch point again max at the end of engagement

- Difficult to manufacture & more costly

Eg: In watches cycloidal tooth used

Note → In involute gear even if centre dist^n is changed ; velocity ratio remain constant

Note → Worm gear is used to obtain a large velocity ratio

(20-2)

$m = 4$

$T = 32$

→ $m = \frac{D}{T}$

$D = 32 \times 4 = \underline{128\ mm}$

w/ Tooth thickness = tooth space

32 tooth thickness + 32 tooth space = pitch circle circumference

64 tooth thickness = πD

1 tooth thickness = $\frac{\pi D}{64}$

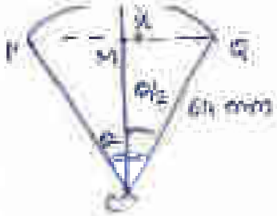
tooth thickness = $\frac{128\pi}{64}$

$\boxed{\text{tooth thickness} = 6.28\ mm}$ = arc (PQ)

angle = $\frac{\text{arc}}{\text{Radius}}$

$\theta = \frac{6.28}{64}$

$\boxed{\theta = 5.622°}$



L→ is working making

$PQ = PM + MQ$

$= 2MQ = 2 \times 64 \sin\theta/_2$

$\boxed{PQ = 5.27 \text{ mm}}$

→ $x = CN - CM = 64 - 64 \cos\theta_2$

$= 0.077$

$b = \text{addendum} + x$

$= 1m + x = 4 + 0.077$

$\boxed{b = 4.1 \text{ mm}}$

$Q^n - 2$ teeth on pinion $t = 16$

$m = 5$ mm

deddendum $= 1\,m$

what should be pressure so that dedendum remain of purely an involute profile ⑤

→ for dedendum consist purely involute profile dedendum overlap the base circle

Radius of deddendum circle = Radius of base circle

$R - \text{deddendum} = R\cos\phi$

$40 - 5 = 40\cos\phi$

$\boxed{\phi = 29°}$

$m = \frac{D}{t} = \frac{2R}{t} = 5$

$\boxed{R = 40}$

Ex two meshing spur gears with 20° pressure angle have module 4 mm, centre distance 220 mm. No. of teeth on pinion is 40. to what value should the centre dist^n be increased so that pressure angle becomes 22° ⑥.

→ centre dist^n $r_1 + R_2 = 220$

$\phi = 20°$
$m = 4$
$c.d = 220$
$t = 40$
$\phi' = 22°$

$m = \frac{d}{t} \Rightarrow 4 = \frac{d}{40}$

$d = 160$

$\boxed{r_2 = 80}$

$\boxed{R = 140}$

→ $r_b = r\cos\phi = (80)\cos 20$

$\boxed{r_b = 75.175 \text{ mm}}$

→ $R_b = R\cos\phi = (140)\cos 20$

$\boxed{R_b = 131.55 \text{ mm}}$

is change

so, $r_b = r' \cos\phi' \Rightarrow 75.175 = r' \cos 22°$

$\Rightarrow \boxed{r' = 81.078 \text{ mm}}$

$R_b = R' \cos\phi' \Rightarrow 131.556 = R' \cos 22°$

$\Rightarrow \boxed{R' = 141.86 \text{ mm}}$

→ New center dist^n $= r' + R'$

$= \underline{222.93} \text{ mm}$

$(VR)_i = \frac{R}{r} = \underline{1.75}$

$(VR)_f = \frac{R'}{r'} = \underline{1.75}$

Note → In involute by changing the center distance velocity ratio does not change in involutes;

Ex Two gears are in mesh have module of 10 mm and pressure angle of 25°, pinion has 20 teeth and gear has 52 teeth. addendum of both gear is equal to 1 m determine

(1) No. of pairs of teeth in contact

(2) angle of action of the pinion and gear

(3) Ratio of sliding velocity to rolling velocity at beginning of engagement at the pitch point and at the end of engagement

→ $m = 10$ mm

$\phi = 25°$

$T = 52$

$t = 20$

$a = 1m = 10$ mm

$m = \frac{d}{t} \Rightarrow d = 10 \times 20$

$= 200$ mm

$\boxed{r = 100 \text{ mm}}$

$m = \frac{D}{T} \Rightarrow D = 52 \times 10$

$\boxed{R = 260 \text{ mm}}$

$R_a = R + a = 260 + 10$

$\boxed{R_a = 270 \text{ mm}}$

$r_a = r + a = 100 + 10$

$\boxed{r_a = 110 \text{ mm}}$

→ Gear & module have same module

Path of approach $= \sqrt{R_a^2 - (R\cos\phi)^2}$

[illegible]

$= \sqrt{(110)^2 - (100\cos 25)^2} - 100\sin 25$

$= 20.07$ mm

→ path of recess $= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{270^2 - (260\cos 25)^2} - (260\sin 25)$

$= 21.925$ mm

① → path of contact = P.O.A + P.O.R.

$= 20.07 + 21.925$

$= 42$ mm

② → contact ratio = $\dfrac{\text{Arc of contact}}{\text{Circular pitch}}$

{ Arc of contact = $\dfrac{\text{path of contact}}{\cos\phi}$

$= \dfrac{42.004}{\cos 25\,(\pi \times 10)}$

$= \underline{1.475}$ (one pair of teeth in complete mesh at pitch pt & for 47.5% of total time contact with one pair in mesh)

③ → velocity @ beginning engagement = vel. of sliding

$= P.O.A\,(\omega_p + \omega_g)$

$\dfrac{V_{sliding}}{V_{rolling}}$ @ beginning engagement $= \dfrac{P.O.A\,(\omega_p + \omega_g)}{r\,\omega_p}$

$= \dfrac{P.O.A}{r}\left(1 + \dfrac{\omega_g}{\omega_p}\right)$ $\left\{ \dfrac{\omega_p}{\omega_g} = \dfrac{R}{r} \right.$

$= \dfrac{20.7}{100}\left(1 + \dfrac{100}{260}\right)$

$= 0.2866$

angle of action

④ → $\delta_{gear} = \dfrac{\text{Arc of contact}}{R} = \dfrac{46.347}{260}$

$= \underline{0.1782 \text{ rad}}$

$\delta_{pinion} = \dfrac{\text{arc of contact}}{r} = \dfrac{46.347}{100}$

$= \underline{0.4634 \text{ rad}}$

$$\frac{}{V_{rolling}}\Bigg|_{@\ \text{pitch point}} = 0$$

3) $$\frac{V_{sliding}}{V_{rolling}}\Bigg|_{@\ \text{end}} = \frac{\text{path of recess } (\omega_p + \omega_g)}{r\omega_p}$$

$$= \frac{41.9732}{100}\left[1 + \frac{\omega_g}{\omega_p}\right] = \frac{41.91}{100}\left[1 + \frac{r}{R}\right]$$

$$= \underline{0.603}$$

Q) Two 20° involute spur gears having module of 6 mm, the larger wheel has 36 teeth, the pinion has 16 teeth. If addendum have 1 m, will the interference occur? What will happen if the no of teeth reduced to 14 teeth

$\phi = 20°$

$m = 6$

$T = 36$

$t = 16$

$m = 1\,m$

$\rightarrow$ $m = \frac{D}{t} \Rightarrow 6 = \frac{d}{16}$

$\boxed{r = 48}$

$6 = \frac{D}{36} \rightarrow \boxed{R = 108}$

→ No. of min of teeth on pinion

$$\lambda = \frac{r}{R} = \frac{t}{T} = 0.44$$

$f = 1$ to avoid interf.

$$t_{min} = \frac{2f\lambda}{\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1} = \frac{2 \times 1 \times 0.44}{\sqrt{1 + 0.44(0.44+2)} - 1}$$

$\boxed{t_{min} = 15}$

⟶ min teeth is 15 and $t_{act} = 16$ Hence interference will not occur $\Rightarrow$ $\boxed{(t_{act} > t_{min}) \Rightarrow \text{to avoid interference}}$ If the act. no of teeth reduced to be 14 which is less than t_{min} $\Rightarrow$ interference will occur

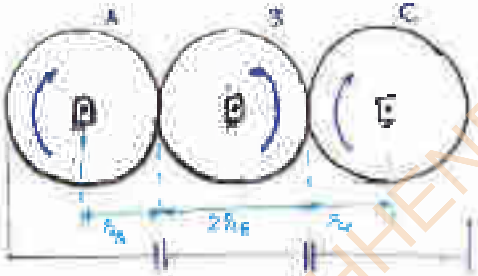
Gear trains

DOF = 1
- each gear on different shaft
  - ↳ simple gear train
- two gears are on same shaft
  - ↳ compound gear train
  - ↳ inverted gear train

DOF = 2
- sun + planet gear train (or) Epicyclic gear train

⇒ Simple Gear train

- In simple gear train all the gears are rotating along on the same plane they are used to connect the shaft which away from each other.



- If the center distance between the shaft is large belt + pulley arrangement is preferred over simple gear train, since the amount of gears is larger than belt + pulley, although there is clipping also

centre distance = $r_A + 2r_B + r_C$

- If even no of gear train are used then 1st & last are rotate in opposite direction
If odd no of gear are used then 1st & last rotate in same direction

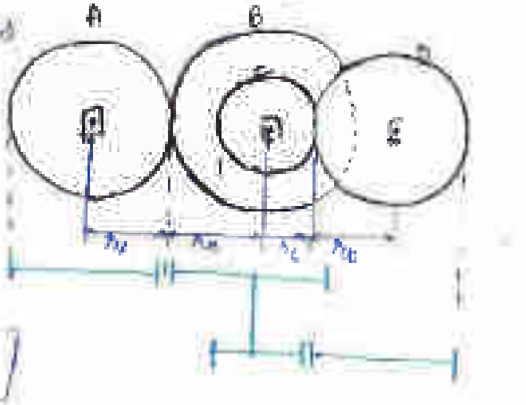
let A/B/C/D gear present

$$\frac{\omega_A}{\omega_D} = \frac{\omega_A}{\omega_B} \cdot \frac{\omega_B}{\omega_C} \cdot \frac{\omega_C}{\omega_D} = \frac{T_B}{T_A} \cdot \frac{T_C}{T_B} \cdot \frac{T_D}{T_C}$$

$$T.V = \boxed{\frac{\omega_A}{\omega_D} = \frac{T_D}{T_A}}$$

- The gears which does not decide the magnitude of train value are known as idlers gear. They are used only direction

- B & C are compound gears
- compound gear train is used to connect the shaft which are parallel & input & output gear will arrange exist on parallel planes



$$\boxed{\begin{aligned}&\text{Centre distance}\\ &= r_A + r_B + r_c + r_c\end{aligned}}$$

T.V :

$$T.V = \frac{\omega_A}{\omega_B} = \frac{\omega_A}{\omega_B}\ \frac{\omega_B}{\omega_C}\ \frac{\omega_C}{\omega_D}$$

{ B & C are same shaft

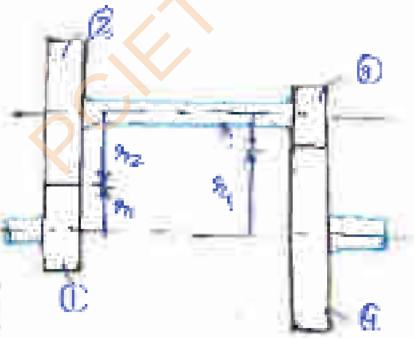
$$= \frac{\omega_A}{\omega_B} \cdot \frac{\omega_C}{\omega_D}$$

$$T.V = \boxed{\frac{\omega_A}{\omega_D} = \frac{T_B}{T_A} \cdot \frac{T_D}{T_C}}$$

$$\boxed{T.V = \frac{\text{product of no of teeth on driven gears}}{\text{product of no of teeth on driver gears}}}$$

→ **Inverted train**
(or)
Reverted train

- If the input & output shaft are co linear & they are connected by reverted gear train

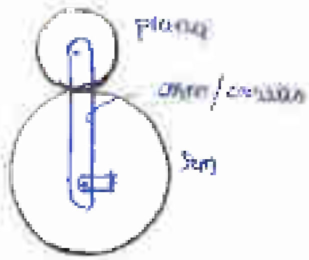


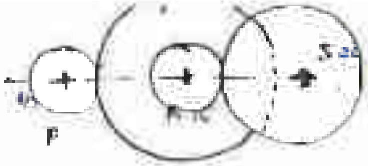
$$\boxed{r_1 + r_2 = r_3 + r_4}$$

$$\Rightarrow \boxed{T_1 + T_2 = T_3 + T_4}$$

→ **Epicyclic Gear Train**
(or) sun & planet gear train

$$\boxed{\text{dof} = 2}$$





$D_Q = 2D_R$

$m_R = 2$

$m_R = \frac{D_R}{T_R} \Rightarrow 2 = \frac{D_R}{15}$

$\boxed{D_R = 30}$

$m_S = \frac{D_S}{T_S} \Rightarrow \boxed{D_S = 40}$

$\rightarrow D_Q = 2(D_R) = 60 \Rightarrow \boxed{D_Q = 60}$

$m_P = m_Q \Rightarrow \frac{D_P}{T_P} = \frac{D_Q}{T_Q} \Rightarrow \frac{D_P}{20} = \frac{\overset{30}{\cancel{60}}}{\underset{2}{\cancel{40}}}$

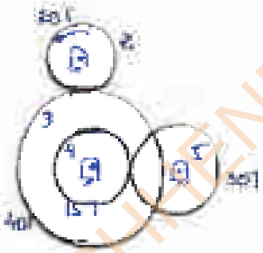
$\boxed{D_P = 30}$

$C.D = r_P + r_Q + r_R + r_S$

$= 15 + 30 + 15 + 20$

$= \underline{80}$ mm

17,



$\frac{N_2}{N_5} = \frac{N_2}{N_3} \times \frac{N_3}{N_4} \times \frac{N_4}{N_5}$ $\left\{ N_3 = N_4 \right.$ same shaft

$= \frac{N_2}{N_3} \times \frac{N_4}{N_5}$

$= \frac{T_3}{T_2} \times \frac{T_5}{T_4} = \frac{\overset{2}{\cancel{40}}}{\cancel{20}} \times \frac{\overset{2}{\cancel{30}}}{\cancel{15}}$

$\frac{1200}{N_5} = 4 \times \Rightarrow \boxed{N_5 = 300 \text{ rpm}}$ (a)

$\boxed{18}$ The T.V eqn which even teeth not come in eqn is idler.

$\frac{N_2}{N_6} = \frac{N_2}{N_3} \times \frac{\cancel{N_3}}{\cancel{N_4}} \times \frac{N_4}{N_5} \times \frac{N_5}{N_6}$

$= \frac{T_3}{T_2} \times \frac{\cancel{T_5}}{T_4} \times \frac{T_6}{\cancel{T_5}}$

$= \frac{T_3}{T_2} \times \frac{T_6}{T_4}$ $\left\{ T_5 \right.$ is not come in T.V eq idler

(13) * [C]

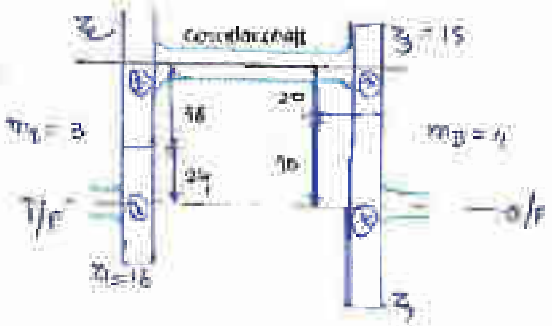
$G.R = \frac{\text{No of teeth gear}}{\text{No of pinion}}$

$i = \frac{Z_2}{Z_1}$

Given, $G.R = 4$

$m_I = 3$, $m_{II} = 4$

overall $G.R = 12$



→ $\frac{Z_1}{Z_2} = \frac{1}{4} \Rightarrow 16 \times 4 = Z_2 \Rightarrow \boxed{Z_2 = 64}$

→ $m_I = \frac{D_1}{Z_1} \Rightarrow D_1 = 3 \times 16 = 48 \Rightarrow \boxed{R_1 = 24}$

→ $m_I = \frac{D_2}{Z_2} \Rightarrow D_2 = \cancel{48}\ 3 \times 64 = 192 \Rightarrow \boxed{R_2 = 96}$

→ $m_{II} = \frac{D_3}{Z_3} \Rightarrow D_3 = 4 \times 15 \Rightarrow \boxed{R_3 = 30}$

$C.C = R_1 + R_2 = 24 + 96 = 120 \Rightarrow \boxed{C.C = 120}$

Now from $G.R = 12$ → total

$\frac{\omega\, i/p}{\omega\, o/p} = 12 \Rightarrow \frac{\omega_1}{\omega_4} = 12$

$\frac{\omega_1}{\omega_2} \times \cancel{\frac{\omega_2}{\omega_3}} \times \frac{\omega_3}{\omega_4} = 12$

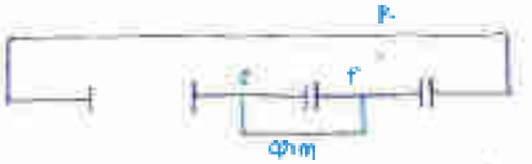
$\frac{Z_2}{Z_1} \times \frac{Z_4}{Z_3} = 12 \Rightarrow 4 \times \frac{Z_4}{15} = 12$

$\boxed{Z_4 = 45}$

14

Sun - planet gear

14



$\left(\frac{\omega_P}{\omega_S} = \frac{T_S}{T_P} \right.$

Condition	Arm	20 Gear S	30 Gear P	80 Gear R
Arm is fixed Gear S rotating with (+x) rpm (CW)	0	+x	$-x\cdot\left(\frac{20}{30}\right)$ $= -\frac{2x}{3}$	$-\frac{2x}{3}\cdot\left(\frac{30}{80}\right)$ $= -\frac{x}{4}$
Arm rotating with +y rpm	y	y + x	$y - \frac{2}{3}x$	$y - \frac{x}{4}$

→ Ring gear fixed

$y - x/4 = 0 \Rightarrow \boxed{x = 4y}$

→ Arm Sun speed $N_{sun} = 100$

$y + x = 100 \Rightarrow 5y = 100$ $\boxed{y = 20}$ (CW)

Shortcut

$\frac{\omega_S}{\omega_R} = -\frac{\omega_S}{\omega_P}\cdot\frac{\omega_P}{\omega_R}$

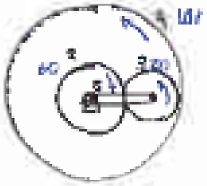
$\frac{\omega_S - \omega_{arm}}{\omega_R - \omega_{arm}} = -\frac{T_P}{T_S}\cdot\frac{T_R}{T_P} = -4$

$\frac{100 - \omega_{arm}}{0 - \omega_{arm}} = -4$

$\boxed{\omega_{arm} = 20}$

(if +ve rotating opposite dirⁿ take (-ve))

22



Shortcut

$\frac{\omega_2}{\omega_4} = -\frac{\omega_2}{\omega_3}\times\frac{\omega_3}{\omega_4}$

$\frac{\omega_2 - \omega_{arm}}{\omega_4 - \omega_{arm}} = -\frac{T_3}{T_2}\times\frac{T_4}{T_3} = -\frac{30}{60}\times\frac{100}{30}$

$\frac{0 - \omega_{arm}}{100 - \omega_{arm}} = -\frac{10}{6}$

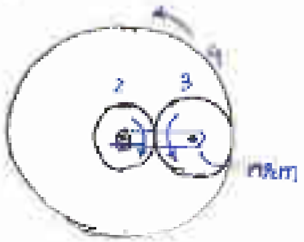
Condition	Arm
~~Arm is on 2nd Gear~~	

$6\omega_{arm} = -1000 - 10\omega_{arm}$

$16\omega_{arm} = 1000$

$\boxed{\omega_{arm} = 62.5 \text{ rpm}}$

(CCW)



Cond^n	Arm 5	Gear 2 60	Gear 3 20	Gear 4 100
Arm is fixed, Gear 2 rotating with (+x) rpm	0	+x	$-x\left(\frac{60}{20}\right)$ $= -3x$	$-3x\left(\frac{20}{100}\right)$ $= -\frac{3x}{5}$
Arm is rotating with +y rpm	y	+x+y	y-3x	$y-\frac{3x}{5}$

Gear 2 fixed

$N_2 = 0 \Rightarrow y + x = 0 \Rightarrow x = -y$

$N_4 = -100$ rpm (CCW)

$\Rightarrow y - \frac{3}{5}x = -100 \Rightarrow y + \frac{3}{5}y = -100$

$\boxed{y = -62.5 \text{ rpm}}$

Another: $\frac{\omega_2}{\omega_4} = -\frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_4}$

$\frac{\omega_2 - \omega_{arm}}{\omega_4 - \omega_{arm}} = -\frac{T_3}{T_2} \cdot \frac{T_4}{T_3}$

CCW

$\frac{0 - \omega_{arm}}{-100 - \omega_{arm}} = -\frac{100}{60}$

$\frac{\omega_{arm}}{100 + \omega_{arm}} = -\frac{10}{6}$

$\omega_a =$ [illegible]

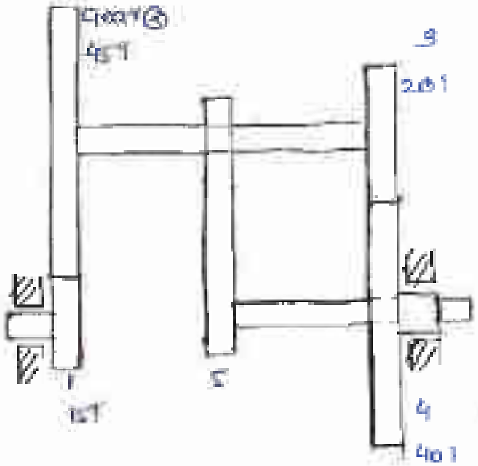
(140-16) #25

$$\frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2}\cdot\frac{\cancel{\omega_2}}{\omega_3}\cdot\frac{\omega_3}{\omega_4}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{T_2}{T_1}\times\frac{T_4}{T_3}$$

$$= \frac{45}{15}\times\frac{40}{20}$$

$$\boxed{\frac{\omega_1-\omega_5}{\omega_4-\omega_5} = 6}$$



b) $$\frac{60-\omega_5}{-120-\omega_5} = 6$$

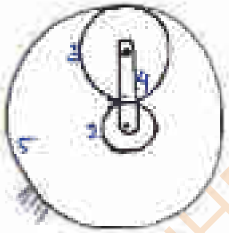
$$\boxed{\omega_5 = -156 \ RPM}$$ (ccw)

[17] $$\frac{\omega_2}{\omega_5} = \frac{-\omega_2}{\omega_3}\cdot\frac{\omega_3}{\omega_5}$$

~~Also~~

$$\frac{\omega_2-\omega_4}{\omega_5-\omega_4} = -\frac{T_3}{T_2}\cdot\frac{T_5}{T_3}$$

$$\frac{\omega_2-\omega_4}{0-\omega_4} = -\frac{T_5}{T_2} = \frac{100}{20}$$



$-60 \to \boxed{\omega_2} - \omega_4 = 5\omega_4$

$6\omega_4 = -60 \Rightarrow \boxed{\omega_4 = -12}$ (ccw)

[18]

condn	arm	20 Gear 2	24 Gear 3, 32 Gear 4	80 Gear 5
arm is fixed, gear 2 rotates with +x radis.	0	$+x$	$-x\left(\frac{20}{24}\right)$ $= -\frac{5x}{6}$	$-\frac{5x}{4}\left(\frac{32}{80}\right)$ $= -\frac{x}{3}$
arm is rotating with +y radis	+y	$y+x$	$y-\frac{5x}{6}$	$y-\frac{x}{3}$

Given $\omega_{shaft} = 50$ rad/s (CCW)

$\omega_r = (?)$

$y + x = 100$ ---(1)

$y = -50$ (CCW)

$x = 150$

→ $\omega_r = y - x/_3 = -50 - 150/_3$

$\boxed{\omega_r = -100}$ (CCW)

Ex 19 → If engine shaft is not transferring any power at that time both the wheel should rotates in opposite direction. If engine shaft is supplying power they will rotates in same direction due to differential gear box

→ Differential gear box is used to provide different velocities to the inner & outer wheel while taking a turn because both the wheel has to travel different distance

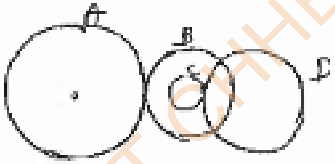
Ex. 20

$T_A = 50$

$T_B = 25$

$\omega_B = 100$ rpm

$\omega_A = (?)$, $T_D = (?)$

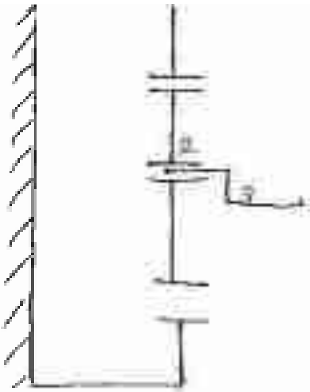


→ $\frac{\omega_A}{\omega_D} = \frac{\omega_A}{\omega_B} \cdot \frac{\cancel{\omega_B}}{\cancel{\omega_C}} \cdot \frac{\omega_C}{\omega_D}$

$\frac{\omega_A}{100} = \frac{T_B}{T_A} \times \frac{T_D}{T_C} = \frac{25}{50} \times \frac{T_D}{T_C} = \frac{25}{50} \times \frac{T_D}{50}$

$\frac{N_A}{100_2} = \frac{25}{50} \times \frac{T_D}{50_2}$ ---(1)

→ $\boxed{N_A = T_D}$ ⇐ 30 rpm & 30 teeth



Cond^n	carrier 3	gear 2 96	gear 104
carrier is fixed gear 2 is rotating (+x) rpm	0	+x	$-x \frac{96}{104}$
carrier is rotating (+y) rpm	+y	find $\boxed{x+y}$	$y + x \frac{96}{104}$

$y + x \frac{95}{104} = 0$

$x + y = 60$

given $\boxed{y = 60 \text{ rpm}}$ (ccw)

$y + x \left(\frac{96}{104}\right) = 0$

$(104)(60) = x(96) = 0$

$\boxed{x = -65 \text{ rpm}}$

Gear 2 = y + x = $\underline{-5 \text{ rpm}}$ (cw)

$\frac{95}{104} x + \cancel{y} = 0$

$x + y = 60$

$\left(\frac{96}{104} - 1\right) x = -60$

$\frac{-8}{104} x = -60$

$\boxed{x = 104}$

[illegible]

The train is in equilibrium;

$\Sigma T_{net} = 0$

$$\boxed{T_c + T_p + T_{arm} + T_A = C}$$

- Hence we can neglect the torque associated with planet

$$\boxed{T_s + T_{arm} + T_A = 0}$$

- Since, planet gear is neither connected to i/p nor to the output, hence we can neglect the torque associated with planet
- There is no power in the system

power @ o/p = power @ i/p

$$\boxed{T_s \omega_s + T_{arm}\omega_{arm} = T_A \omega_A}$$

- The torque applied when a gear is fixed is known as fixing torque

Ex. In a gear train, gear DE and FG are compound gear. If the wheel A is fixed and the arm makes 80 revolution clockwise, find the revolution of B & C. If the arm is applied a turning moment of 1 kN m determine the turning moment on the shaft supporting the wheel C

→ $T_A = 60$

$T_B = 120$

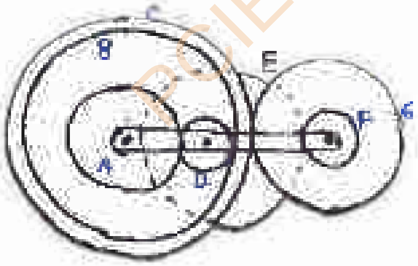
$T_C = 135$

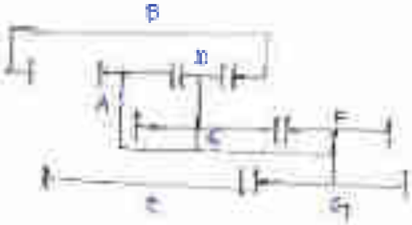
$T_D = 30$

$T_E = 75$

$T_F = 30$

$T_G = 60$





Condition	Arm	60	20 / Gear C 75	120	30 / 60	135
Arm is fixed & A rotates with (x) rev	0	$+x$	$-x\left(\frac{60}{30}\right)$ $= -2x$	$-2x\left(\frac{30}{120}\right)$ $= -\frac{x}{2}$	$2x\left(\frac{75}{30}\right)$ $= +5x$	$-5x\left(\frac{60}{135}\right)$ $= -\frac{20x}{9}$
Arm is rotating y rev	y	$+x+y$	$-2x+y$	$-\frac{x}{2}+y$	$+5x+y$	$-\frac{20x}{9}+y$

A wheel fixed

$N_A = 0 \Rightarrow x + y = 0$

given $y = 20$ (given) $\boxed{x = -20}$

$N_c = -\frac{20}{9}(-20) + 20 \Rightarrow \boxed{N_c = 64.44 \text{ rev}}$

⇒ equilibrium eq^n $\Sigma T_{net} = 0$

$T_{arm} + T_A + T_B = 0$

For net power loss = 0

power @ i/p = power @ o/p

$T_{arm}\omega_{arm} + T_A\omega_A = T_C\omega_C$

since gear A is fixed $\boxed{\omega_A = 0}$

$T_{arm}\omega_{arm} = T_c\omega_c$

$1 \times 20 = T_c \times 64.44$

$\boxed{T_c = 0.310 \text{ kN m}}$

Braking torque

$1 - T_A + 0.310 = 0 \Rightarrow \boxed{T_A = -1.310 \text{ kN m}}$

speed of gear C, if the wheel D is fixed & arm rotates at 400 revolution clockwise.

$T_A = 52$, $T_C, T_D = ?$

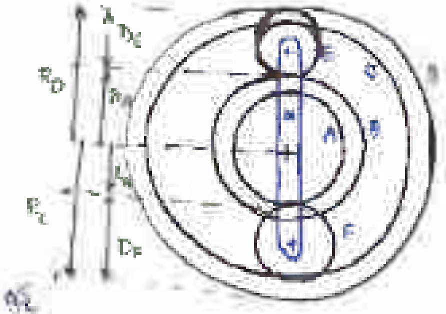
$T_B = 56$,

$T_E = T_F = 36$

wheel D fixed

$N_{arm} = 200$ Rev

$N_D = (?)$



~~$\frac{\omega_A}{\omega_F} = \frac{\omega_A}{\omega_B} \times \frac{\omega_B}{\omega_E} \times \frac{\omega_E}{\omega_C} \times \frac{\omega_C}{\omega_D} \times \frac{\omega_D}{\omega_E}$~~

~~$\frac{N_E}{1} \quad \frac{400}{N_E} = \frac{T_E}{T_B} = \frac{36}{56}$~~

~~$N_E = 311.11$ rev~~

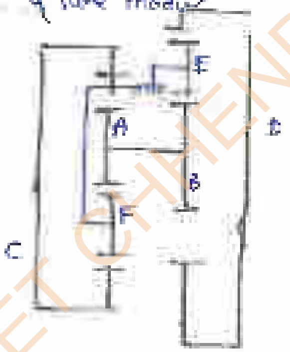
$\rightarrow \quad R_D = D_E + R_A \Rightarrow \quad D_D = 2D_E + D_A$

$\frac{D_D}{m} = \frac{2D_E}{m} + \frac{D_A}{m}$ {same module

$T_D = 2T_E + T_A$

$T_D = 2(36) + 52$

$\boxed{T_D = 124}$



$\rightarrow \quad R_A + D_F = R_C \quad \Rightarrow \quad \frac{D_A}{m} + \frac{2D_F}{m} = \frac{D_C}{m}$

$T_A + 2T_F = T_C$

$52 + 2(36) = T_C$

$\boxed{T_C = 124}$

cond^n	Arm	Gear A/Gear B 52 56	Gear F 36	Gear C 124	Gear E 36	Gear D 124
	0	$+x$	$-x\left(\frac{52}{36}\right)$	$-\frac{52}{36}x \times \left(\frac{36}{124}\right)$ $= -\frac{52x}{124}$	$-x\left(\frac{56}{36}\right)$	$-x\left(\frac{56}{36}\right)\left(\frac{36}{124}\right)$ $= -x\frac{56}{124}$
	1	$x + y$	$y - x\left(\frac{52}{36}\right)$	$y - \frac{52}{124}x$	$y - \frac{56}{36}x$	$y - x\left(\frac{56}{124}\right)$

$y = 7100$ [illegible]

$x = 457.16$ [illegible]

$N_c = y - \frac{x}{126} \times 58$

$\boxed{N_r = 8.27 \text{ [illegible]}}$

B. Fig (a) OA = 640 mm
Fig (b) EA = 450 mm, FE = 160 mm, angle θ = 30° were co
Show each speed of rotation is same in both cases,
calculate % change in speed ~~for~~ 50 mm rise in the
level of governor balls